MODEL REDUCTION APPLIED TO A HEATING FLOOR COUPLED WITH A BUILDING

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1. Introduction

The thermal modelling of the building has rapidly developed in the last decade, thanks to progress in computer science and engineering.

The theory and mathematical tools for a rigorous description of very complex phenomena are available to the researcher. This approach results in putting the problem in the form of equations with partial derivates. Higher the complexity and accuracy of the model is needed, greater the size of the system of differential equations to solve is, and consequently the computing time increases. Taking into account of the dynamics highly increases the computing and numerical problems. These remarks justify alone the interest of using reduced models. A further advantage of this type of models derives from the facility of modelling a subsystem in detail and then including this subsystem, after reduction, in an environment of modelling capable to process the functions of a complex thermal system.

This paper presents techniques used for achieving a reduced state model of a water heating - floor and its coupling with a building model, obtained with the help of the TRNSYS software. The dynamic thermal behavior of the floor is modelled by integrating the equation of the heat in two steps. First, there is the discretisation of space using the finite volume method. The different matrices (the state, command, observation and direct coupling ones) are thus computed for the reference model which is order 215 in the example processed. By linear aggregation, a technique used for model reduction, one obtains a reduced model (of order 13 for this example). Secondly, having the reduced model achieved, it is included as a new type (subroutine) in TRNSYS, the software which numerically integrate the differential equations, and coupled with other components of the thermal model, especially the building model.

2. The heating floor and its reduced model

The heating floor, chosen in this example, comprises 4 layers (fig. 1). The thermal characteristics are indicated in the table 1.

For symmetry reasons, it is considered half the distance between two successive tubes. The problem of two-dimensional heat conduction has been described using Heat2 [1] software. The area has been discretised in 215 control volumes for the example shown in figure n°1.

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The energy balance for each control volume yields in the system of following coupled differential equations:

$$[K1]\dot{T}(t) = [A1]T(t) + [B1]U(t) \quad (1)$$

$$Y(t) = [C1]T(t) + [D1]U(t) \quad (2)$$

where:
[A1] : internal heat - transfer matrix;
[B1] : external heat - transfer matrix;
[C1] : state-output coefficients matrix;
[D1] : direct transmission matrix;
[K1] : thermal capacity matrix;
T(t) : vector of the temperatures
U(t) : input vector; U (t)={Tu,Tf,Td}, Tu is the upper surface temperature, Tf is the fluid temperature and Td is the lower surface temperature of the floor.
Y(t): output vector; Y(t)={Φu, Φf, Φd}, Φu is the upper surface heat flow, Φf is the fluid heat flow, Φd is the lower surface heat flow of the floor.

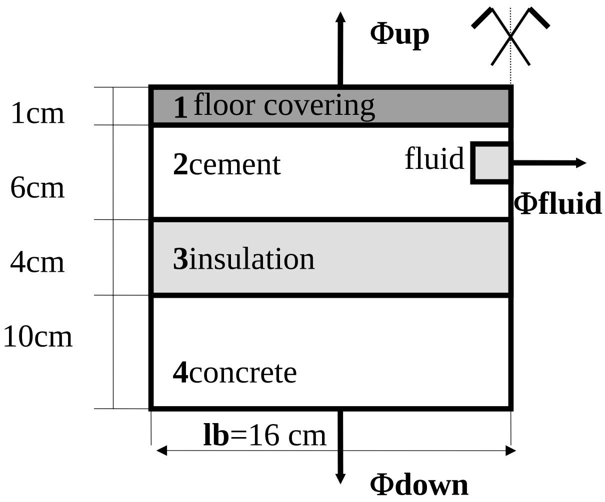

Fig.1 Heating floor scheeme

Table 1.

N° layer	thermal conductivity (W/mK)	thermal capacity (10^6 J/m^{3}K)
1	1.04	1.05
2	1.15	1.64
3	0.04	0.05
4	1.50	2.21

The state equation (1) has a large order as the state matrix [A1] is 215 by 215. The observation equation (2) connects the desired outputs, the surface heat flows Y(t), with the temperature vector T(t).

This system was transformed by changing the basis and diagonalizing the matrix $[K1]^{-1}[A1]$. This step of calculation is relatively tedious but interesting because, after changing the basis, differential equations are uncoupled and consequently one has the possibility to solve each of them independently of the others.

In the modal basis, the thermal behavior of the floor is then governed by matrix equations:

$$\dot{Z}(t) = [W]Z(t) + [G]U(t) \quad (3)$$

$$Y(t) = [\Omega]Z(t) + [K]U(t) \quad (4)$$

where:

[W] : diagonal matrix of eigenvalues λ_{ii} of $[K1]^{-1}$ [A1]; dim(n,n)
[G] : command matrix; dim(n,p)
[Ω] : observation matrix; dim(q,n)
[K] : direct transmission matrix; dim(q,p)
Z (t) : state vector in the modal basis; dim(n)

The vector Z is connected to the vector of state T (that contains temperatures in the grid points) by the matrix [P], by changing from the initial basis to the modal basis: T(t) = [P]Z(t).

The advantage of changing the basis is that there are methods developed to define a linear model which has the same steady-state and dynamic behavior but a lower order than the initial one [2].

In this paper, the linear aggregation method is chosen. The principle of this method consists in imposing a linear relationship between the states of the initial model and the ones of the reduced model, of the type: X(t) = [L] Z(t), with: X(t) - vector of state of the reduced model and [L] - matrix of aggregation.

After reduction, systems (3) and (4) become :

$$\dot{X} = [A]X + [B]U \tag{5}$$

$$Y = [C]X + [D]U \tag{6}$$

where:
X : state vector of the reduced model; dim(nr) nr<<n;
$\dot{X}$: time derivative of the state vector.

The diagonal matrix [A] of the aggregated model preserves eigenvalues that correspond to the selected dominant modes of the initial model. Its eigenvalues are chosen among the eigenvalues associated with the dominant modes of the matrix $[K]^{-1}$ [A1]. The selection of dominant modes is based on their energy contributions G(i,i) obtained on the basis of the analysis of the gap between the state of the system to a given instant and its state resulted for Dirac or Heaviside pulse inputs entries [2,3]. The energy contribution of the first 13 dominant modes being 98%, one has considered that a reduced model of order 13 would be sufficient to reproduce the behavior of the system.

With G(i,i)=E(i,i)/ ΣE(i,i), the expression of E(i,i) is as follows, r depending the chosen typical solicitation (r=1 for Heaviside pulse, r=0 for Dirac pulse):

$$E(i,i) = -\frac{\sum_{l=1}^{q} C_{li}^2 \sum_{k=1}^{p} B_{ik}^2}{2\lambda_i \lambda_j^{2r}} \tag{7}$$

The [B] matrix results as a partition of the command matrix [G] of the original model.

Matrices [C] and [D] are determined, in the approach retained in this work, by the minimization of errors between outputs of the complete model and those of the reduced model. The direct transmission matrix [D] is obtained by writing the steady-state energy balance of the complete model and the reduced one.

The reduced model is therefore represented by quadrupling matrixes [A] (dim(13,13)), [B] (dim(13,3)), [C] (dim(3,13)) and [D] (dim(3,3)) that represents the space-state model. These matrices are calculated for limit conditions that allow us to

connect the model to other components of the thermal system (control system, building, etc .) in TRNSYS software environment.:
- null flows on axes of symmetry (Neuman homogeneous),
- temperature imposed in surface (Dirichlet conditions),
-coefficient of heat exchange between the heating fluid and the slab $h_f = 5000 Wm^{-2}K^{-1}$

The [A], [B], [C] and [D] matrices once calculated, are stored in a file and characterize then 1m length of tube of the heating floor configuration chosen . We can therefore obtain to this stage a reduced model library of heating floors.

3. The new TRNSYS type of “heating floor”

TRNSYS[6] is a simulation software; its current applications are made especially on the building thermal and energy system analysis. It is a modular software, each component or subsystem being modelled in the form of a module (“type”). It is very convenient to develop new models; it allows the access to the source code, written in FORTRAN and to include new modules by connecting them with others by inputs and outputs. The simulation is described thanks to a file of simulation, called “deck”, in which are established connections between modules and their time function. The program iterates between the “types” so that the outputs of one subsystem become the inputs for others, until the required error is achieved.

The new created “type” is an interface between the model developed using HEAT2 [1] and the “type 19 ZONE” of TRNSYS. Thus, it has been conceived to provide the heat flow of a non-ASHRAE wall for a building having the “type 19” model.

Using the four matrices from a file library, the “type heating floor” achieves the time integration of the equation of state (5) and the estimation of heat flux calculation as given by (16). If we use the exponential matrix for the integration of (5) and we assume that U(t) is constant between t and t+Δt, we have :

$$X^{n+1} = [e^{A\Delta t} - I]\, A^{-1} B U^{n+1} + e^{A\Delta t} X^{n} \qquad (8)$$

with:
$X^n = X(t)$: state vector at time $t = n.\, \Delta t$
$X^{n+1} = X(t + \Delta t)$: state vector at time $t + \Delta t = (n+1)\, \Delta t$.
I : unit matrix
U^{n+1} : input vector at time $t + \Delta t$.

For the starting the computation for “type heating floor” a steady-state simulation is used, with:

$$X^{0} = [A]^{-1} B U^{0} \qquad (9)$$

For the simulation of the thermal behavior of the heating floor, in given meteorological condition, we have considered the subroutines from TRNSYS library: weather report data, solar radiation processor, zone and types of exploitation, connected in the framework of a “deck”, written according to figure 3. Practically, a uniform distribution of the temperature of the floor (or the ceiling) is considered.

Figure 2 gives the block diagram usually used for describing a type in TRNSYS:

Parameter:
P(1) : The length of the pipe
Inputs
U(1) : Temperature of the lower face of the floor
U(2) : Heating water temperature
U(3) : Temperature of the upper face of the floor

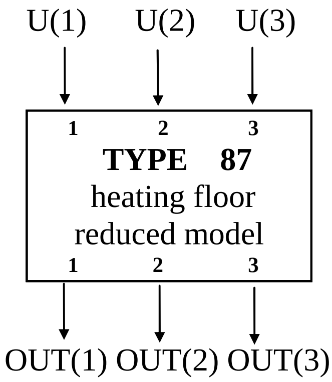

Fig.2 Block diagram of type 87.

A "type control system" delivers a temperature of the heating water correlated with the outdoor temperature (open loop control). For a constant water flow, the temperature of the hot water has a linear variation according to the external temperature. Another control strategy that we have considered supposes a variation of the temperature of the fluid according to the temperature equal the external air, that integrates the solar radiation. Coupling the type «heating floor» with the «building» one give some problems of solutions' stability, especially when the "mode 2" of functioning of the building model (free variation of the internal temperature of the zone) and for time steps smaller than an hour.

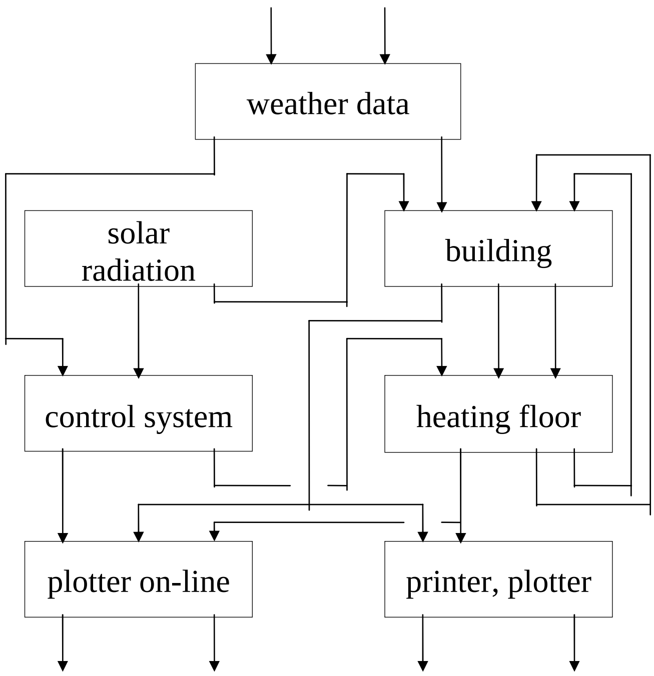

Fig.3 Block diagram of the deck.

An analysis of the subroutine has pointed out that the internal iteration of this "type", achieved for wall temperatures is in accordance with the convective heat flow of the room that satisfies the internal balance, without taking into account the changing in heat flow due to a wall defined as a "non ASHRAE wall" (floor and possible ceiling). To eliminate this problem, we have modified the "TYPE19 ZONE" giving priority to external iterations in comparison to internal iterations, which are still necessary. Acting in this way, we have eliminated all convergence and stability problems of the building model coupled to the reduced model of the heating floor. Note that, despite these modifications, it is always recommended to make simulations with a time step higher than 0.125h. With shorter time step, the TYPE19 has a bad functioning. This is linked to the utilization of the z transform, particularly to the number of coefficients and to the time step (1h) used to calculate them.

4. Simulation and results

Several simulations have been achieved for an apartment of 72 m ^{3} in volume, situated on one floor of a building. The south wall has a glazed surface of 7,5m². The internal and external walls are typical ASHRAE ones and they have been chosen from the library of the software. The floor and the ceiling are described by the presented reduced model, the ceiling being the lower surface of the floor of the apartment above.

The used meteorological file corresponds to the first days of the month of January of Lyon (France). The external temperature oscillates around 0°C.

The temperature of the hot water has been controlled, being kept constant according to simulations. The extreme controlled temperature (corresponding to the external basis temperature) has been chosen so that the temperature of surface of the floor does not exceed 28°C.

The results of the variation of the temperature of the room are given in fig. 4. Data correspond to:

Test 1: heating water temperature controlled by the outdoor temperature;

Test 2: heating water temperature controlled by the outdoor equivalent temperature;

Test 3: constant heating water temperature.

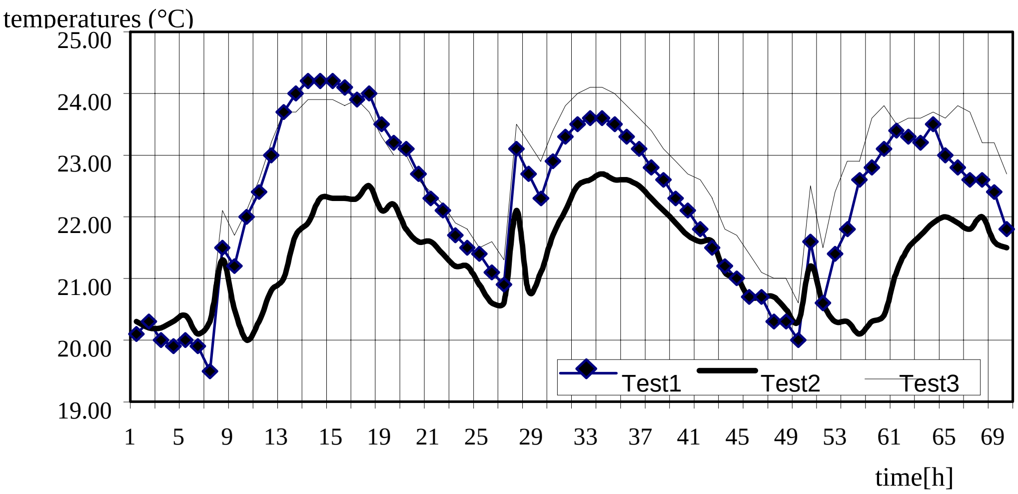

Fig.4 Zone air temperature.

One can notice that the room air temperature has almost the same shape in all cases. The variation of the temperature has a smaller amplitude as the control takes into account to the solar radiation by means of an equivalent temperature.

For above described situations, the total thermal flow (convective and radiative) emitted by the heating floor is represented in fig. 5. Negative flows are those entering the floor and they correspond to hours of high solar radiation. One can see that the consumption of lower energy takes place in the case in which the control is considering the outdoor temperature and the highest one is when there is no control at all.

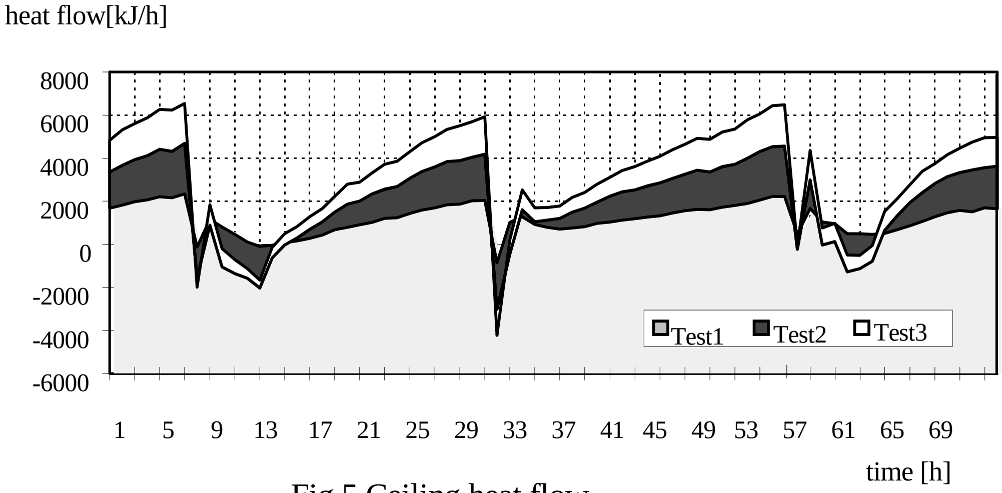

Fig.5 Ceiling heat flow.

To emphasize the interest of the floor heating in a cold climate, we have chosen, for the same building, the climate of Denver (USA). In order to not exceed the temperature of 24°C for the surface of the floor, the maximal temperature of the hot water has been fixed to 50°C. The functioning in mode 2 of the "type building" shown that the internal temperature oscillates around 20°C. In mode 1, for a fixed required temperature, the software gives an output flow needed to insure this temperature. The fig. 6 shows the flow emitted by the floor (hffloor) and the extra flow (hfair) required to maintain an internal temperature of 20°C. We can see that this flow is in the order 300 W, what shows the possibility of utilization of the floor heating in this situation.

The computing time devoted to the simulation has been divided approximately by ten for all processed cases and differences with the complete model are generally less than to 1%.

6. Conclusions.

The study gives an interesting example of the utilization of reduced models of thermal subsystems and its integration in a simulation software. It is a typical case for which the reduced model presents great advantages concerning calculation times and the processing of great thermal system.

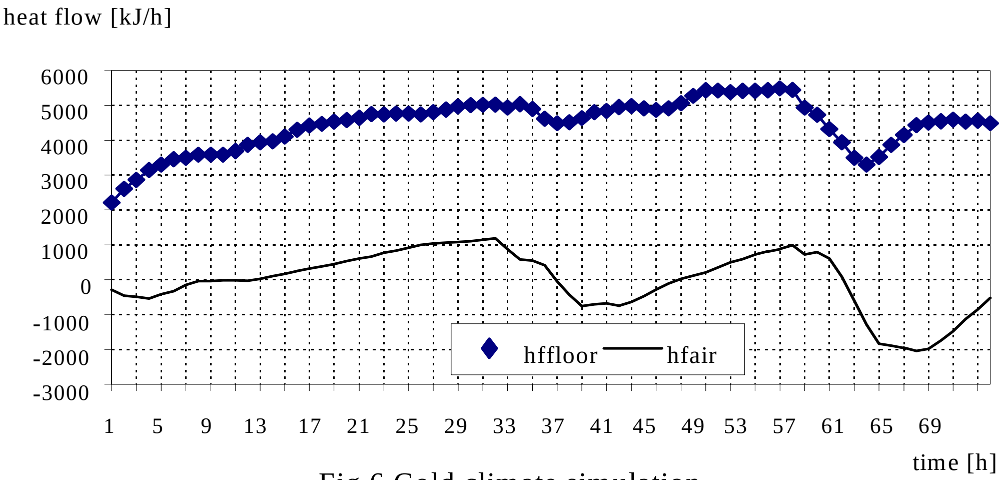

Fig.6 Cold climate simulation.

The heating floor module that we have developed shows the possibility to realize a connection between specialized software, such that HEAT2 and TRNSYS by bringing interesting new elements for modelling components that are not yet in the library of TRNSYS.

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