Simulation of Transient Response of Floor-Heating Systems for Middleastern Construction Materials and Style

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(Main Topic:-Modelling and Measurement/Application to Domestic Building)

ABSTRACT

A mathematical model of transient response of floor heating systems to ambient temperature variation is developed based on equivalent thickness method. The model is then resolved numerically using Rungge-Kutta technique and results are presented for various floor and screed materials and styles used in Jordan and the Middleast. Search is then conducted for the optimum configuration with lowest time constant. Furthermore, the effect of various controlling techniques on transient response is then simulated. The numerical results of time constants are in good agreement with practical observation both in Jordan and Europe. A comparison between results for Jordanian and European constructions is conducted for different temperature settings and pipes spacing. Such investigation was not performed previously and is of great importance to Engineering applications.

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LIST OF SYMBOLS

αL	Global heat transfer coefficient from floor surface (both radiation and convection) (W m^2/°C).
ρi	Density of each material in the screed (kg/m^3).
ρp	Density of pipe material (Kg/m^3).
λi	Therma conductivity of each material in the screed (W/m°C).
λp	Therma conductivity of each material in the screed (W/m°C).
Af	Floor area (m^2).
Aw	Wall area (m^2)
Cp	Equivalent therma capacity of the screed per unit area (J/Kg m^2°C)).
Cpa	Thermal conductivity of air (J/(Kg °C)).
Cpi	Thermal capacity of the each screed material per unit area (J/(Kg m^2°C)).
Cpw	Thermal conductivity of water (J/(Kg °C)).
Di	Pipe inner diameter (m).
Do	Pipe outer diameter (m).
$\overset{o}{m}$	Mass flow rate of water in heating pipes (Kg/sec)
qc	Convective heat output from the floor (W/m^2).
qrad	Radiation heat output from the floor (W/m^2).
qL	Total heat output from floor (W/m^2).
Rk	Total thermal resistance of structure below heating pipes (m^2 °C/W).
RL	Total thermal resistance of structure above heating pipes (m^2 °C/W).
s	Spacing, distance between heating pipes (m).
tA	Ground temperature (°C).
Tas	Screed temperature at center line of heating pipes (°C).

TL	Floor surface temperature (°C).
Ti	Room air temperature (°C).
to	Temperature of heating pipes external surface (°C).
Twi	Inlet temperature of water to heating pipes (°C).
Two	Outlet temperature of water from heating pipes (°C).
Tamb	Outside ambient temperature (°C).
Tw	Wall temperature (‘C).
U	Heat transfer coefficient (W m^2/°C).
Uav	Average heat transfer coefficient for the room’s walls (W m^2/°C).
UMRT	Unheated Mean Radiant Temperature (°C).

$UMRT = (\Sigma AwTw) / (\Sigma Aw)$.

I- Introduction

The use of floor heating (F.H.) systems in Jordan and other Middle Eastern countries is increasing by an accelerating rate. Although the idea of panel heating is not quite new to the area, since such heating systems can be found in ancient palaces, the principle is recently being rediscovered by local architects and heating designers. The many advantages of floor heating (F.H.) are found to be suitable for the region's climate especially since there is no sever weather changes. As such no complicated control mechanisms are needed. The comfort provided by this radiative heating method is being appreciated by Jordanian house owners while the development of plastic thermal pipes made F.H. systems affordable to moderate size residences.

Nonetheless, the designing of F.H. systems is not a straightforward procedure. Compared to the designing of the traditional central heating systems using radiators, F.H. is much more complicated. For traditional systems, once the heating load is calculated the selection of radiators is a matter of choice, while for F.H. systems the designer has to design the radiator, which in this case is the floor and the pipe mesh. In fact when the system was first introduced in Jordan designers relied totally on European based charts to select the spacing between pipes without taking into consideration the variation of floor structure and construction materials between Middle Eastern and European practices. That resulted in many fault designs when the system was first introduced. Although designers modified their practice by trial and error procedure, it was only recently that Al-Maaitah [1] have put forward a complete set of design charts for local floor structure and construction materials. He showed that when the required spacing between pipes is 35 cm for European floor and screed structure and materials, the same conditions require a maximum spacing between pipes of 25 cm when Jordanian structure and materials are used.

However, the calculations made in Ref. [1] were for steady state situation. On the other hand, a well known problem of F.H. systems is their slow time response due to the fact that they poses thermally heavy structures. Variation of system configuration and materials properties would alter the time response of these systems. Nevertheless, experimental data of such variation is lacking since it is quite expensive to build and rebuild F.H. systems. Moreover, not many theoretical investigation of the system is available in the literature due to the fact that a complete model of these systems is quite complicated. As such, more simplified models have to be constructed. Mazza and Oliaro [2] introduced a model based on electrical systems analogy. They did not produce any calculations and their model does not take the interaction between water temperature and screed temperature. In fact their model requires heat supply by heating water to be consider as an input. However, they presented an excellent procedure to calculate the equivalent thermal capacity of the screed which will be used in the present investigation; more recently, Hayari [3] attempted to build a time step model to calculate transient response for Jordanian constructions. That work suffered from many numerical and analytical mistakes along with singularity problems. The time step in Ref. [3] was too large and variation of inlet water temperature with time is not considered.

In the present work we present a simple, yet sound, model to simulate transient response of F.H. systems. The present model takes into consideration the interaction between screed and water temperature along with the room's atmosphere temperature. As such, prediction of room’s air temperature, floor surface temperature, and water outlet temperature can be conducted. The effect of spacing between pipes and floor structure is investigated for various ambient and initial conditions.

II- Mathematical Model

Consider F.H. screed similar to the one presented in Figures 1.a or 1.b. The configuration presented in Figure 1.a is for European screed and floor structure while that of Figure 1.b is for Jordanian one. Nonetheless, the model presented here is of general nature and is valid for any type of screed and floor structure. The model is based on heat balance between supply and return water from one side, and heat absorbed by the room’s atmosphere passing through the screed and floor materials from the other side. Heat is transferred from floor surface to room atmosphere in two modes; namely radiation and convection. Nearly 52% of heat is transferred by radiation while around 48% is by convection. Here we present two methods for the calculation of heat transfer from floor surface to room’s atmosphere: the first is the ASHRAE method [4] which distinguishes between the two modes of heat transfer as follows;

Heat transfer by radiation per floor area (qrad) can be given by the following formula [5]

$$qrad = 0.4476[\{(1.8\ TL+492)/100\}^{4} - \{(1.8\ UMRT + 492)\}^{4}] \qquad (1)$$

where TL is the floor surface temperature in (°C) and UMRT is the unheated mean radiant temperature of the space defined as

$$UMRT = \Sigma tw\ Aw/\Sigma Aw \qquad (2)$$

Tw is wall temperature (°C)

and Aw is wall area (°C).

Moreover the convective heat transfer per floor area (qc) is given by [5]

$$qc = 4.596(TL-Ti)^{1.12} \qquad (3)$$

where Ti is the room’s atmosphere temperature (°C).

The second method which will be used in the present analysis is the DIN standard method [6] which lumps the two modes of heat transfer in one equation as follows:

$$qL = 8.92\ (TL - Ti)^{1.1} \qquad (4)$$

where qL is the total heat transfer from floor surface per floor area.

Equation (4) can be represented in a different form as

$$qL = \alpha L\ (TL - Ti) \qquad (5)$$

where $\alpha L = (TL - Ti)^{0.1}$ (6)

αL is defined as the global heat transfer coefficient. The presentation of equation (6) is also used in Ref. [7].

Next we will proceed in describing the mathematical model. In this model the following assumptions are made:

- Heat transfer across the screed is one dimensional. This is a fair assumption since pipes are distributed uniformly in the F.H. mesh.
- The change of rate of screed and floor temperatures is on a faster scale than that of the room's atmosphere. This is due to the fact that heat transfer in the screed is by conduction while that to the room's air is by convection.
- The temperature variation cross the screed with time is spatially uniform (i.e. the rate of change of temperature with time does not vary from one point to another across the screed).

Based on these assumptions consider the heat balance of a screed similar to that of Figures 1.a and 1.b over an infinitesimal interval of time dt. If the temperature of any point in the screed is T_j then after infinitesimally small period dt the point temperature will become ($T_{j\,+}$ d T_j). As such, the heat balance over the shown control volume is as follows:

Heat supplied by heating water = Heat absorbed by screed + Heat transferred through upper screed and the floor to room's atmosphere + Heat transferred through lower screed to base underneath insulation`

(7)

Similarly the heat balance for the room's air is as follows

Heat transferred from floor surface = Heat absorbed by room's air + Heat transferred to ambient. (8)

By integrating heat flux over time Equation (7) can be represented as

$$\int \overset{o}{m}\ Cpw\,(Twi - Two)\,dt = Af\,\{ \int (Tas - Ti)/RL\,dt + \int (Tas - tA)/RK\,dt$$
$$+ \int Cp_i\,\rho\,dT \tag{9}$$

For definition af all variables please see the list of symbols.

By taking the limits of this integral over an infinitesimal time period (from t to t+dt) the integrands can be considered as constants. Rearranging the resulting equation and noting that dT/dt is assumed to be the same for all points of the screed the following equation results:

$$\frac{dTas}{dt} + Tas\,\{1/(Cp\,RL) + 1/(CP\,RK)\} -$$
$$\{Ti/RL + tA/RK + \overset{o}{m}\,Cpw(Twi - Two)/Af\}/Cp = 0 \tag{10}$$

The equivalent thermal resistance of the upper structure is given as

$$RL = 1/\alpha L + \Sigma di/\lambda i + RP \tag{11}$$

Where λi is the thermal conductivity of the material of each layer in the upper screed and di is the thickness of each layer.

Furthermore $RP = 1/(2\pi\lambda p) \ln(Do/Di)$

Similarly the equivalent thermal resistance of the lower screed structure is given as

$$RK = \Sigma\, di/\lambda i + 1/(2\pi\lambda p) \ln(Do/Di) \tag{12}$$

On the other hand, the equivalent thermal capacity of the F.H. screed and floor structure is given by Ref. [2] as

$$Cp = (dl - \pi\, s\, Do^2 - Di^2/4)\, \rho l\, cpl + \pi\, (Do^2-Di^2)\, \rho p\, Cpip + \Sigma\, \rho i\, di\, Cpi \tag{13}$$

In a similar fashion equation (8) can be represented as

$$Af\, \alpha L\, (TL - Ti) = (Af + Aw)\, Uav\, (Ti - Tamb) + \rho a\, Cpa\, dTi/dt \tag{14}$$

By discritizing the term dTi/dt using finite deference it becomes

$$Dti/dt \approx (Ti\text{\`{}} - Ti)/dt \tag{15}$$

where Ti\` is room's air temperature after dt interval of time.

Then equation (14) becomes

$$Ti\text{\`{}} = dt\{Af\, \alpha L(TL-Ti) - (Af + Aw)\, Uav(Ti - Tamb)\} / \rho a\, Cpa + Ti \tag{16}$$

For this model to be mathematically well posed the values of Twi and Two need to be defined. The variation of water inlet temperature with time depends, among many other factors, on the Boiler capacity. In the present model we assume this variation to be linear with time until (Twi) reaches a final temperature (Twf). It is assumed that boiler outlet temperature reaches Twf after 30 minutes. That is

$$Twi(t) = \begin{array}{ll} Twoi + 0.02\, t & \text{if } Twi \langle Twf \\ Twf & \text{if } Twi = Twf \end{array}$$

where Twoi is the initial water temperature.

On the other hand, the outlet water temperature from F.H. mesh (Two) can be determined once the temperature of the pipe external surface (to) is found (see Ref. [7]). It can be shown that

$$(Two + Twi)/2 = to + [\, (1/RL + 1/RK)\, RL\, \alpha L\, (TL-Ti) - (tA-Ti)/RK]\, s\, Rp \tag{18}$$

Approximating (to) by using steady state analysis we get

$$to = RL\ \alpha L\ (TL-Ti)/\eta + Ti \tag{19}$$

Where η is the fin efficiency defined as

$$\eta = 2\tanh(k\ s/2)/(k\ s) \tag{20}$$

and

$$k = \sqrt{2/\pi}\sqrt{\{(1/RL 1/RK)(\lambda l\ Do)\}} \tag{21}$$

Moreover, the relation between to and Tas can be defined as [7]

$$to = (Tas-Ti)/\eta + Ti \tag{22}$$

The model described by equations (7) - (22) is now defined. The procedure by which these equations are solved is explained hereafter.

III-Solution Procedure

To apply the model described in section II certain parameters and initial conditions should be specified. The floor and exposed walls area are two inputs to the model. The average heat transfer coefficient of the rooms exposed structure must be specified along with ambient temperature. The present work is concerned with Jordanian buildings which according to Jordanian codes [8] must have a maximum U value for the ceiling of 1 W/m² °C while that of side walls (including windows) is 1.8 W/m² °C. Hence, we took the average value of U (Uav = 1.4 W/m² °C). An other input parameter is the mass flow rate of water in the F.H. mesh m. This is defined by defining the pipe inner diameter Di and the velocity of water in the pipes. For residential buildings this velocity has a lower limit of 0.6 m/sec and an upper limit of 1.2 m/sec, in the present calculations, however, we chose this velocity to be 0.9 m/sec. In addition to that pipes dimensions and spacing between pipes need to be specified along with material thermal properties which can be given in references like reference [9]. Next the problem of predicting the time response of Ti, Tas, TL, and Two is resolved as follows:

1. The initial temperatures of screed, floor, room's air, and water in pipes are set.
2. After a time step dt = 0.5 sec the new inlet water temperature (Twi) is calculated from equation (17).
3. Equation (10) is then resolved using fourth order Rungge Kutta procedure and (dt) of 0.5 sec hence the next Tas at time = t + dt is calculated.
4. The values of (to) and (TL) are updated using equations (22) and (21) respectively.
5. The new value of (Ti) is then calculated using equation (16). Consequently the new value of (Two) is calculated using equation (18) and the updated values of Tas, Ti, TL, Twi, and RL.

6. The calculation for the next time step is then conducted by returning to step 2 where all the updated values are now the initial values.

IV-Results and Discussion

The mathematical model described earlier can be used to simulate the transient response of a space to floor heating. Although the model presented can be used for any space geometry, certain configurations should be specified to be implemented in the model. Here we will consider a room of 20 m² area, its height is 3 m and its ceiling is exposed to ambient atmosphere. Furthermore, the area of the exposed side walls is 20m².

The results presented are for two types of floor and screed constructions, the first is the construction usually used in Jordan and the Middle East referred to as the Jordanian screed and floor. The other is the one used in Europe as explained in reference [7]. As it is clear from Figure 1 that Jordanian and European screeds are nearly similar while the main difference is in floor construction. Jordanian floor is at least 35 mm thicker than the European one. Moreover, the insulation of European screed is nearly three times that used in Jordan. Materials properties and pipes configurations are listed in Table 1.

After specifying the initial temperatures of screed, water, and room atmosphere in addition to the ambient temperature, the program can be run to predict the temperature variation with time for floor surface, room atmosphere, screed at pipes center line, and water output. Figure 2 shows the variation of floor surface and room atmosphere temperature with time when spacing between pipes (s) is 25 cm, initial water and screed temperature is 5 °C, ambient temperature is 3° and room temperature is 4°C. Water inlet temperature (Twi) is also shown in the Figure. For Jordanian construction the transient response is relatively slow. The room's air reaches an acceptable temperature of around 20°C after approximately 270 minutes (4.5 hours). This is in agreement with practical observation for Jordanian residences. The final screed temperature is around 26°C and water outlet temperature is around 40°C. This also agrees with practical observation for temperatures in Jordanian buildings of similar configuration.

For European floor and screed construction, on the other hand, Fig. 3 shows that for the same configuration and conditions the time response is considerably faster. The rooms air temperature reaches 20°C after nearly 130 minutes. That is due to the fact that the thermal inertia of the European construction is considerably less than that of the Jordanian screed and floor construction. The time constant in this case is also in agreement with observations reported in references [7]. The final floor temperature is nearly 27°C while the final room temperature is around 21°C. Although for these calculations it is assumed that the room has the same Uav for European and Jordanian floor and screed constructions, actually the average U factor set by European standards is considerably less than that of the Jordanian one.

When spacing between pipes is reduced to 20 cm, Figure 4 shows that the time response for Jordanian floor is considerably improved for the same conditions of Figure 2. The room atmosphere temperature reached 20°C after 200 minutes. The final floor temperature reached 27 °C while the final room temperature reached 21°C. When a European construction is used instead of Jordanian one the time required for the room's air to reach 20°C is reduced to 115 minutes as shown in Figure 5. It is clear that time response for European construction is not affected by reducing (s) from 25 to 20 cm as it is the case for Jordanian construction. The final floor surface temperature is 28°C while that of the room's atmosphere is 22°C.

When the operation is started at cooler initial conditions and cooler ambient temperature the time response is much slower as shown in Figure 6. In this case the initial temperature room's atmosphere is 1°C while the ambient temperature is 0°C. The spacing between pipes is 20 cm (same as that of Figure 4). The room's air reached 20°C after nearly 270 minutes. The final temperature of the floor is 26°C and the room atmosphere never exceeds the neighbourhood of 20°C. For European screed of 25 cm spacing and the same conditions of Figure 6, the results shown in Figure 7 demonstrate that the room reached a temperature of 20°C after 200 minutes. For the same conditions and pipe spacing of Figure 7, Figure 8 shows that using Jordanian floor would cause the room's atmosphere temperature to have an upper limit of 19°C with a very slow time response. As mentioned in Table 1 the final inlet water temperature in Figures 2-8 is 45°C. For the sake of comparison we show in Figure 9 the transient response of floor heating system for the same (cold) conditions of Figure 8 and same spacing but with (Twf) of 49°C. In this case the room reached a final temperature of 20°C after 220 minutes.

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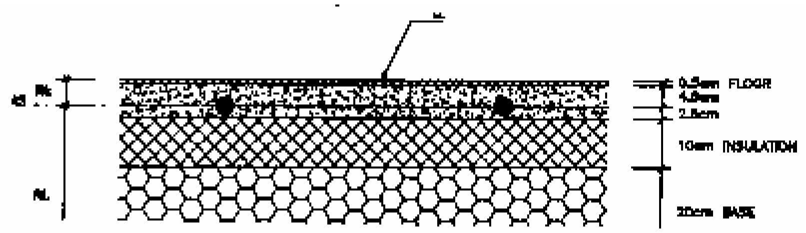


Figure 1.a. European floor and screed construction.

Figure 1.b. Jordanian floor and screed construction

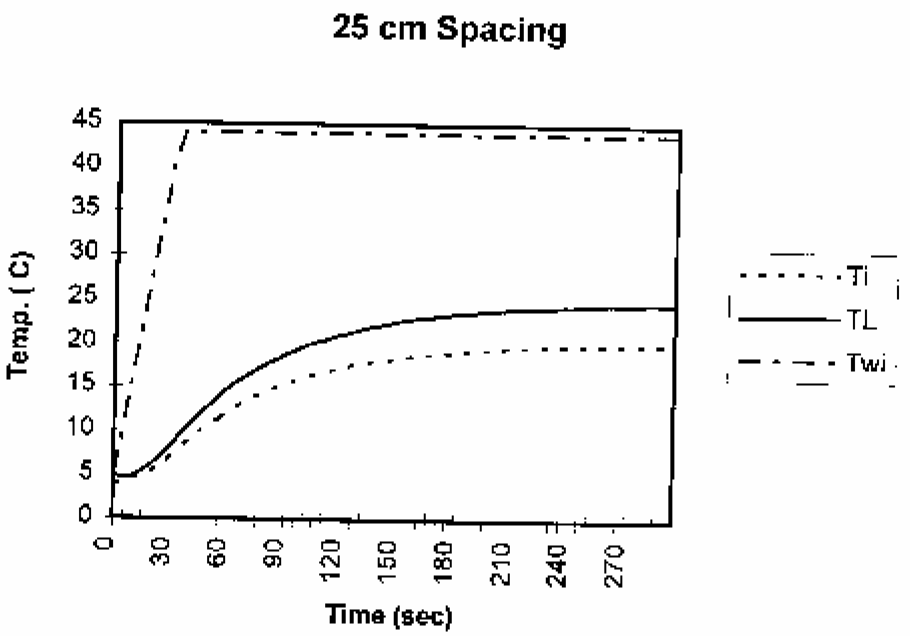


Figure 2. The variation of Ti, TL and Twi with time for Jordanian floor construction and 25 cm spacing between pipes. Ambient Temperature is 3°C, screed initial temperature is 5°C, and initial room temperature is 4°C.

Eurobean Screed

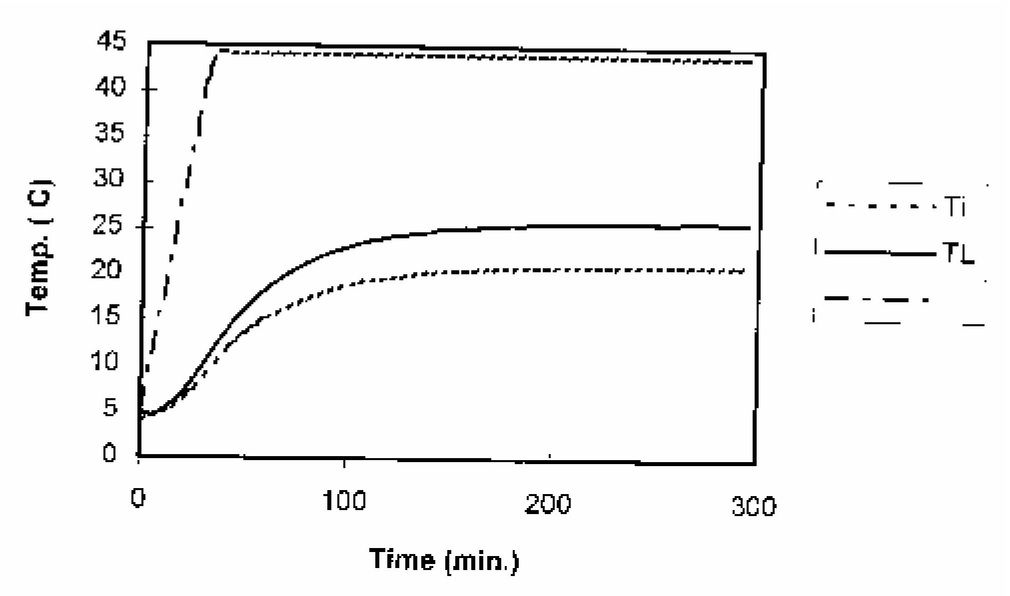
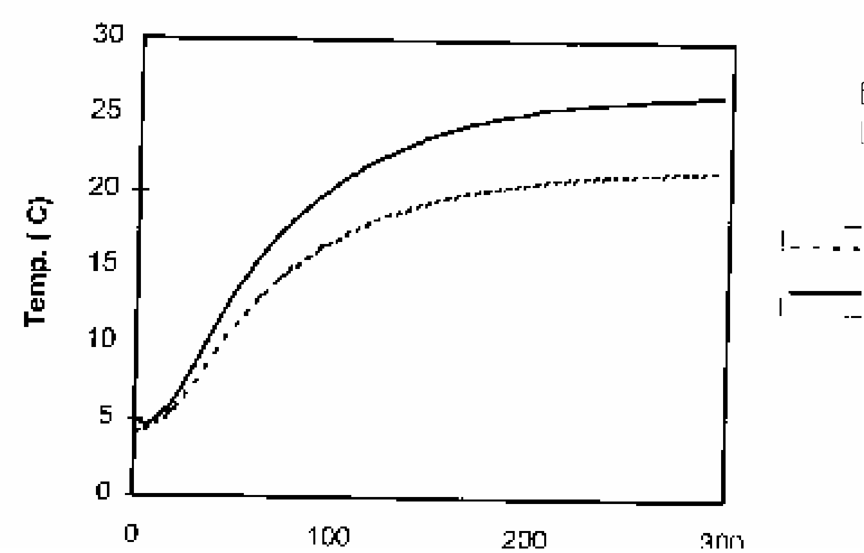


Figure 3. The variation of Ti, TL and Twi with time for European screed construction and 25 cm spacing between pipes. Ambient Temperature is 3°C, screed initial temperature is 5°C, and initial room temperature is 4°C.



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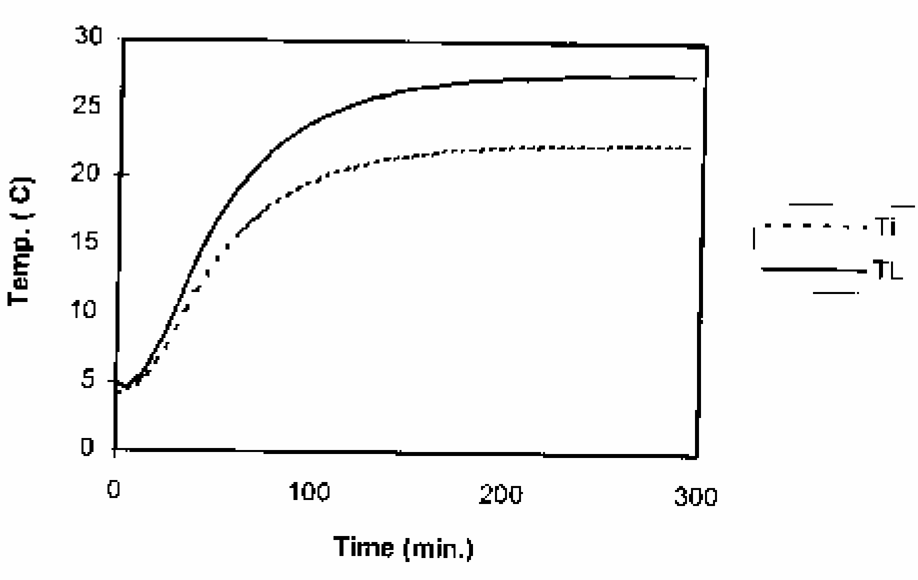

Figure 5. The variation of Ti, TL and Twi with time for European floor construction and 20 cm spacing between pipes. Ambient Temperature is 3°C, screed initial temperature is 5°C, and initial room temperature is 4°C.

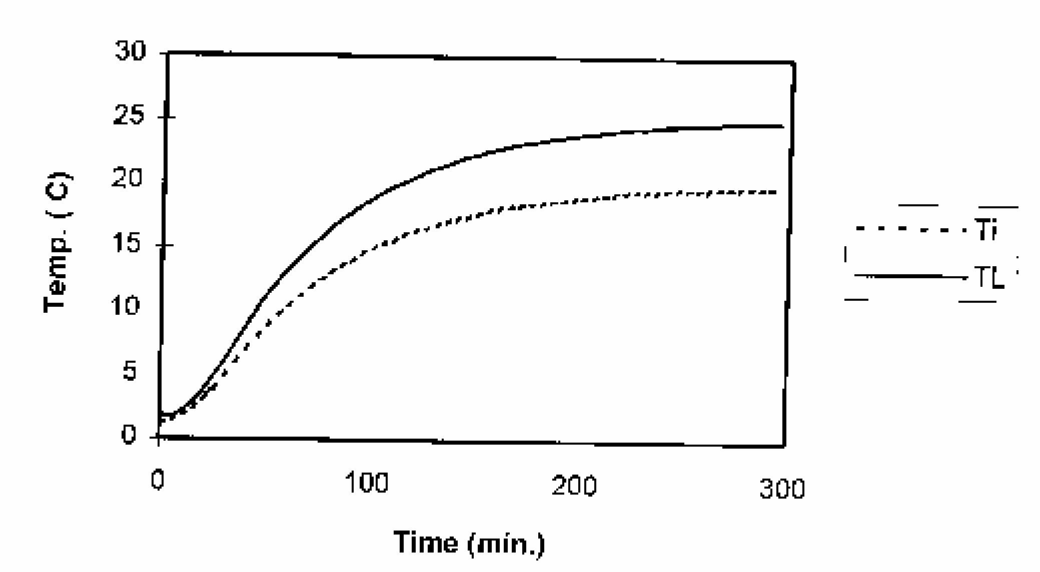

Figure 6. The variation of Ti, TL and Twi with time for Jordanian floor construction and 20 cm spacing between pipes. Ambient Temperature is 0°C, screed initial temperature is 2°C, and initial room temperature is 1°C.

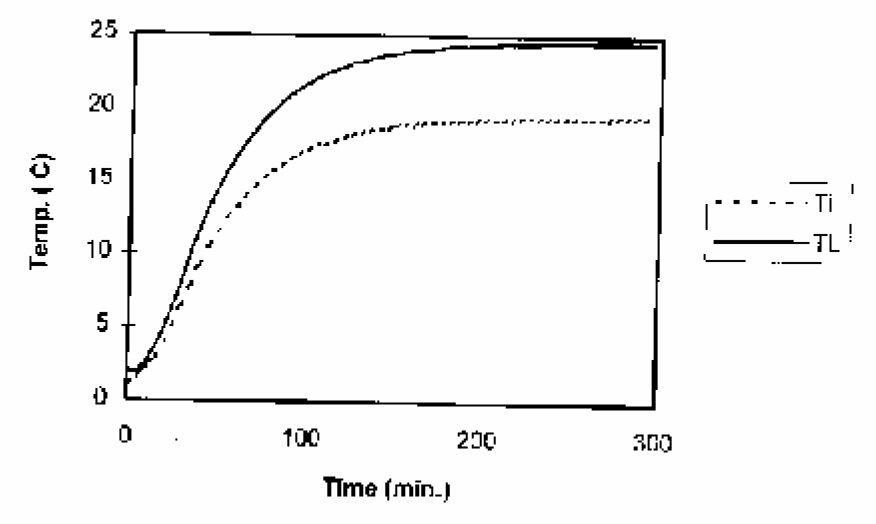

Figure 7. The variation of Ti, TL and Twi with time for European floor construction and 25 cm spacing between pipes. Final Twi is 45°C, ambient Temperature is 0°C, screed initial temperature is 2°C, and initial room temperature is 1°C.

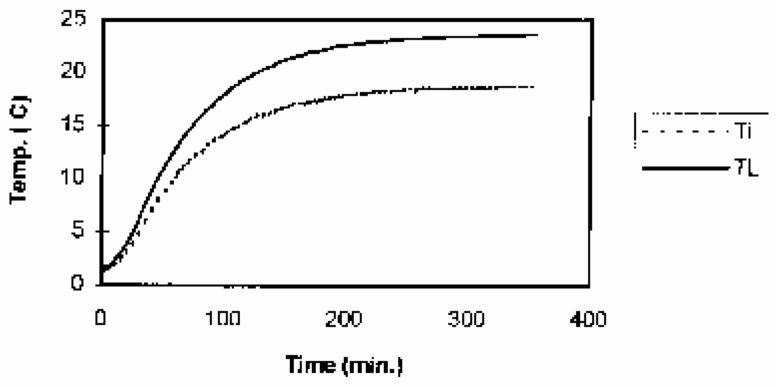

Figure 8.0. The variation of Ti, TL and Twi with time for Jordanian floor construction and 25 cm spacing between pipes. Final Twi is 45°C, ambient Temperature is 0°C, screed initial temperature is 2°C, and initial room temperature is 1°C.

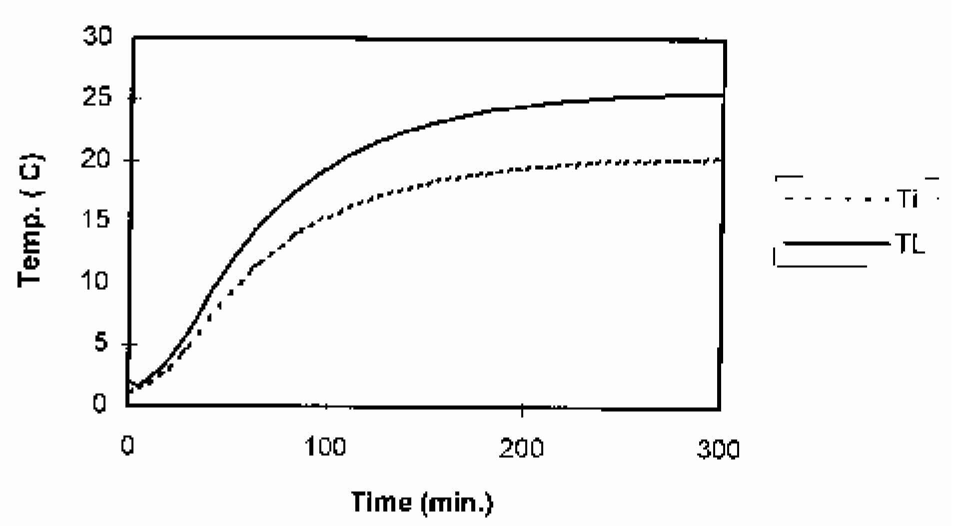

Figure 9. The variation of Ti, TL and Twi with time for Jordanian floor construction and 25 cm spacing between pipes. Final Twi is 49°C, ambient Temperature is 0°C, screed initial temperature is 2°C, and initial room temperature is 1°C.