

**Ventilation for Energy Efficiency and Optimum
Indoor Air Quality
13th AIVC Conference, Nice, France
15-18 September 1992**

Poster 25

**An Analytical Study of Branched Connections: Its
Implications to Multizone Air Flow Measurements.**

L. Shao, S. Sharples, I. C. Ward, H-G. R. Kula

**Building Science Unit, School of Architectural
Studies, University of Sheffield, P O Box 595, The
Arts Tower, Western Bank, Sheffield, S10 2UJ,
United Kingdom**

SYNOPSIS A branched connection is a single air flow passage connecting more than two zones. Its existence in a building has not been a critical issue for the measurement of air flows of single zones, as far as the validity or accuracy of the measurement techniques is concerned. However, with the ever increasing sophistication of building air flow measurement techniques --- which include tracer gas and multifan pressurisation techniques --- and the ever increasing use of them in multizones, it becomes increasingly desirable to examine the effect of branched connections. This paper presents an analytical study of the validity of the multizone air flow measurement techniques, as they are applied to buildings containing branched connections. It is found that the multifan pressurisation techniques have embedded inadequacies, which could lead to large flow rate measurement errors, if the techniques are applied to buildings containing branched connections. It is also found that all tracer gas techniques are valid regardless of the types of connections present. However, the interpretation of their results is much more restricted than in the case where only direct connections exist in the tested building.

List of Symbols

C	tracer concentration	a,b	tracer species
P	pressure	C	point C in Fig. 1
ΔP	pressure difference	D1	pressurisation fan for zone r
δP	ΔP across air flow passage section	e,0	outside
Q	flow rate	m	pressure ring defined by [1]
Q _i	flow rate; i=2,3,4,e; (Fig. 2)	meas	measured flow rate
Q _{ij}	flow rate from zone i to zone j	o	beginning of test
q	flow rate through a passage section	true	actual flow rate
R	flow resistance of a passage section	superscript	
subscripts		'	a condition at which $\Delta P = \Delta P'$
1,2,3,4,r	zone number as seen in Fig. 1 & 2		

1. Introduction

A connection is here defined as an air flow passage linking otherwise air tight zones --- rooms, corridors and staircases. Connection can be well defined as for an open door or window or they can be poorly defined as for background leakage cracks. The connection can also be classified into the direct connection if it connects only two zones or the branched connection if it connects three or more zones. Branched connections are common in buildings. Under floor or behind-wall wiring, gas supply tubing, central heating tubing and general plumbing all create branched connections between zones. Branched connections may also be found at prefabricated panel joints and room partitioning board joints. Cavity walls plus cracks between building bricks, too, helps creating branched connections.

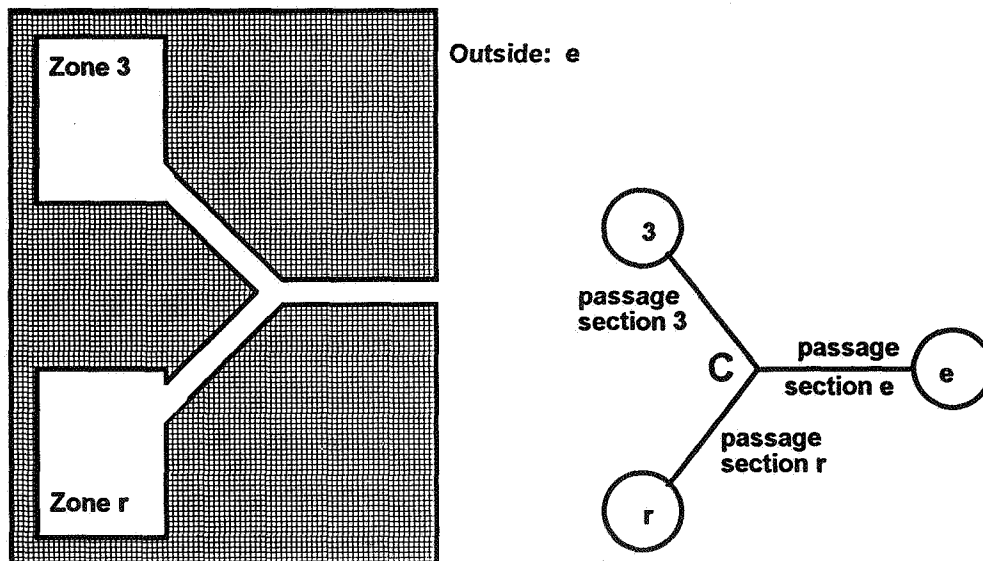


Fig. 1. Schematic of a branched connection linking two zones and the outside.

An example of the simplest branched connections is shown in Fig. 1. The branched connection with three arms links zone *r* with zone 3 and the outside which is denoted as "e". This arrangement was also depicted using a "circle-bar" diagram, also in Fig. 1, to simplify its graphical presentation and to facilitate the concentration of attention on the essential features of the connection. In this diagram a zone or the outside is represented by a circle and an above mentioned "arm" by a bar. The latter is referred to as a passage section defined as a section of a connection in which there is no branches.

Building air flows has been measured using, predominantly, the pressurisation and the tracer gas techniques. Both methods were first developed for air flow measurements in single zones. They were applied either to single zones or multizones transformed into single zones by, for example, opening the doors of the zones. For these types of applications the validity or accuracy of the two techniques was not in any way related to the presence of branched connections.

In the past few years both the above techniques have become increasingly sophisticated. Multi-tracer gas and multi-fan pressurisation techniques have appeared and have been applied increasingly widely to multizone air flow measurements. However, there has been little research into whether these techniques are valid or accurate when applied to multizone buildings containing branched connections, the answer to which is not as obvious as in the case of single zone air flow measurements.

In the following an analytical work is presented, which was carried out as a step towards answering the above question. One version of the multifan pressurisation method, the deduction technique, and one version of the multi-tracer method, the tracer concentration decay method, is to be examined in the following in terms of their accuracy when the branched connections are present. The other versions of both methods were also examined, details of which will not be discussed in this paper since both the examination method and the conclusions are the same as those presented below. However, more information can be found in Ref. [2].

2. The examination of the multifan pressurisation method

There are two versions of this method, i.e., the deduction technique and the guarding zone technique. As explained in the introduction, the following discussion will be confined to the deduction technique. In addition, the nomenclature used in the original paper[1] on this technique has been adopted for clarity and consistency.

2.1 Brief description of the technique

The multifan pressurisation method was devised for and has been applied to measurements of leakage distributions in multizone buildings. Referring to Fig. 2,

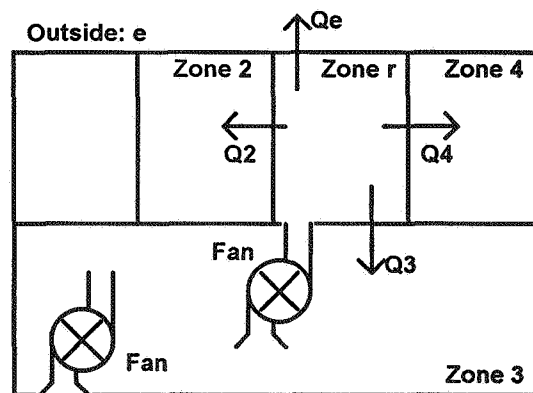


Fig. 2. Schematic of a multi-fan test arrangement.

to measure the leakage distribution for zone *r* is to obtain the $Q - \Delta P$ curve or the function $Q(\Delta P)$ for all the flows Q_e , Q_2 , Q_3 and Q_4 over a ΔP range of 0 to a practically likely maximum, typically 50Pa. Also, the zone flow to which is to be measured is referred to as the flow recipient zone. E.g., if $Q_2(\Delta P)$ is to be measured then zone 2 is called the flow recipient zone for that measurement.

In applying the deduction technique, the pressure in zone r (P_r) is kept constant while the pressure in the air flow recipient zone, which is also the pressure ring in this technique, P_m , is varied in descending steps from P_r to 0. The flow rates Q_e , Q_2 , Q_3 and Q_4 are measure at each of the steps of pressure differences ΔP , ($\Delta P = P_r - P_m$), thereby obtaining the $Q(\Delta P)$ functions. Some of the above flow rates can be obtained directly, while others are found by subtracting one flow rate from another. e.g. to obtain $Q_3(\Delta P)$ zone r is pressurised to P_r , the pressure in zone 3 (the now pressure ring) reduced in steps and the pressure in all other zones kept at 0 by opening windows or doors. The zone r pressurisation fan flow rate Q_{D1} is measured at each step. One then has $Q_3(\Delta P) = Q_{D1}(P_r - P_m) - Q_{D1}(P_r - P_{m0})$. Note $P_{m0} = P_r$, so

$$Q_3(\Delta P) = Q_{D1}(\Delta P) - Q_{D1}(0). \quad (1)$$

The other $Q(\Delta P)$ functions are obtained in a similar manner. Details can be found in Ref. [1].

2.2 The examination

Consider the simplest multizone configuration, a building of two zones, linked to each other and the outside via a branched connection as shown in Fig. 1. It is assumed that there are no other connections between zone 1 and 2, for the sake of analytical simplicity and clarity, although there must be other, direct connections between each of the two zones and the outside, because otherwise there can not be flows to or from the zones, on a sustained basis. The three passage sections of the connection are assumed in this section 2.2 to be identical and each being a long narrow type crack. These passage sections are assumed to have such large length to height ratios that the entrance effect becomes negligible and the relationship between the flow rate through such a passage section (q) and the pressure difference across it ΔP is linear:

$$\Delta P/q = R = \text{constant}$$

where the ratio R is referred to as the resistance of the passage section. The above assumption was made primarily to give the following analysis a greater degree of clarity and simplicity. As shown later, the conclusions thereby obtained by no means only apply to connections consisting of the above type of passage sections. In fact, it holds true for all practical existing types.

The resistances of the three passages in Fig. 1, leading to zones r, zone 3 and the outside are assumed to be, respectively, $R_r = R_3 = R_e = R$, and the pressure in zone r assumed to be P . The place where the passage sections meet is denoted as node C.

Apply the deduction technique to the building and measure $Q_3(\Delta P)$. At the point of $\Delta P = 0.5P$ (i.e. $P_r = P$; $P_3 = P_m = 0.5P$; $P_e = 0$), according to Eq. 1

$$Q_3(0.5P) = Q_{D1}(0.5P) - Q_{D1}(0) \quad (2)$$

First, consider $Q_{D1}(0)$. It is known at that moment, $P_r = P$; $P_3 = P$ and $P_e = 0$ and according to mass conservation,

$$\begin{aligned} Q_{D1} &= q_r \\ q_r + q_3 &= q_e \end{aligned} \quad (3)$$

where q_r , q_3 and q_e are the rates of flow through the passage sections against the corresponding resistance R_r , R_3 and R_e , respectively. Eq. 3 can be transformed into

$$\frac{P_r - P_c}{R_r} + \frac{P_3 - P_c}{R_3} = \frac{P_c - P_e}{R_e} \quad (4)$$

substituting the resistances and the pressures in Eq. 4 with their values, one has $P_c = 2P/3$ and thus

$$Q_{D1}(0) = q_r(0) = \frac{P_r - P_c}{R_r} = \frac{1}{3} \frac{P}{R}$$

$Q_{D1}(0.5P)$ can be obtained in a similar manner:

$$Q_{D1}(0.5P) = \frac{1}{2} \frac{P}{R}$$

and thus from Eq. 2 one obtains

$$Q_3(0.5P) = \frac{1}{2} \frac{P}{R} - \frac{1}{3} \frac{P}{R} = \frac{1}{6} \frac{P}{R}$$

In other words, if the meter readings for Q_{D1} are absolutely accurate, the measured flow rate for Q_3 is $P/6R$. The true value for Q_3 can be calculated using again Eq. 4. Remembering $P_r = P$; $P_3 = 0.5P$; $P_e = 0$; one obtains $q_r = P/2R$; $q_e = P/2R$; as well as

$$q_3 = 0 \quad (5)$$

Eq. 5 shows that there is no air flow going from zone r to zone 3. The true or practical value of Q_3 is zero. At this particular pressure difference, the relative error of measurement is infinite.

Apply the above analysis method to every point in the ΔP range to obtain the $Q_3(\Delta P)$ function, it is found that the $Q_3(\Delta P)$ measured using the deduction technique, assuming all readings are absolutely correct, is

$$Q_3(\Delta P)_{meas} = \frac{\Delta P}{3R} \quad (6)$$

while the true or practical function is

$$Q_3(\Delta P)_{true} = \begin{cases} \frac{2\Delta P - P}{3R}; & \Delta P > 0.5P \\ 0; & \Delta P \leq 0.5P \end{cases} \quad (7)$$

therefore, the relative error caused by the inadequacy of the deduction technique for this particular case is

$$error = \frac{Q_3(\Delta P)_{meas} - Q_3(\Delta P)_{true}}{Q_3(\Delta P)_{true}} = \begin{cases} \frac{P - \Delta P}{2\Delta P - P}; & \Delta P > 0.5P \\ \infty; & \Delta P \leq 0.5P \end{cases} \quad (8)$$

As seen above, the relative error increases with decreasing ΔP and approaches infinity as ΔP approaches $0.5P$. The large errors are solely due to the inadequacy of the deduction technique in dealing with branched connections, since instrument and operator errors were excluded from the above analysis. Therefore, the deduction technique cannot be relied upon in testing multizone buildings, if they consist of branched connections or the type of connection in them is not known.

2.3 Discussion

The type of sufficiently "long and narrow" passage sections were used in the above examination. This is purely for clarity and simplicity purposes, since their $Q(\Delta P)$ functions are in the simple linear form. However, the use of such a type of passages is not a necessary condition for the above analysis. The conclusions from the above analysis holds true when the common types of passage sections, whose $Q(\Delta P)$ functions are in the power law or quadratic forms[3], i.e., $Q = K \Delta P^n$ or $\Delta P = K_1 Q^2 + K_2 Q$, are used to replace the linear type passage sections. However, there are still some types of passage sections, which are not even described by the above two equations [3]. Their $Q(\Delta P)$ functions can be represented, in most cases, by the following type of equation:

$$q = f(\delta P) \quad (12)$$

where f is a monotonously increasing function. In other words, the flow rate through the passage section in question, q , increases with the pressure difference across it (δP). The conclusion from the above analysis holds true also for this quite general type of passage. This point can be illustrated by the example of further examining the application of the deduction technique to the two zone building in Fig. 1. The only difference between the building used here and that in the above analysis is that the identical linear passage sections are replaced by three identical passages of the general type described by Eq. 12. Again, we focus on a particular point ($\Delta P'$) in the ΔP range, at which $P_3 = P_c$ and therefore $q_3 = 0$ and $Q_3 = 0$. (The existence of this point is obvious and easily proven.) According to the deduction technique (Eq. 1), there is

$$Q_3(\Delta P') = Q_{D1}(\Delta P') - Q_{D1}(0) \quad (13)$$

Since $Q_3(\Delta P') = 0$, if the deduction technique is correct or valid, then there is $Q_{D1}(\Delta P') = Q_{D1}(0)$ or, noting $Q_{D1} = q_r$

$$q_r(\Delta P') = q_r(0). \quad (14)$$

Denote the condition at which $\Delta P = \Delta P'$ by superscript ' and $\Delta P = 0$ by subscript "o". Eq. 14 is then written as

$$q'_r = q_{ro} \quad (15)$$

From Eq. 15 and Eq. 12, it can be obtained that $\delta P'_r = \delta P_{ro}$. In addition, P_r is kept constant and $P'_r = \delta P'_r + \delta P_e'$; $P_{ro} = \delta P_{ro} + \delta P_{eo}$. So, one has

$$\delta P_e' = \delta P_{eo} \quad (16)$$

From Eq 16 and 12

$$q'_e = q_{eo} \quad (17)$$

Because $q_3 = 0$, then from the law of mass conservation, it can be obtained

$$q'_r = q'_e \quad (18)$$

Combining Eq. 15, 17 and 18, one has

$$q_{eo} = q_{ro} \quad (19)$$

However, in reality, the relation between the two flow rates is

$$q_{eo} = 2q_{ro} \quad (20)$$

because $q_{3o} = q_{ro}$ due to passages being identical and, $q_{3o} + q_{ro} = q_{eo}$.

The contradiction between Eq. 19 and 20 is due to the assumption that the deduction technique represented by Eq. 13 is valid or correct. Therefore it is demonstrated again, in more general terms, that the above named technique is not reliable for testing buildings containing branched connections.

The same has been found true for the guarding zone method. The analysis method was only slightly different to that above and the details can be found in Ref. [2].

The multifan pressurisation technique in general can only be applied to directly linked multizone buildings. It breaks down when air passages to three or more zones cross each other. This is not too surprising since the technique was devised assuming, implicitly, that there are only direct connections. This assumption must be upheld, if they are to be successfully applied to buildings with branched connections. This practically means that each place at which the air passages cross each other should be treated as a zone, included or excluded from the controlled pressure ring or guarding zone, just like the well defined zones like rooms and corridors. This is not the case in the current multifan pressurisation techniques, and consequently they are not valid. Treating a passage junction as a zone and controlling its pressure is not easy in practical terms, unless it is, e.g. a wall cavity with a fairly large internal space. There is also the practical difficulty of identifying branched connections and locating the junctions, which will be discussed later. The tremendous difficulty in improving the multifan pressurisation techniques to cope with branched connections is obvious.

3. The examination of the multi-tracer gas method

The basic principle for the tracer gas method is that the rate of tracer consumption, either in the form of concentration decay or injection rate, in a zone is directly linked to the airflow rate there. For multizone air flow measurements, more than one tracer has to be used, hence the multi-tracer method. There are several versions of the method including the decay technique, the constant concentration technique and the constant injection technique. The first technique was used more often because less monitoring/controlling equipment are required. As explained in the introduction, the following discussion will be confined to the decay technique.

3.1 Brief description of the technique

The description will be given in the context of a two zone, zone 1 and zone 2, building. That for a N zone building can be found in Ref. [2]. Normally the procedure begins with injecting two different species of tracer gases, species "a" and "b", into zone 1 and 2, respectively. The tracer gases are then uniformly mixed with the air in their corresponding zones. Subsequently the tracer concentration decays due to the dilution effect of the interzonal air flows and those between the zones and the outside are measured. The rate of these air flows can then be calculated based on the following equations.

$$V_1 \frac{dC_{a1}}{dt} = -C_{a1}Q_{10} - C_{a1}Q_{12} + C_{a2}Q_{21} \quad (26)$$

$$V_1 \frac{dC_{b1}}{dt} = -C_{b1}Q_{10} - C_{b1}Q_{12} + C_{b2}Q_{21} \quad (27)$$

$$V_2 \frac{dC_{a2}}{dt} = C_{a1}Q_{12} - C_{a2}Q_{21} - C_{a2}Q_{20} \quad (28)$$

$$V_2 \frac{dC_{b2}}{dt} = C_{b1}Q_{12} - C_{b2}Q_{21} - C_{b2}Q_{20} \quad (29)$$

$$Q_{01} - Q_{10} + Q_{21} - Q_{12} = 0 \quad (30)$$

$$Q_{02} - Q_{20} + Q_{12} - Q_{21} = 0 \quad (31)$$

where for $m \in (0, 1, 2)$, $n \in (0, 1, 2)$, V_m is the internal volume of zone m ; C_{am} or C_{bm} are the tracer concentrations in zone m for species a and b respectively; Q_{mn} is the air flow rate from zone m to zone n . Note the outside is here conveniently referred to as zone 0.

3.2 The examination

Consider again the building used in section 2.3 and illustrated in Fig. 1. The three passage section comprising the connection are assumed identical and of the general type described by Eq. 12. In order to conform to the normal nomenclature of tracer techniques, the subscripts 0, 1 and 2 will be used for denoting the outside and the zones, replacing subscripts e, 3 and r respectively. Zone 1 and 2 are both assumed to have an internal spatial volume of V .

Suppose that air is blown, e.g. by using a fan, into zone 2 at a flow rate of "q". Consequently, the flow rates through the passage sections are $q_0 = q/2$, $q_1 = q/2$ and $q_2 = q$. So the true rates of the flow between the zones and between the zones and the outside are $Q_{01} = 0$, $Q_{10} = q/2$, $Q_{02} = q$, $Q_{20} = q/2$, $Q_{12} = 0$ and $Q_{21} = q/2$.

The above six flow rates can also be obtained by applying the tracer decay technique, which in this case utilises two tracers (a and b). At the beginning of the test, tracer a is only present in zone 1 and tracer b in zone 2 and their initial concentrations are C_{a10} and C_{b20} . The validity of the tracer decay technique is then assessed by comparing the measured data with the true data.

As described in the last section in the decay technique, the following are to be measured: V_1 , V_2 , C_{a1} , C_{a2} , C_{b1} and C_{b2} . If the measurement instrument readings are absolutely accurate, then

$$V_1 = V_2 = V \quad (32)$$

$$C_{a2} = 0 \quad (33)$$

The absolute accurate measurement of the other three concentration decay history can be worked out as follows.

For zone 2, based on the mass conservation of tracer b , there is

$$V_2 \frac{dC_{b2}}{dt} = -q_2 C_{b2} \quad (34)$$

Integrating the above equation, noting $V_2 = V$, $q_2 = q$ and the initial condition of $C_{b2} = C_{b20}$ at $t=0$, one obtains the C_{b2} history

$$C_{b2} = C_{b2o} e^{-\frac{qt}{V}} \quad (35)$$

For zone 1, the mass conservation of tracer a requires

$$V_1 \frac{dC_{a1}}{dt} = -q_1 C_{a1} \quad (36)$$

and the mass reservation for tracer b requires

$$V_1 \frac{dC_{b1}}{dt} = -q_1 C_{b1} + q_1 C_{b2} \quad (37)$$

Eq. 36 is solved in the same was as for Eq. 34, to obtain

$$C_{a1} = C_{a1o} e^{-\frac{qt}{2V}} \quad (38)$$

Eq. 37 can be transformed into the form below

$$\frac{dC_{b1}}{dt} + \frac{q}{2V} C_{b1} = \frac{q}{2V} C_{b2o} e^{-\frac{qt}{V}} \quad (39)$$

with the initial condition of $C_{b1} = 0$ at $t=0$. The solution for this equation is

$$C_{b1} = C_{b2o} e^{-\frac{qt}{2V}} - C_{b2o} e^{-\frac{qt}{V}} \quad (40)$$

Substituting Eq. 32, 33, 35, 38 and 40 into the two zone tracer decay equations Eq. 26-31, solving them simultaneously, one obtains the measured flow rates: $Q_{01} = 0$, $Q_{10} = q/2$, $Q_{02} = q$, $Q_{20} = q/2$, $Q_{12} = 0$ and $Q_{21} = q/2$, which are exactly the same as the true flow rates obtained at the beginning of this section. Therefore, the tracer decay technique is perfectly valid in this application.

3.3 Discussion

That the tracer decay technique is successful in the above test case is not at all accidental. The technique is represented, in the above case, by Eq.26-31. They are based on the principle of mass conservation for the tracer gases and the air, which is a universal principle. For example, Eq. 26 interpreted in physical terms means that in zone 1 the rate of tracer a increase (represented by the term on the left side) equals the rate of influx of tracer a (represented by the third term on the right) minus the rate of out flux of tracer a (represented by the first and second terms on the right). In addition, the representation of tracer increase rate by the left side term and the representation of influx and out-flux rates by the right side terms are always correct, for direct or branched connection alike. The former is obvious enough. The latter is principally because the definition of interzonal flow rates Q_{ij} , $i, j \in (0, 1, 2)$, is independent of the path through which Q_{ij} arrives, thus the influx rate of tracer a will be $C_{a2}Q_{21}$ whether Q_{21} comes through a branched connection or direct connection and the same is true for the out-flux terms. The same reasoning can be applied to the other five equations Eq. 27-31. Thus the equations representing the two zone tracer decay technique are always correct regardless of the connection types present. Therefore if V_1 , V_2 , C_{a1} , C_{a2} , C_{b1} , C_{b2} are accurately measured, then by solving Eq. 26-31, the interzonal flow rates will be accurately obtained. The validity of the technique observed previously is guaranteed from here.

The analysis and the conclusions presented so far in this section 3 has been extended to the other two multizone tracer gas techniques --- the constant concentration and the constant injection techniques -- and to buildings containing more than two zones. Details of this work can be found in Ref. [2].

It has been shown above that the tracer techniques are valid for measuring interzonal flow rates Q_{ij} in buildings with branched connections . However, when interpreting the tracer gas results, one must be aware of the possibility and the implication of the fact that Q_{ij} might have come through a branched connection linking others zones as well as zone i and j . In such a situation Q_{ij} not only depends on conditions in zone i and zone j , but also on those of the other zones that the branched connection links. Consequently, Q_{ij} measured under a certain set of zone conditions may not be the same as that from another measurement, even if the conditions in zone i and j are exactly reproduced in the latter test. Indeed, since there is currently no method for knowing which and how zones are linked by branched

connections, it cannot be guaranteed that Q_{ij} measured now can be repeated later, unless the conditions in each and every zone in the building are reproduced. In other words, information on interzonal flow rates obtained using tracer techniques are safely used only under a set of zone conditions identical to those under which the information were obtained.

The restriction brought in by the branched connection in terms of tracer measurement interpretation is much too great, for, in addition to "flow from zone i to zone j is Q_{ij} when they pressure difference between them is ΔP_{ij} " one now has the attached string of "and the pressure differences among the other eight zones in the buildings are.....". This is particularly serious for the setting up of databases for interzonal flows. One has to carry out a set of tests under each and every likely combination of conditions in the zones. The alternative would be to devise a method for detecting branched connections so as to reduce the number of combinations of zone conditions to be tested. A piece of research work on this can be found in Ref. [2]

4 Conclusions

An analytical study of the validity of the multizone air flow measurement techniques, in the presence of branched connections, has been carried out.

It is found that the multifan pressurisation method which includes the deduction technique and the guarding zone technique has embedded inadequacies, which could lead to large measurement errors, if the techniques are applied to buildings containing branched connections.

All versions of the tracer gas method are found to be valid regardless of the types of connections present. However, the interpretation of their of their results is much more restricted than in the case where only direct connections exist in the tested building.

The importance of branched connections is apparent. A survey of their presence in buildings and their likely forms and dimensions would be most useful. For that purpose, a method for detecting branched connections is clearly needed, and it is in this area that research by the authors is proceeding.

Acknowledgement

This study was part of an air infiltration research project funded by the UK Science and Engineering Research Council (Grant No. GR/F28397).

References

1. Furbringer, J.-M. and Roulet, C.-A. "Study of the errors occurring in measurement of leakage distribution in buildings by multifan pressurization." *Building and Environment*, Vol. 26, No.2, pp111-120, 1991.
2. Shao, L., Sharples, S. and Ward, I.C. "Branched connections: its implications to multizone air flow measurements and its detection." Internal report, Building air infiltration research group, Building Science Unit, University of Sheffield, UK, Nov., 1991.
3. Baker, P.H., Sharples, S. and Ward, I.C. "Air flow through cracks." *Building and Environment*, Vol. 22, No. 4, 1987, pp293-304.

