Quantifying ventilation rates in heterogeneous rooms based on point measurements of carbon dioxide

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SUMMARY

Introduction: Indoor air quality (IAQ) is increasingly accepted as a leading factor in human health, and the ventilation of our indoor spaces is a key modifier of IAQ as the principal means by which indoor pollutants are diluted. Knowledge of the ventilation rate is essential for understanding and modelling our indoor environment, yet quantifying the ventilation rate for regular operational spaces remains a challenge. Measurements of carbon dioxide (CO_2) concentrations indoors are established as a reasonable general indicator of IAQ in occupied spaces and are the basis of many regulations and standards, e.g. ASHRAE (2022). We present a robust method to better exploit point CO_2 measurements to estimate the daily mean per-person ventilation rate $\bar{Q}_{pp}$, requiring very limited other contextual information.

Current widely applied methods rely on special-case solutions to the CO_2 mass balance equation (Sherman, 1990; Batterman, 2017). These methods include: the decay rate, for which the space must be unoccupied, and the ventilation rate is assumed constant during the decay; and the steady-state method, for which the balance between the source and ventilation must remain constant for a sufficiently long time so concentration becomes constant. Both methods further assume the room to be well-mixed. The present method overcomes some limitations of these methods since it makes no assumptions regarding the ventilation provision throughout the day, nor requires the room to be in a steady state, nor the air within to be well-mixed. Importantly, the method facilitates ventilation estimates in operational spaces during normal use, not requiring the room to be unoccupied. This is significant since ventilation provision can significantly differ during occupied and unoccupied periods (Godwin, 2007).

Theory & Methodology: Beginning with the general CO_2 mass balance equation for a room, it can be shown that the daily average per-person ventilation rate $\bar{Q}_{pp}$ can be expressed precisely as

$$\bar{Q}_{pp} = K \frac{F_{\mathrm{occ}} \overline{\langle G \rangle_N}}{\overline{C_X}}, \qquad (1)$$

where F_{occ} is the fraction of time which the given room is occupied during the day (from first occupancy until CO_2 decays to background at the end of the day), $\overline{\langle G \rangle_N}$ is the time average of the per-person average CO_2 generation rate, and $\overline{C_X}$ is the time averaged excess CO_2 concentration at the sensor location, which is measured. Details of the dimensionless coefficient K and the full derivation of Eq. (1) are given in Finneran & Burridge (2024). The coefficient K captures effects of ventilation rate variability during the day, and the effects of heterogeneity of CO_2 in the room. Heterogeneity causes a discrepancy between the room averaged excess CO_2 $\langle C \rangle_V$ and that measured by the sensor C_X. Importantly, $K = 1$ if the ventilation rates (total and per-person) are constant, and if the room is well-mixed, but we make no such assumptions. The challenge is that for a given scenario, the coefficient K and the average per-person generation rate $\overline{\langle G \rangle_N}$ are unknown, but we show that the uncertainty in these parameters is sufficiently low, enabling meaningful estimates of $\bar{Q}_{pp}$ with very little contextual information.

Taking UK secondary school classrooms as a case study, simulations of CO_2 levels in classrooms with a high degree of variability were conducted using Monte Carlo methods. Reasonable probability distributions (informed by available data) were assigned to all relevant parameters, including: the classroom volume, the ventilation rate profile, the occupancy levels and duration, the CO_2 generation rate of occupants, and the CO_2 heterogeneity in the

room. From these randomly generated inputs, the excess CO$_2$ at the sensor location C_X (that would be measured) can be calculated. Approximately 10^7 classroom days were simulated and results of one illustrative example simulation are shown in Figure 1.

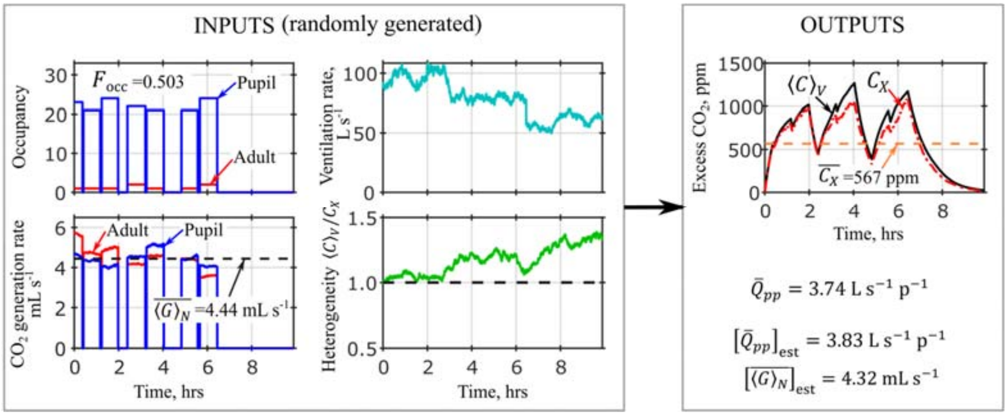

Figure 1: Illustrative example of classroom day simulation. The inputs are randomly generated with data-informed stochastic models. The precise value of $\bar{Q}_{pp} = 3.74$ L s^{-1}p^{-1} is known within the simulation and can be estimated accurately using Eq. (2) as $[\bar{Q}_{pp}]_{\text{est}} = 3.83$ L s^{-1}p^{-1}. Room volume = 190 m^3.

Results & concluding discussion: It was found that the coefficient K is remarkably constrained and close to a value of unity, despite the large variability of input parameters. Uncertainty in K can be reduced with knowledge of spatial CO$_2$ distributions (e.g. using multiple sensors), and/or knowledge that the ventilation rate is reasonably constant (e.g. mechanical ventilation). Also, since classrooms typically contain large numbers of occupants of a similar age, uncertainty in $\overline{\langle G\rangle_N}$ is relatively constrained. These findings provide justification for the estimate of $\bar{Q}_{pp}$ via

$$[\bar{Q}_{pp}]_{\text{est}} = \frac{F_{\text{occ}}[\overline{\langle G\rangle_N}]_{\text{est}}}{\overline{C_X}}, \tag{2}$$

requiring only the measured CO$_2$ levels to calculate $\overline{C_X}$, the occupied time fraction F_{occ}, and an estimate for the average per-person generation rate $[\overline{\langle G\rangle_N}]_{\text{est}}$. An estimate $[\overline{\langle G\rangle_N}\,]_{\text{est}}$ can be informed by knowledge of the age of occupants and classroom activities. The accuracy of this calculation tool, Eq. (2), is shown in Figure 2. For the worst-case scenario of no contextual information (hence $[\overline{\langle G\rangle_N}]_{\text{est}} = 4.32$ mL s^{-1} is used as the average over all occupants and activities) uncertainty is approximately $\pm 22\%$. However, for the high-information scenario (e.g. gathering information of occupants age and room activities, and using multiple CO$_2$ sensors), uncertainty is reduced to approximately $\pm 12\%$, with 95% confidence.

This calculation tool, Eq. (2), can be applied to CO$_2$ data to calculate the average per-person ventilation rate. Uncertainty has been rigorously quantified for the case study of UK secondary school classrooms. It is expected that this method can be extended to many other building types, and it is a challenge for future work to explore uncertainties in other applications.

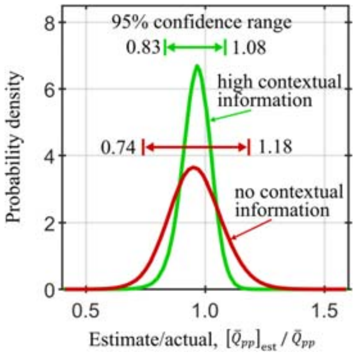

Figure 2: Probability density function for estimated $\bar{Q}_{pp}$ from Eq. (2) compared to the true value, for no and detailed contextual information scenarios

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