THE FUZZY SET THEORY FOR ANALYSING THERMAL COMFORT (°)

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ABSTRACT

Researchers involved in studying problems concerning the built-up environment face an objective complexity, of easy instrumental evaluation, and a subjective complexity which is psycho-physiological concerning human beings with different individual reactions to the same exposure conditions. We are dealing with quantitative data, made up of measured physical quantities, with qualitative data made up of the comfort status at an individual level, giving the correct relevance to the latter, if the comfort requirements of people to whom the built-up environment is addressed, are seen as priority.
Considering the thermohygrometrical aspect within the environmental comfort evaluation, this paper looks into the possibility to apply the fuzzy set theory to deal with subjective data, which are imprecise and non-absolute in themselves. In fact, the fuzzy set theory allows use of linguistic variables that fit better than numerical variables in representing human behaviours.
Application is proposed to a real case concerning a university building, for which environmental data, collected during the summer and winter seasons, and subjective data, collected by questionnaires, are available. The aim of the present study is to compare the results of a fuzzy analysis with the results obtainable from a traditional analysis, to favour the qualitative approach compared to the quantitative one in diagnosing indoor thermal comfort.

Topic: 3 (applications in community buildings)
Scope: thermal comfort analysis
Originality: fuzzy set theory application
Completeness: a real application is considered

Keywords: Fuzzy Set Theory,
Moderate Thermal Environment,
Thermal Comfort.

1. INTRODUCTION

The invitation to publish field-collected data that ISO Standard 10551[1] [1] puts to researchers working on thermal comfort is the stimulus for this contribution, although currently there seems to be a greater interest in problems connected with indoor air quality (IAQ), which would suggest if not the exhaustion of research on thermal comfort, at least an arrest.

In the assessment of thermal comfort the innovative approach recently introduced is that based on the "subjective" thermal environment evaluation, subject of the above-mentioned Standard, in which researchers of different competences, such as engineers, ergonomists and psychologists, are involved.

According to the latest trend, the well-established approach of the predictive kind has to be supplemented by direct determination of the subjective experience on the climatic environment, that could perhaps replace the objective method, if sufficiently validated. At this stage of the research, it seems therefore obvious to still make reference to the predictive approach, of objective validity, in the evaluation of results obtainable through the psychological method, at the same time improving reliability of acquisition tools and data treatment techniques.

In this framework the conclusions which several researchers have come to must be considered (as a reference, see [2]). On the basis of the results obtained by applying the two methodologies in parallel, they have often underlined the need to review the microclimatic prescriptions of the current standards on thermal comfort of moderate thermal environments, as they are based on laboratory tests.

2. FIELD STUDY ON DIRECT THERMAL COMFORT ASSESSMENT

In this field study a building located in Southern Italy was used. The destination of the building is for university education, so it is frequented by students who were involved in the thermal assessment survey on a voluntary basis. Within the building thermal homogeneous zones were identified, with a number of subjects in each of them significant to achieve a mean subjective response from the distribution of a questionnaire. This means of acquisition and recording of personal subjective judgements was prepared [3] following the instructions contained in ISO Standard 10551, so that the survey can also be considered an application of this Standard.

The study was conducted in summer 1995 and winter 1996, in which microclimatic measurements were performed, following ISO Standard 7726 [4], in the course of 78 visits concerning 12 different rooms, in which more than 1400 interviews were carried out. In table 1 data associated with each measurement are reported, identified by an abbreviation: R= room, WM=winter measurement, SM=summer measurement; so that, for example, R1SM1 is measurement n.1 in room n.1 performed in summer.

The measurements have been listed following the increasing operative temperatures, reporting relevant data such as: number of participants, N; mean value of estimated intrinsic thermal clothing insulation including seat contribution, I_{cl}; air temperature, T_a, mean radiant temperature,T_{mr}, air relative velocity ,v_{ar}, measured in the room and associated to the N subjects; operative temperature calculated from air temperature and mean radiant temperature,

[1] ISO. (1995) - ISO 10551: Ergonomics of the Thermal Environment - Assessment of the influence of the thermal environment using subjective judgement scales. Geneva, International Standard Organisation.

T_o; relative humidity, measured in the room, RH; mean thermal sensation obtained from the questionnaires, TS; percentage of dissatisfied people obtained from the questionnaires according to ISO Standard 10551, PD; predicted mean thermal sensation, obtained by applying ISO Standard 7730 [5], PMV, and finally predicted percentage of dissatisfied people obtained according to ISO Standard 7730, PPD.

Table 1: Summary of mean values of measured physical quantities and measured and predicted subjective judgements.

	N (-)	I_{cl} (clo)	T_a (°C)	T_{mr} (°C)	v_{ar} (m/s)	T_o (°C)	RH (%)	TS (-)	PD (%)	PMV (-)	PPD (%)
R2WM1	17	1.24	13.4	15.9	0.20	14.6	61	-1.9	88.2	-1.4	47.6
R4WM1	28	1.08	15.0	16.1	0.16	15.1	45	-1.9	85.7	-1.4	47.4
R3WM1	27	1.11	15.2	15.3	0.15	15.3	41	-1.7	85.2	-1.4	45.9
R7WM1	11	0.99	16.2	17.9	0.12	17.1	63	-1.0	72.7	-1.0	27.1
R2WM2	21	1.04	15.4	19.1	0.24	17.1	45	-1.8	85.7	-1.4	44.3
R2WM3	21	1.05	15.6	19.1	0.24	17.2	47	-1.1	85.7	-1.3	41.9
R7WM2	12	1.00	16.9	18.1	0.13	17.5	61	-0.9	66.7	-0.9	24.3
R2WM4	15	0.99	16.9	18.6	0.12	17.7	52	-0.6	66.7	-0.9	27.8
R2WM5	21	0.99	17.0	18.5	0.12	17.8	52	-0.3	42.9	-0.9	26.6
R1WM1	17	0.96	17.8	18.4	0.11	18.1	58	-0.9	82.3	-0.9	24.7
R3WM2	32	0.93	17.2	19.1	0.15	18.1	54	-0.9	75.0	-1.1	31.5
R3WM3	31	0.90	17.3	19.2	0.17	18.2	53	-0.7	71.0	-1.2	36.3
R3WM4	23	1.01	17.2	19.4	0.16	18.3	54	-0.5	56.5	-0.9	22.3
R3WM5	26	0.89	16.8	20.3	0.24	18.4	45	-0.5	46.1	-1.4	45.8
R3WM6	33	0.99	17.4	19.3	0.17	18.4	44	-0.4	60.6	-1.0	26.8
R2WM6	26	0.89	16.9	20.2	0.41	18.5	44	-0.8	57.7	-1.6	57.7
R3WM7	23	1.01	17.5	19.5	0.16	18.5	52	-0.5	56.5	-0.9	21.5
R3WM8	30	0.99	17.4	20.0	0.21	18.7	45	-0.3	60.0	-1.0	27.6
R9WM1	52	1.07	18.0	19.6	0.11	18.8	63	-0.4	59.6	-0.5	12.2
R9WM2	48	1.08	18.4	19.8	0.11	19.1	63	0.0	45.8	-0.4	10.4
R8WM1	15	1.03	18.7	19.8	0.12	19.3	49	0.5	33.3	-0.6	14.0
R8WM2	15	1.03	18.6	20.1	0.11	19.4	49	0.5	38.9	-0.5	12.6
R6WM1	13	0.93	18.1	20.8	0.13	19.5	53	-1.0	92.3	-0.7	17.3
R6WM2	18	0.97	18.8	20.4	0.14	19.6	51	0.2	38.9	-0.6	15.2
R3WM9	18	0.91	18.7	20.8	0.22	19.7	56	-0.6	66.7	-0.8	21.9
R6WM3	10	1.04	18.8	20.9	0.82	19.8	53	0.3	40.0	-1.1	33.0
R6WM4	10	1.04	18.7	20.9	0.14	19.8	53	0.0	40.0	-0.4	10.4
R3WM10	18	0.91	19.0	21.1	0.20	20.0	54	-0.1	38.9	-0.8	19.4
R1WM2	17	1.00	19.8	20.3	0.11	20.1	50	-0.7	58.8	-0.4	12.3
R3WM11	36	0.95	19.0	21.3	0.16	20.2	46	-0.2	47.2	-0.6	13.9
R7WM3	11	0.99	20.2	20.3	0.12	20.3	50	0.6	63.6	-0.4	8.9
R1WM3	17	0.96	19.9	20.8	0.11	20.4	66	-0.8	76.5	-0.3	9.8
R3WM12	35	0.95	19.2	21.6	0.17	20.4	47	-0.5	68.6	-0.6	13.0

Table 1 (continued).

R7WM4	18	0.95	20.7	21.7	0.11	21.2	48	0.7	55.6	-0.2	7.3

R7WM5	18	0.95	20.7	22.0	0.12	21.4	48	0.2	33.3	-0.2	7.1
R1WM4	16	0.97	21.1	22.0	0.11	21.5	56	0.1	31.2	-0.1	7.9
R8WM3	10	0.93	19.6	23.5	0.14	21.5	52	-0.4	60.0	-0.3	8.9
R8WM4	21	0.82	20.3	23.1	0.14	21.7	51	0.6	38.1	-0.5	13.2
R5WM1	19	0.88	21.1	22.5	0.17	21.8	45	0.2	31.6	-0.3	8.7
R8WM5	10	0.93	19.8	23.7	0.17	21.8	51	-0.1	60.0	-0.3	9.6
R8WM6	19	0.80	20.5	23.5	0.15	22.0	50	0.3	42.1	-0.5	13.5
R8WM7	15	1.03	20.8	23.1	0.13	22.0	48	0.5	40.0	0.0	6.0
R5WM2	19	0.88	21.3	23.0	0.18	22.1	45	0.0	84.2	-0.3	8.0
R7WM6	14	0.98	21.7	22.5	0.13	22.1	46	0.7	28.6	0.0	5.6
R5WM3	35	0.89	21.6	23.2	0.22	22.4	42	0.3	28.6	-0.3	9.2
R8WM8	15	1.03	21.3	23.9	0.14	22.6	48	0.3	46.7	0.1	6.1
R5WM4	34	0.90	21.9	24.1	0.25	22.9	41	0.4	55.9	-0.2	7.8
R8WM9	14	1.05	21.6	24.2	0.16	22.9	48	0.4	35.7	0.2	6.3
R10SM1	15	0.53	23.8	25.9	0.20	24.6	57	-0.3	33.3	-0.2	6.3
R10SM2	10	0.51	24.6	26.1	0.14	25.3	41	0.7	40.0	0.0	5.3
R10SM3	17	0.56	26.0	26.6	0.12	26.3	48	0.4	58.8	0.5	10.5
R4SM2	13	0.54	25.6	27.3	0.30	26.3	65	1.2	61.5	0.2	5.8
R10SM4	8	0.47	26.1	27.0	0.12	26.5	54	1.2	87.5	0.5	9.8
R12SM1	11	0.54	26.2	26.8	0.12	26.5	63	0.6	54.5	0.6	13.7
R3SM13	18	0.50	25.8	27.5	0.36	26.5	62	0.7	44.4	0.2	5.5
R12SM2	17	0.54	26.3	27.1	0.22	26.7	59	0.7	52.9	0.4	9.8
R2SM7	11	0.57	26.0	28.1	0.38	26.8	60	0.8	63.6	0.4	8.5
R4SM3	16	0.53	26.1	28.8	0.50	27.1	60	0.7	56.2	0.2	6.7
R11SM1	10	0.54	26.9	27.6	0.11	27.3	49	0.5	36.4	0.8	18.0
R11SM2	4	0.56	27.6	27.9	0.16	27.7	48	0.5	50.0	0.9	20.5
R11SM3	4	0.56	27.6	28.1	0.15	27.8	50	0.5	25.0	0.9	22.5
R11SM4	10	0.54	27.8	28.1	0.19	27.9	47	0.9	40.0	0.8	19.8
R3SM14	17	0.46	26.9	29.5	0.45	27.9	55	1.1	70.6	0.4	9.2
R11SM5	15	0.65	27.7	28.7	0.18	28.2	62	1.4	80.0	1.2	34.0
R11SM6	9	0.52	27.9	28.7	0.19	28.3	43	1.4	77.8	0.9	21.8
R11SM7	14	0.61	27.8	29.0	0.23	28.4	51	1.3	78.6	1.0	27.4
R2SM8	12	0.54	26.9	30.4	0.29	28.4	51	0.8	66.7	0.9	20.6
R5SM5	19	0.54	27.9	29.9	0.22	28.6	47	1.7	89.5	1.0	28.0
R5SM6	10	0.54	27.6	29.7	0.27	28.6	48	0.9	70.0	0.9	22.3
R8SM10	21	0.50	27.9	30.0	0.37	28.7	43	1.0	61.9	0.8	18.9
R11SM8	16	0.61	28.7	29.2	0.19	28.9	49	1.7	87.5	1.2	36.7
R8SM11	23	0.48	28.8	29.5	0.14	29.1	44	2.6	100.0	1.2	34.8
R8SM12	24	0.48	28.3	30.4	0.33	29.2	41	1.4	70.8	0.9	24.1
R5SM7	9	0.49	29.1	30.7	0.35	29.4	40	2.0	100.0	1.2	33.5
R11SM9	16	0.60	29.3	29.8	0.24	29.5	52	1.0	81.2	1.4	45.5
R6SM5	16	0.58	29.1	30.4	0.16	29.7	45	2.1	100.0	1.4	47.9
R8SM13	27	0.49	29.5	30.9	0.33	30.1	42	1.2	74.1	1.3	41.3
R5SM8	5	0.48	29.5	30.6	0.14	30.1	39	1.8	100.0	1.4	47.4

In the calculation of the predicted mean vote PMV, the metabolic rate assumed for all the subjects, M = 1.2 met, was estimated the value being considered as corresponding to a sedentary activity, typical for offices, schools, and laboratories.

The analysis of the thermal conditions of real environments was performed referring with priority to the most commonly used predictive index of thermal sensation, that is Fanger's Predicted Mean Vote, PMV, to which field measured thermal sensation index, TS, corresponds. TS index was measured on the seven point ASHRAE thermal sensation scale, here used discontinuously.

In figure 1 the indices TS and PMV are correlated, pointing out most negative deviations between predictive and diagnostic methods or deviations between regression lines of TS vs. PMV and TS=PMV. The correlation is characterised by the following regression equations:

Winter	TS = 1.16 PMV+0.47 (r = 0.76)
Summer	TS = 0.92 PMV+0.37 (r = 0.67)
Winter+Summer	TS = 0.99 PMV+0.34 (r = 0.88)

Data treatment considering individual votes rather than grouped means gives lower correlation coefficients, resulting for Winter r = 0.33, for Summer r = 0.35 and for all the data r = 0.58.

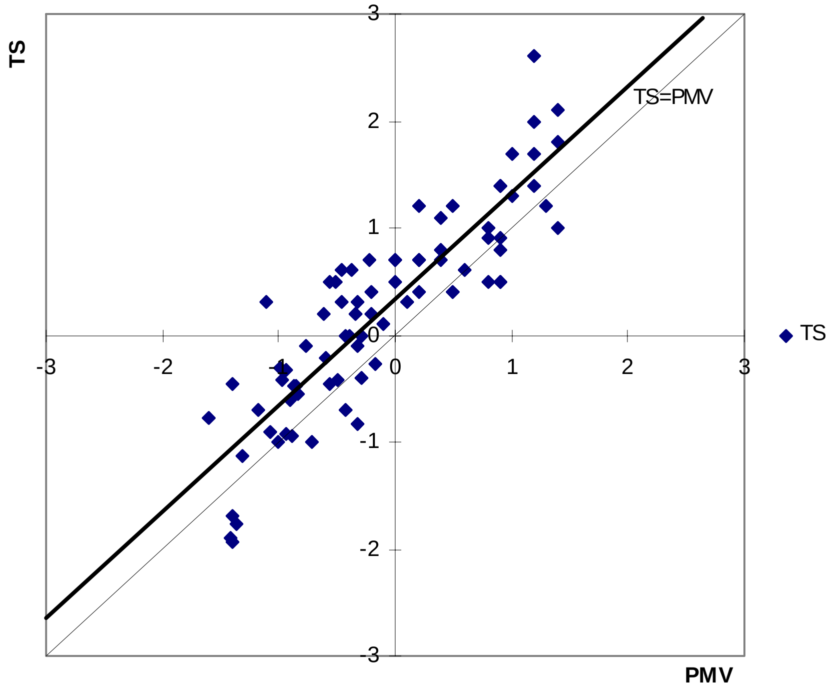

Fig.1: Winter+Summer data of measured TS vs. predicted PMV.

Considering the difference seen in the field of thermal insulation characteristics of clothing between the seasons, separate regressions of PMV and TS vs. T_o were calculated, as shown in figures 2 and 3, where it is possible to locate neutral temperatures, measured (**T_{nm}**) and predicted (**T_{np}**) respectively, in correspondence to the points where the curves cross the neutral thermal sensation line.

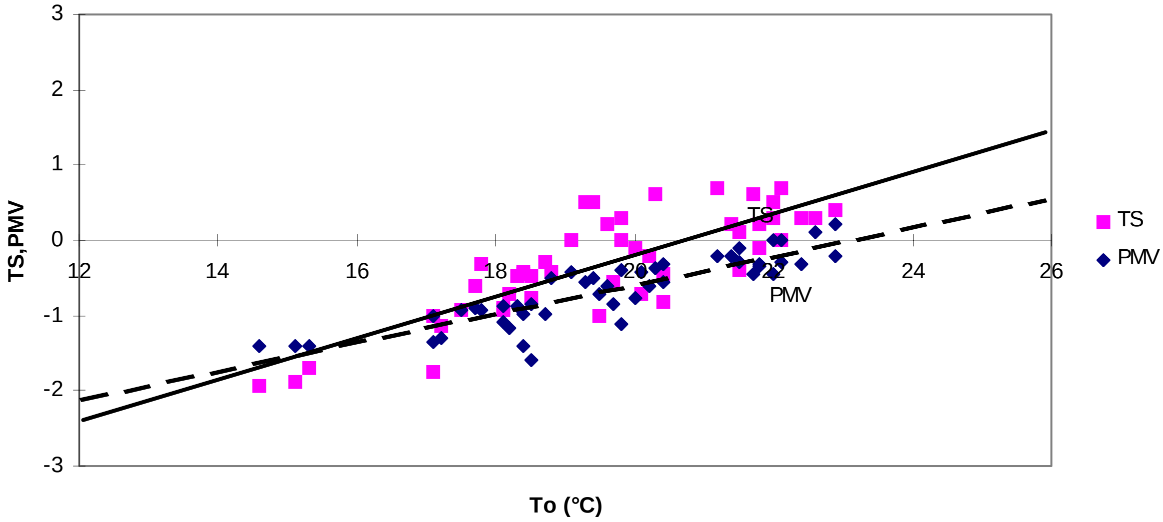

Fig.2: Winter data of measured TS and predicted PMV vs. operative temperature T_o.

The regression equations obtained are:

Winter	TS = 0.28 To - 5.71 (r = 0.82)
Winter	PMV= 0.19 To - 4.41 (r = 0.87)
Summer	TS = 0.30 To - 7.39 (r = 0.71)
Summer	PMV= 0.29 To - 7.21 (r = 0.92)

The neutral temperatures result:

	T_{nm} (°C)	T_{np}(°C)
WINTER	20.4	23.2
SUMMER	24.6	25.1

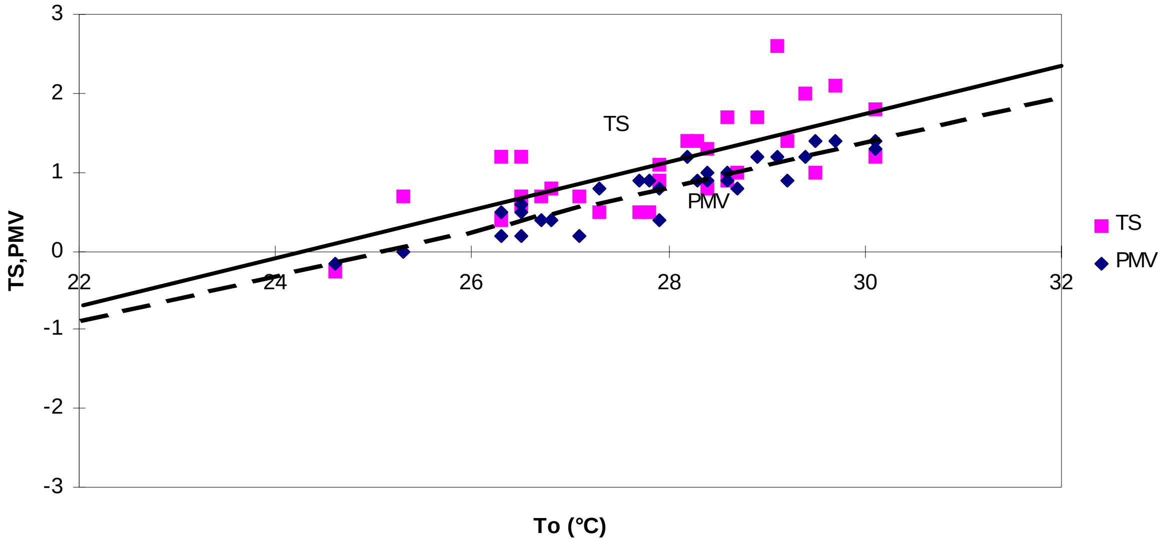

Fig.3: Summer data of measured TS and predicted PMV vs. operative temperature T_o.

3. INTRODUCTION TO THE FUZZY SET THEORY

Studying the real world is usually carried out through a description of phenomena, i.e. a representation, based on models which approximate reality in a more or less satisfying way. However representation techniques are fundamentally based on numbers, thus modelling reality is made through a "precise" language, although in many cases real phenomena do not have features of precision, due to the complexity of the variables themselves and of interrelations among variables that determine them. Moreover, a description is an interpretation of a phenomenon, thus affected by "subjectivity" features, even if some elements may be shared by researchers in the same disciplinary field. And when describing a phenomenon, human beings refer to their own sensations, make correlations, make deductions, use a communication language, all of which are imprecise. In reality, there are some precise statements, such as: *aircraft are a means of transportation* and *birds sing*; however, even though both can fly, birds are not a means of transportation and aircraft cannot sing. But, if for example the dimension of an aircraft or the pleasure in listening to the birds singing is considered, immediately imprecision margins of the description can be noticed: *I flew on a really big plane*, or *very big*, or *quite big*, or *not big*, or *not very big*; and *the singing of a canary is very pleasant,* or *pleasant*, or *not pleasant at all*. Not only do the definitions hold imprecision in themselves, but also the reference words -dimension, sensation- are not explained. These examples show how human beings think in terms of sets with "fuzzy" rather than precise boundaries, which moreover fit the complexity of the real world better. Consequently, representation models cannot be considered realistic if limits for a fuzzy interpretation are excluded from the description of real world problems. Allowing for approximate reasoning [6], fuzzy logic offers a move in this direction, thus assuring better correspondence of the model to the real world.

The fuzzy set theory was introduced by L.A. Zadeh [7] as a conceptual reference for managing fuzzy logic, and can be considered as a new set theory, i.e. fuzzy, which generalises and includes in itself the classical set theory. The basic definition of the fuzzy set theory is that if U is a universe of discourse, and u is a generic variable member of U, a **fuzzy set** A in U is characterised by a membership function $\mu_A(u)$ which associates a real number in the [0,1] interval to each element u in U:

$$A=\{(u, \mu_A(u))|u\in U\} \quad 1)$$

where the $\mu_A(u)$ value is the **degree of membership** of u in A: 1 means the full membership of u in A, 0 means the full non-membership of u in A, all the values included in the 0-1 interval mean intermediate degrees of membership between the two extremes. In such a way, the moving of an element from membership to non-membership state in a set is gradual rather than discrete, as, on the contrary, it is in the classical bi-valued logic. For example, U is the universe of aircraft, and A the set of the really big aircraft in U. To simplify, four elements of U are considered: u_1, a 747 Boeing, u_2, an Airbus 300, u_3, a DC 9, u_4, an ATR 42. Following classical bi-valued logic, the ordinary set $\underline{A}$ in U will include only the u_1 element:

$$\underline{A}=\{u_1\} \quad 2)$$

while using fuzzy sets, the fuzzy set A in U will include all the elements considered with different degrees of membership:

$$A=\{(u_1, 1), (u_2, 0.7), (u_3, 0.2), (u_4, 0)\} \quad 3)$$

where 1, 0.7, 0.2, and 0 are the degrees of membership of the 747 Boeing**,** the Airbus 300, the DC 9, and the ATR 42, respectively to the fuzzy set A of the really big aircraft. The example shows that the fuzzy set A includes within itself the ordinary set A.

It is possible to make a series of operations on fuzzy sets -union, intersection, complement, negation, fuzzy relationships- for which it is possible to refer to the specific bibliography [8].

Thus, through the fuzzy set theory, variables can be considered fuzzy, i.e. variables which cannot be defined in a univocal and precise way, and can be associated to fuzzy sets in a universe of discourse, if assigned the values of their membership function. This feature is of fundamental importance, since among fuzzy variables there are **linguistic variables**, i.e. variables whose values are words or sentences in natural language, thus allowing the possibility of evaluating qualitative, not only quantitative, aspects of the phenomena described. A typical example of linguistic variable is the temperature: in evaluating the water temperature of a shower, people express themselves in terms of *cold*, *cool*, *quite cool*, *good*, *quite warm*, *warm*, *hot*, and the moving between two adjacent sets is gradual rather than abrupt. In traditional logic, precise boundaries of distinction are defined between sets; for example, it is possible to establish that water is good if the temperature is 30°C, quite cool from 25°C to 30°C, quite warm from 30°C to 35°C, and so on. Therefore at a temperature of 29.9°C water will be quite cool and at 30.1°C quite warm, in spite of the fact that temperature values are very close to the good one. This is a precise and quantitative evaluation of the temperature variable, whereas with the fuzzy set theory it is possible to attribute a degree of membership for each element to the considered sets, thus the same element can belong to more than one set at the same time, so that, a temperature of 5°C can have a membership degree of 1 to the fuzzy set of cold temperature and a membership degree of 0 to the other fuzzy sets which the temperature variable has been divided into; and a temperature of 26°C can belong to the fuzzy set of quite cool temperature with a 0.8 degree of membership and to the fuzzy set of good temperature with a 0.2 degree of membership. This is a qualitative evaluation, which does not exclude the quantitative one, and allows for imprecision, not excluding precision. Figure 4 shows the graph of the example reported.

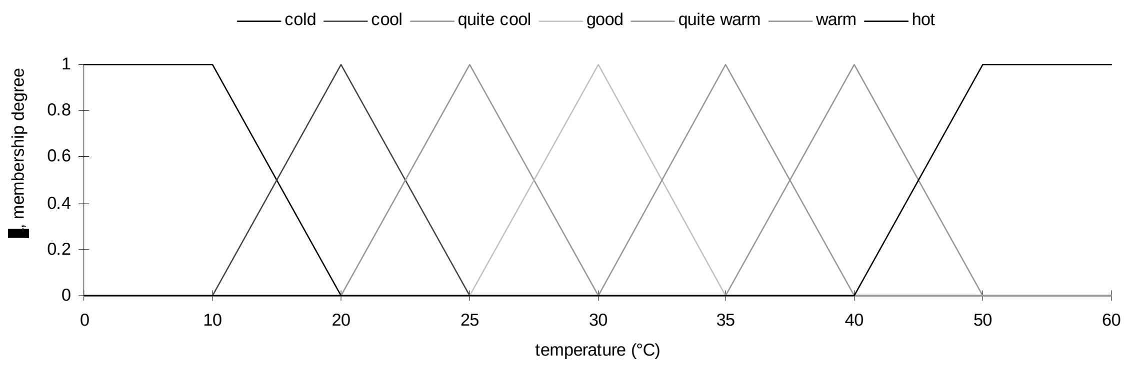

Fig. 4: A fuzzy division of the universe of discourse temperature of a shower.

Thus, the fuzzy set theory uses imprecision in terms of vagueness rather than lack of knowledge and provides a mathematical reference framework through which even vague conceptual phenomena can be studied with precision and rigour: “there is nothing fuzzy about fuzzy set theory” [9].

4.1 A FUZZY ANALYSIS OF THERMAL COMFORT

In the sector of thermal comfort evaluation, the fuzzy set theory can be applied advantageously. When data collected in field are analysed, researchers deal with both quantitative data related to the environment and physically measurable by instruments - temperature, humidity, air velocity, etc.-, and qualitative data, surveyed by questionnaires handed out to the users of an environment, measurable with difficulty by numerical entities - thermal sensation, thermal preference, thermal environment acceptability, etc.- Thus, the analysis univocally correlates subjective data expressed by individuals in qualitative terms, with a scale of numerical values subdivided into intervals, each one equal to the other and defined by whole numbers, that do not have any relationship with the reference variables; for example, the thermal perception of individuals is often correlated with the thermal perception scale, subdivided into seven intervals ranging from -3 to +3, values which do not directly refer to and/or derive from the operative temperature, about which individuals had expressed their own sensations. Thus, data change from qualitative to quantitative and as such, they are utilised in the whole evaluation: the degree of arbitrariness in the qualitative-quantitative moving of datum should be obvious to anybody, although all researchers in the sector apply the same kind of reference scale. If the fuzzy set theory is introduced, it becomes possible to manage the typical imprecision of human expressions and so the approach to data analysis changes so that the situations studied correspond better to the real complexity.

Taking this point of view as the starting point, this paper proposes a re-interpretation of the seven point thermal sensation reference scale, with the aim of referring the interval limits to the operative temperature and of obtaining variable widths of the intervals, more suitable to human sensation. In such manner, the thermal sensation reference scale does not vary between the values -3 and +3, but between a minimum and a maximum value of operative temperature, and the intervals are not constant and with a unitary width, but variable.

For a data fuzzy analysis, first of all universes of discourse must be recognised, that is the intervals within which variables in input can range. Since those variables are usually made by numerical values, a **fuzzification** procedure is applied which allows an evaluation of numerical values in linguistic terms [10]. The number of fuzzy sets in which the universes of discourse are subdivided and the type of the membership function become significant elements of the analysis.

Fuzzification must be followed by a **defuzzification** procedure that, operating in the opposite way to the previous one, returns a numerical value from the fuzzy functions. The available techniques for the defuzzification operation are varied [11] and they are evaluated and selected each time according to the problem under consideration and the goal to achieve.

In the case of thermal comfort, a fuzzy analysis proves to be relatively facilitated, since the variables in input are already available in their own linguistic interpretation -thermal sensation, thermal preference, thermal environment acceptability, etc.- and moreover, the number of intervals in which to subdivide the universes of discourse can correspond to the number of intervals of the reference warmth scales utilised in traditional analysis. The choice of a membership function can be simply but effectively determined by the ratio of the number of people giving a certain answer to the number of all the people answering. In such a way, membership function itself is discrete rather than continuous.

As a defuzzification technique it is possible to use the COA (center of area) method which returns its centroid as the defuzzified value of a fuzzy set. It is one of the most popular methods, of immediate comprehension and with good experimental results [12]. For a universe of discourse U and a discrete membership function $\mu(u)$, the result returned by the COA method is:

$$u^{COA}=\Sigma u_i \mu(u_i)/\Sigma \mu(u_i) \quad (i=1....n) \qquad 4)$$

4.2 APPLICATION

The fuzzy experimentation carried out first used data available from the winter period survey. The operative temperatures of the different measurements and the related answers of people interviewed in the questionnaires concerning their thermal sensation were considered. Thus, the universe of discourse U is constructed by the operative temperatures; there are seven fuzzy sets A_i of U and are made up by the operative temperatures expressed in their own degree of membership to the seven categories of judgement for the thermal sensation: *cold, cool, slightly cool, neutral, slightly warm, warm, hot*. If the answers of all the people interviewed, taken at a certain temperature, are considered, then the membership degree of the operative temperature, for example to the cold operative temperature set, can be calculated as the ratio of the number of people answering “cold” to the total number of answers; all the same for the other intervals.

Available data were all indiscriminately used, initially referring only to the thermal sensation scale. The fuzzification procedure applied to these data produced very irregular membership functions and quite low membership degrees of the different functions (figure 5), features that make the procedure unsatisfactory and unusable.

This pronounced irregularity induced reflection on the data used in an indiscriminate fashion, without exercising any form of control in selecting homogeneous, thus comparable, data, moreover, then referring them to only one judgement scale. Thus, selection among the data was made, using only those that: i) presented direct correspondence between the thermal sensation and the thermal preference scales, ii) concerned the last measurement taken for each surveyed environment, iii) were related to the judgement of individuals wearing clothes corresponding to the standard clothes worn in the winter season ($0.96<I_{cl}<1.05$). The first condition is aimed to exclude inconsistent answers concerning the thermal perception: the individual expressing a thermal sensation of *cold* should later express a thermal preference of *hot*, or at least of *warm*, but presumably not for the other categories. The second condition is aimed to exclude the answers which are more affected by uncertainty, since the individuals already knew the questionnaire in the previous measurements and thus they should have a more precise cognition, even if personal, of what they are requested to express. The third condition is aimed to exclude extreme answers due to clothing too light or too heavy for the seasons under considerations, this being a chance factor in the survey.

The fuzzification procedure carried out on the selected data shows membership degrees higher than the first experimentation but membership functions are still irregular which do not allow a clear fuzzy subdivision of the universe of discourse considered.

Thus, the reflection on data became deeper and some relevant elements of the survey were singled out: the sample examined was most irregular, the environments surveyed greatly differed from each other, a linguistic explanation of the meaning of the intervals which an answer was requested for, was lacking. Consequently, instead of the procedure being unsuitable, it must be deduced that the data available from the survey did not fit the experimentation proposed here.

While waiting for homogeneous data collected in the field but following control criteria, it was decided to test the procedure on data collected by Fanger [13], which certainly assure the necessary homogeneity. In fact, the fuzzification procedure applied to Fanger's data produced a significant result, since a fuzzy subdivision of the universe of discourse of operative temperatures was successful (table 2, figure 6).

Table 2: Operative temperatures of Fanger's data and their membership degrees to the seven judgement categories for thermal sensation.

t_i operative temp.	$\mu_1(t_i)$ cold	$\mu_2(t_i)$ cool	$\mu_3(t_i)$ slightly cool	$\mu_4(t_i)$ neutral	$\mu_5(t_i)$ slightly warm	$\mu_6(t_i)$ warm	$\mu_7(t_i)$ hot
18.90	0.4250	0.3625	0.1625	0.0375	0.0125	0.0000	0.0000
20.00	0.2875	0.4875	0.1500	0.0750	0.0000	0.0000	0.0000
21.10	0.1250	0.3056	0.4514	0.1042	0.0139	0.0000	0.0000
22.20	0.0875	0.3375	0.3125	0.2625	0.0000	0.0000	0.0000
23.30	0.0556	0.1111	0.4722	0.3333	0.0208	0.0069	0.0000
24.40	0.0000	0.0375	0.2250	0.6250	0.1125	0.0000	0.0000
25.60	0.0069	0.0208	0.1806	0.5764	0.1875	0.0278	0.0000
26.70	0.0000	0.0000	0.0375	0.5750	0.3125	0.0500	0.0250
27.80	0.0000	0.0000	0.0417	0.3889	0.4097	0.1458	0.0139
28.90	0.0000	0.0000	0.0000	0.1750	0.4125	0.3000	0.1125
30.00	0.0000	0.0000	0.0000	0.2500	0.4500	0.2625	0.0375
31.10	0.0000	0.0000	0.0000	0.0875	0.3875	0.3625	0.1625
32.20	0.0000	0.0000	0.0000	0.0500	0.1625	0.5000	0.2875

The defuzzification procedure followed fuzzification, applying the COA method, as already reported in 4.1. For each of the seven fuzzy sets (j) in which the operative temperature universe of discourse was subdivided, by the 4), a defuzzified value was obtained, meaningful for the interval:

$$t_j^{COA}=\Sigma t_i\mu_j(t_i)/\Sigma\mu_j(t_i) \quad (i=1.....13) \qquad 5)$$

These values (table 3) can be interpreted as the central points of seven intervals on a new scale for thermal sensation.

Table 3: Results of the defuzzification procedure, by the COA method, on Fanger's data.

$\Sigma t_i\mu_j(t_i)$	19.8347	34.5782	45.8076	91.0588	70.9988	50.1808	19.7411
$\Sigma\mu_j(t_i)$	0.9875	1.6625	2.0333	3.5403	2.4819	1.6556	0.6389
t_j^{COA}	**20.0858**	**20.7989**	**22.5283**	**25.7208**	**28.6061**	**30.3106**	**30.8991**

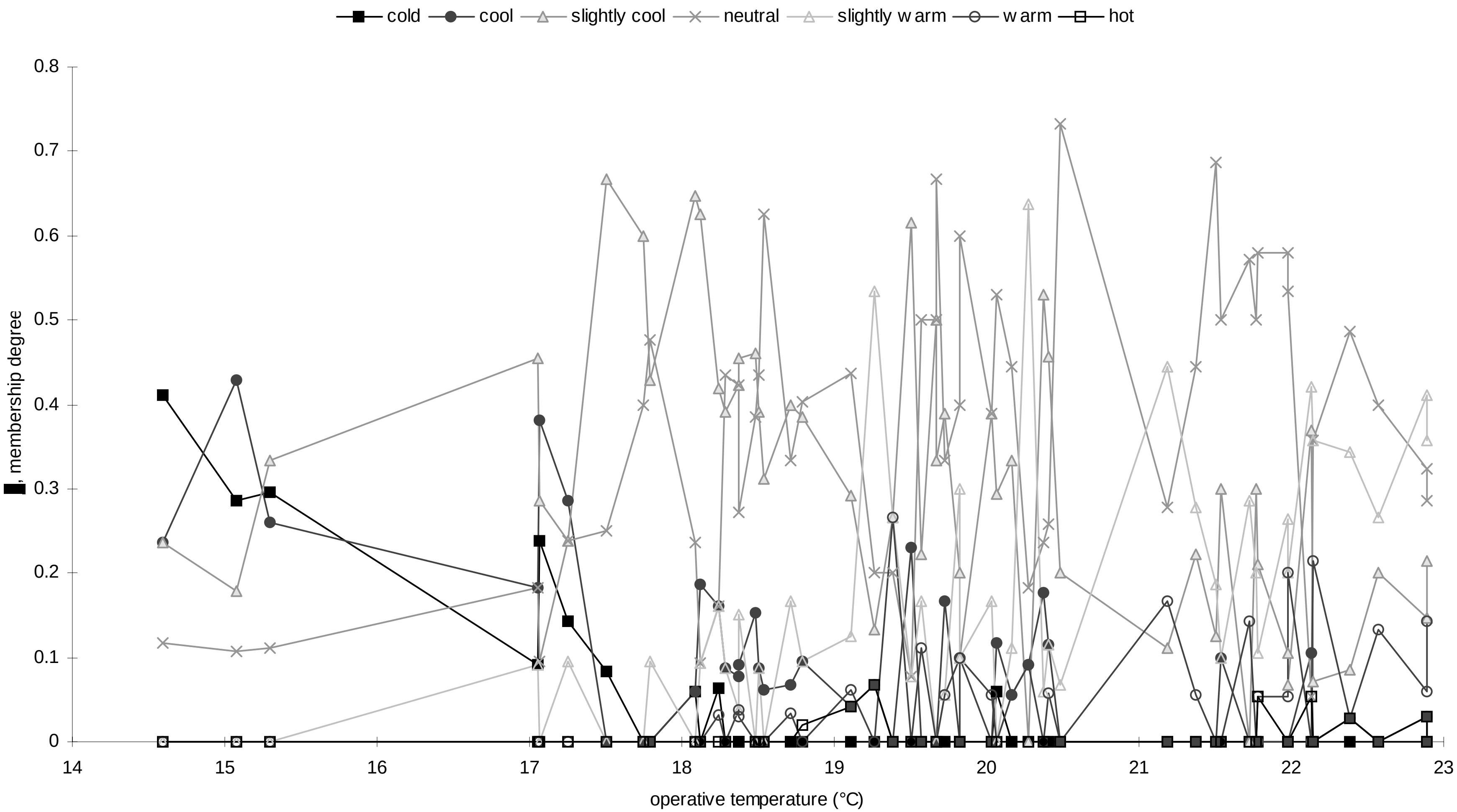

Fig. 5: Fuzzification procedure on all winter data.

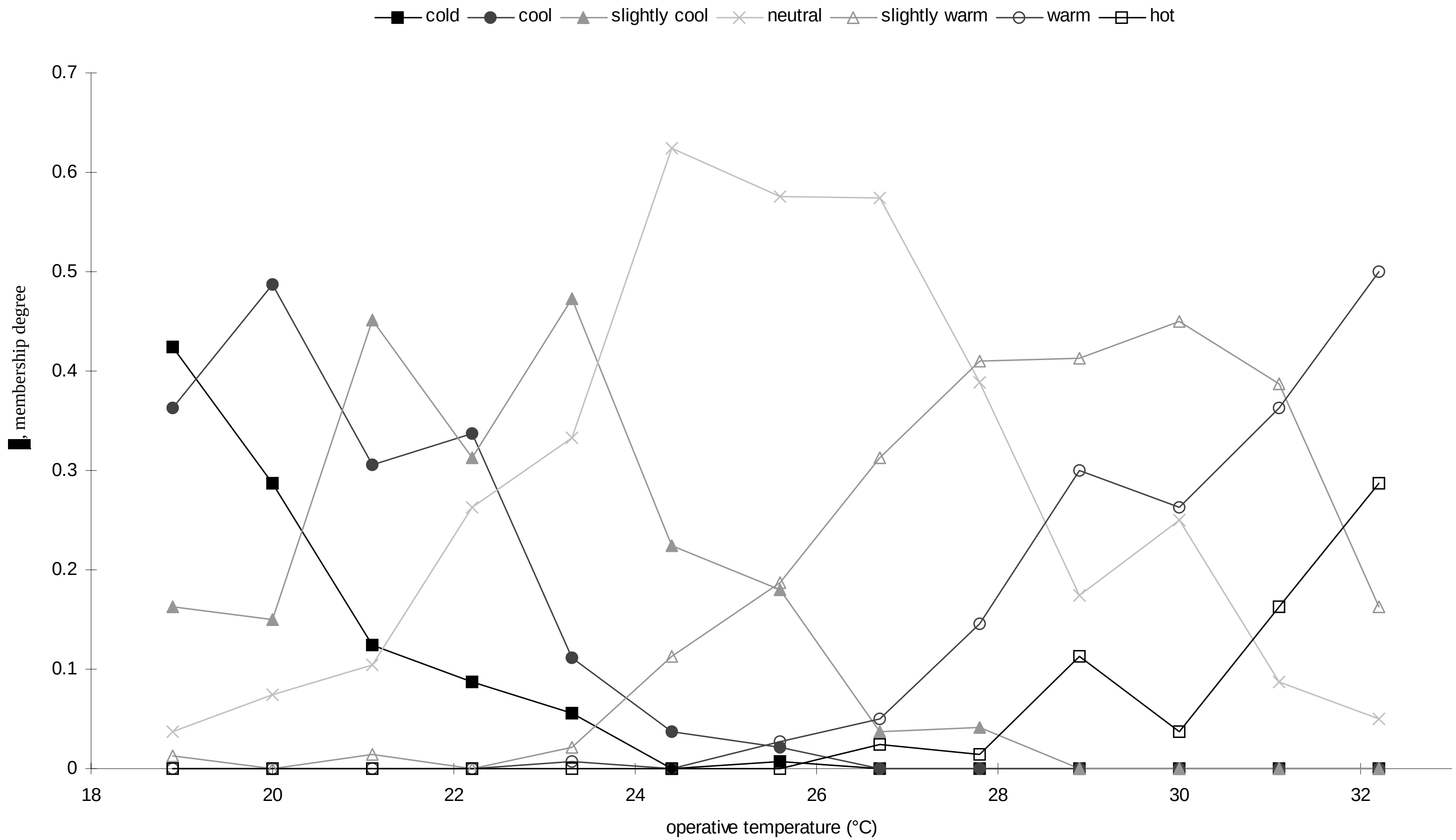

Fig. 6: Fuzzification procedure on Fanger’s data.

5. DATA ANALYSIS BASED ON A NEW REFERENCE SCALE

The application of the fuzzy set theory to Fanger's data produced a new reference scale for thermal sensation that can be denominated **Perceived Temperature Scale**, to which it is possible to associate a **Perceived Temperature Index**, PT, if measured in field and a **Predicted Perceived Temperature Index**, PPT, if calculated by a new thermal comfort equation.

As for the moment there is no possibility to modify the thermal comfort equation that constitutes the basis of the predictive method, we considered comparing our field data again re-interpreting the originary data of TS and PMV on a reference seven point scale, ranging between -3 and +3, characterised by intervals of variable widths, obtained proportionally to the correspondent intervals on the new perceived temperature scale.

Figure 7 represents the scale of PPT, obtained from the fuzzy analysis of Fanger's data (the values are rounded off to the first decimal figure); the modified scale, PPT', obtained by subtracting the value of +25.7 from each of the seven points of the PPT scale, with the aim of having the 0 value corresponding to the central point of the scale; the seven point ASHRAE thermal sensation scale of PMV; and the modified seven point scale, PMV', obtained from the previous scale by varying the interval widths proportionally to the scale of PPT'. To show how the last scale was derived, it can be said, for example, that the point +1.67 was obtained as (2.9/5.2)*3.

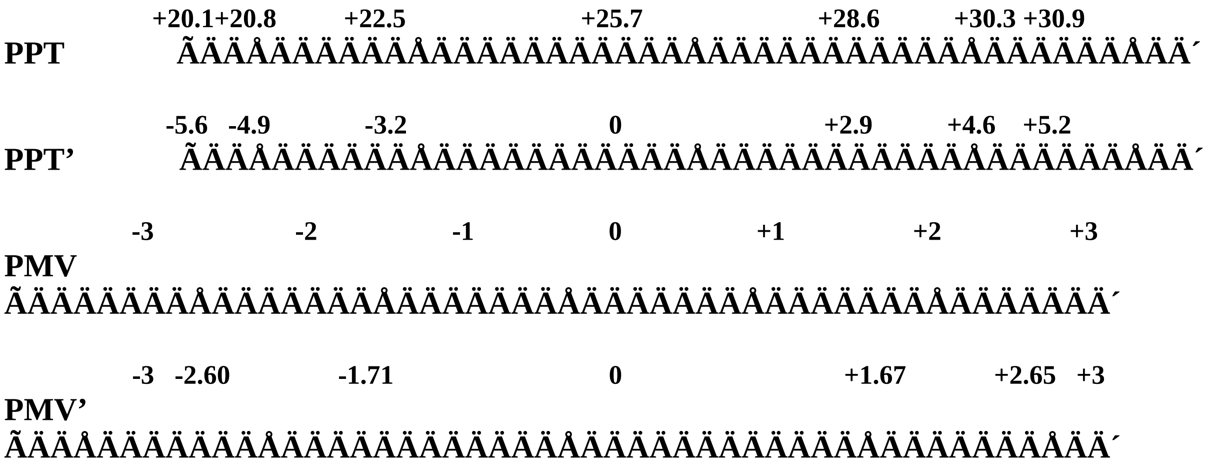

Fig. 7: Scales of PPT, PPT', PMV, PMV'.

In conclusion the new widths of the seven categories of thermal sensation judgements, compared to the traditional ones, follow a scale ranging from -3 to +3:

Thermal sensation judgement	Modified seven point ASHRAE Scale	Traditional seven point ASHRAE Scale
COLD	PMV'<-2.8	PMV<-2.5
COOL	-2.8≤PMV'<-2.15	-2.5≤PMV<-1.5
SLIGHTLY COOL	-2.15≤PMV'<-0.85	-1.5≤PMV<-0.5
NEUTRAL	-0.85≤PMV'≤+0.85	-0.5≤PMV≤+0.5
SLIGHTLY WARM	+0.85<PMV'≤+2.15	+0.5<PMV≤+1.5
WARM	+2.15<PMV'≤+2.8	+1.5<PMV≤+2.5

HOT	PMV'>+2.8	PMV>+2.5

Considering all the measured and predicted field data, collected in summer and winter, on the traditional scale of PMV it resulted that for 49% of the measurements values corresponding to TS and PMV belonged to the same judgement class, even if with discrepancy within that class. The same data re-interpreted on the modified PMV' scale with variable intervals, resulted as belonging to the same judgement class for 77% of the measurements. To explain how this result was obtained, consider, for example, the first measurement (R2WM1) of table 1 to which TS = -1.9 and PMV = -1.4 correspond. These values interpreted on the traditional PMV scale lie in the categories of thermal sensation *cool* and *slightly cool* respectively. But by making reference to the modified PMV' scale they are in agreement as both lie in the *slightly cool* category, being comprised between -2.15 and -0.85 points. Examining all measurements (78) we found 38 judgements which agreed and thus the percentage 38/78 = 49 when referring to PMV scale, but 60 judgements which agreed with the percentage 60/78 = 77 when referring to PMV' scale.

This result can be considered a refinement of the diagnostic tool of thermal comfort evaluation.

6. CONCLUSION

Using the fuzzy set theory for thermal comfort evaluation seems to offer a more realistic approach to the problem than what has been proposed so far. The diagnostic method, which uses questionnaires, though managing qualitative data in input, has always been based on a classical statistical analysis of the data quite arbitrarily expressed in numbers, thus not diverging very much from the predictive method layout. On the contrary, the approach proposed in the present paper supplies a different perspective for thermal comfort evaluation, both in the predictive and diagnostic methods, since it refers to a thermal sensation scale whose intervals are variable and defined by real operative temperature values, rather than numbers without any relationship with reality.

The achievement of this first objective shows an improvement in correlation between the psychological and predictive methods, opening future work perspectives for re-formulating the thermal comfort equation and potential new fuzzy applications in this domain of knowledge.

(°) The paper is the result of collective work. The individual contributions are articulated as follows: paragraphs 1, 2, and 5 are by I. Fato; paragraphs 3, 4, and 6 are by E. Conte.

REFERENCES

[1] ISO. (1995) - ISO 10551: Ergonomics of the Thermal Environment - Assessment of the influence of the thermal environment using subjective judgement scales. Geneva, International Standard Organisation.

[2] Schiller, G.E. (1990). A comparison of measured and predictive comfort in office buildings. In *Transaction ASHRAE*, 96, Vol.1.

[3] Alfano, G., Cirillo, E., d'Ambrosio, F.R., Fanelli, C., Fato, I., Fattorini, E., Leonardis, C., Riccio, G., Strambi, F., and Valentini, F. (1993). Proposta di questionario per la valutazione soggettiva del benessere termoigrometrico. In Proceedings of *the 5° Congresso Nazionale della Società Italiana di Ergonomia*, D2, Palermo.

[4] ISO. (1985) - ISO 7726: Thermal environments - Instruments and methods for measuring physical quantities. Geneva, International Standard Organisation.

[5] ISO. (1994) - ISO 7730: Moderate thermal environments - Determination of the PMV and PPD indices and specification of the conditions for thermal comfort. Geneva, International Standard Organisation.

[6] Zadeh, L.A. (1992) The Concept of a Linguistic Variable and its Application to Approximate Reasoning. Selected Papers by L.A. Zadeh. John Wiley & Sons Inc, New York, USA, 219-366.

[7] Zadeh, L.A. (1965). Fuzzy Sets. *Information and Control*, Vol. 8, 338-353.

[8] Zimmermann, H.J. (1991). *Fuzzy set theory and its applications*. Kluwer, Dordrecht, The Netherlands.

[9] Zimmermann, H.J. (1987). *Fuzzy Sets, Decision Making, and Expert Systems*. Kluwer, Boston, USA.

[10] Mendel, J.M. (1995). Fuzzy Logic Systems for engineering: a tutorial. In *Proceedings of the IEEE*, Vol. 83, n. 3, 345-378.

[11] Yager, R.R., Filev, D.P. (1993). On the issue of defuzzification and selection based on a fuzzy set. *Fuzzy Sets and Systems*, Vol. 55, 255-272.

[12] Yager, R.R., Filev, D.P. (1994). Essential of Fuzzy Modeling and Control. John Wiley & Sons Inc, New York, USA.

[13] Fanger, P.O. (1972). *Thermal Comfort*. McGraw-Hill, New York, USA.