Simplified Methods for Combining Natural and Mechanical Ventilation

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ABSTRACT

In determining ventilation rates, it is often necessary to combine naturally-driven ventilation, such as infiltration, with mechanical systems. Modern calculation methods are sufficiently powerful that this can be done from first principles with time varying flows, but for some purposes simplified methods of combining the mechanical and natural ventilation are required—we call this "superposition". An example of superposition would be ventilation standards that may pre-calculate some quantities within the body of the standard. When there are balanced mechanical systems, the solution is simple additivity because a balanced system does not impact the internal pressure of the space. Unbalanced systems, however, change internal pressures and therefore can impact natural ventilation in such a way as to make it sub-additive. Several sub-additive superposition models are found in the literature. This paper presentments the results of millions of hours of simulations of the physically correct solution, which span a broad range of weather, leakage and structural conditions. This wide range of data allows for the comparison of three superposition models from the literature and two new ones. The results showed that superposition errors can be reduced significantly by using the appropriate model(s), from the 20% over-prediction from simple linear addition to 1%, or less.

KEYWORDS

Unbalanced ventilation, infiltration, REGCAP simulation, empirical models, superposition

1 INTRODUCTION

Most homes are ventilated by the form of natural ventilation known as "infiltration", which is defined in ASHRAE Standard 62.2 (ASHRAE 2013) as the "uncontrolled inward leakage of air through cracks and interstices in any building element and around windows and doors of a building". In order to decrease energy consumption, house envelopes are getting tighter. Combined with potential increases in pollutant sources in indoor living environments, this raises concerns for indoor air quality (IAQ). Since people spend an average of 90% of their time inside, more houses are using a mechanical ventilation system to maintain a good air quality.

Infiltration is caused by two driving forces, namely the wind and stack effects. The wind raises the pressure on the windward side of the building, and lowers the pressure on the other sides in proportion to the square of wind speed. The stack effect is due to density differences between indoor and outdoor air. In winter the heated air inside the building is less dense than the cold air outside resulting in pressure differences across the envelope with higher inside pressure at the top of the building and lower inside pressure at the bottom of the building. The reverse happens in summer when the outside temperature is greater than the inside.

If a balanced ventilation system is installed, the impact on the infiltration will not be significant because the balanced system does not change the pressures across the building leaks. As a result the total ventilation rate (Q_t) is simply the addition of the fan flow (Q_f) and the natural infiltration (Q_{inf}).

Unbalanced mechanical ventilation systems modify the indoor pressure of the building, which is interacting with the wind and stack induced flows, making the combination of the flows sub-additive. Exhaust fans depressurize the building which increases the air flow in through the building envelope. The greater the fan flow, the higher proportion of the building envelope experiences inflow. The opposite effect occurs with supply-only systems.

In order to avoid both excessive energy consumption and poor IAQ, it is necessary to predict the total flow rate resulting from the combined natural and mechanical ventilation. This can be done using mass flow balance physical and mathematical models to find the internal pressure that balances the incoming and outgoing mass flows. This approach is powerful but requires many computational inputs and can be too time consuming for some purposes such as ventilation standards or simplified parametric modelling. An alternative is to use a simple empirical model for estimating the total ventilation rate Q_t from Q_f and Q_{inf}. These models are generically called "superposition" models. A few models were suggested and tested a few decades ago but the results are sometimes contradictory and there is no consensus on the best one to be used.

In this study we used the REGCAP air flow mass balance model to simulate millions of hours of the physically correct solution, with a broad range of weather, leakage and structural conditions. Then we compared this data with three superposition models from the literature as well as two new empirical models based on the simulation results. The objective was to determine the uncertainty of existing models and to develop improved models that retain the ideal of simplicity.

2 BACKGROUND

2.1 Previous work on superposition

In the eighties and early nineties a number of models for empirically combining the natural infiltration flow and unbalanced mechanical ventilation were suggested. A summary is presented in the Appendix. However, many of these were optimized for limited situations, such as the Palmiter and Bond (Palmiter & Bond, 1991) method, referred here as the half-fan model, which was developed for stack only natural infiltration.

Li (Li, 1990) tested ten models by comparing them with a flow model over a range of wind speeds (0 to 8 m/s) and temperature differences (-20 to 20°C) with open and closed exterior doors and two different exhaust fan speeds. His conclusion was that the quadrature combination of natural and mechanical ventilation worked best. This result is in agreement with the earlier work of Modera and Peterson (Modera & Peterson, 1985) who also used a mass balance ventilation model.

Field tests with tracer gas measurements by Kiel and Wilson (Kiel & Wilson, 1987) and later by Wilson and Walker (Wilson & Walker, 1990) found that for strong exhaust mechanical ventilation (four times the natural rate), simple linear addition was the most acceptable model. Unlike Li, these studies showed large under-predictions using quadrature. This could be due to different building envelope leakage, weather conditions, leakage distributions and strength of mechanical ventilation but it mainly underlines the necessity of new studies.

2.2 REGCAP model

REGCAP is a two zone ventilation model combined with a heat transfer model and a simple moisture transfer model. The two zones are the house and the attic above it and interact through the ceiling. The ventilation rate is found by determining for each zone the internal pressure required to balance the incoming and outgoing mass flows resulting from the natural and mechanical ventilation driving forces.

The model uses an envelope airtightness measurement (ACH_{50}) and a description of the leakage distribution. The leakage for the home is split between walls, floor, ceiling and open flues/chimneys. In this study the leakage distribution was varied with the number of storeys and the type of foundation. Each leak is defined by its flow coefficient, pressure exponent, height above grade, wind shelter and wind pressure coefficient taken from wind tunnel tests. An iterative method is used to solve the non-linear mass balance equations. The attic temperature is not regulated and will therefore both be affected by the ventilation rate and affect the infiltration flow due to the stack effect. In addition, REGCAP includes models for the HVAC equipment in the home and operates on one-minute time steps. The ventilation and heat transfer models are coupled and the combined solution is also found iteratively. A more detailed discussion of REGCAP, including validation compared to measured field data, was done by Walker et al. (Walker, Forest, & Wilson, 2005).

2.3 Applications

Each simplified model can either be used for forward or inverse calculations. The forward model predicts the total ventilation airflow (Q_t) as a function of the natural infiltration (Q_{inf}) and the fan flow (Q_f), whereas the inverse model gives Q_f as a function of Q_t and Q_{inf}. They can be applied to hourly or annual calculations, which results in four different cases:

- <u>Hourly, Forward Case:</u> for the hourly air change rate prediction; useful for estimating energy loads and needed for relative pollutant exposure calculations.
- <u>Annual, Forward Case:</u> predicting the annual effective ventilation given the effective infiltration and a fixed (or effective) fan flow; for indoor air quality (IAQ) purposes.
- <u>Hourly, Inverse Case:</u> when one wants to vary the fan size each hour to compensate for varying hourly infiltration in order to keep the total ventilation constant.
- <u>Annual, Inverse Case:</u> for finding the fixed fan size that will combine with effective infiltration to produce a desired total ventilation; useful for standards such as the 62.2.

3 APPROACH

3.1 REGCAP simulations

We used REGCAP to create a data based on a wide range of weather and housing conditions. The range of inputs is presented in Table 1 and results in 720 combinations. The number of storey changes but the floor area is constant and equals to 1900 ft² (176 m^2). For each set of inputs, we ran the model for the 525 600 minutes of a year and we hourly averaged the outputs, which results in more than 6.3 million of points of comparison for the superposition models.

Table 1: Range of inputs for the REGCAP simulations

Parameters	Values
Envelope airtightness (ACH_{50})	0.6; 3; 5; 7; 10
Mechanical ventilation type	Exhaust ; supply
Number of storey	1; 2; 3
Foundation type	Slab on-grade; crawlspace; basements
Climate zones	Miami; Houston; Memphis; Baltimore; Chicago; Burlington; Duluth; Fairbanks

We calculated the flow through the exhaust or supply fan (Q_f) according to ASHRAE Standard 62.2 which includes the infiltration credit. We used REGCAP to calculate first the infiltration flow through the envelope (Q_{inf}) due to the stack and wind effect, with no mechanical ventilation operating. We repeated the simulations with supply or exhaust fans operating to obtain the total flow (Q_t). Then we compared the results for each superposition method for combining Q_f and Q_{inf} to Q_t.

For the annual calculations, the fan flow is still the same as it is a constant over the year, but Q_{inf} and Q_t are effective annual average infiltration rates. The effective values differ from the averaged ones. As defined in ASHRAE Standard 62.2, they correspond to “the constant air infiltration rate that would result in the same average indoor pollutant concentration over the annual period as actually occurs under varying conditions”. This annual approach can be particularly useful when one wants to size the fan to the total ventilation required by ventilation standards.

3.2 Simplified models

The equations describing the five simplified superposition models are presented in Table 2.

The three first models come from the literature described earlier. The **additivity** model, which is a simple addition of the flows, is in the current ASHRAE 62.2 Standard, and has been experimentally verified by Kiel and Wilson. **Simple quadrature** is the current model in the ASHRAE Handbook of Fundamentals, and has been verified by both Modera and Peterson and Li. The **Half Fan** model was used in earlier editions of ASHRAE Handbook of Fundamental and has been established experimentally by Palmiter and Bond. For each of these models the forward and inverse forms are equivalent. For all three models, verification was for a narrow range of conditions and the current study aims to investigate their performance over a much wider range of homes and conditions.

In addition we developed other models for this study in order to reduce the uncertainties associated with the existing models. We used the simulation results to approximate a sub-additivity coefficient (Φ) weighting the infiltration contribution to either the total ventilation (forward) or the fan sizing (inverse):

$$Q_t = Q_f + \Phi_{\mathrm{fw}} Q_{inf} \qquad (1)$$

$$Q_f = Q_t - \Phi_{\mathrm{inv}} Q_{inf} \qquad (2)$$

Table 2: Forward and inverse equations of the simplified models compared to the REGCAP results

Model	Forward	Inverse
Additivity	$Q_t = Q_f + Q_{inf}$	$Q_f = Q_t - Q_{inf}$
Simple quadrature	$Q_t = \sqrt{{Q_f}^2 + {Q_{inf}}^2}$	$Q_f = \sqrt{{Q_t}^2 - {Q_{inf}}^2}$
Half-fan	$Q_t = \max\left(Q_f\ , Q_{inf} + \frac{Q_f}{2}\right)$	$Q_f = \min\left(Q_t\ , 2(\mathrm{Q_t} - \mathrm{Q_{inf}})\right)$
Exponential sub-additivity	$Q_t = Q_f + \exp\left(-k_{fw}\frac{Q_f}{Q_{inf}}\right)Q_{inf}$	$Q_f = \mathrm{Q_t} - \exp\left(-k_{inv}\left(\frac{Q_t}{Q_{inf}} - 1\right)\right)Q_{inf}$
Simple inverse sub-additivity (SISA)	$Q_t = \frac{Q_f}{2} + \sqrt{\frac{{Q_f}^2}{4} + {Q_{inf}}^2}$	$Q_f = \mathrm{Q_t} - \frac{{Q_{inf}}^2}{Q_t}$

Table 3: Values of the k coefficient for the exponential model

	Forward	Inverse
Hourly	$k_{fw,h} = \frac{2}{3}$	$k_{inv,h} = 1$
Annual	$k_{fw,a} = \frac{4}{9}$	$k_{inv,a} = \frac{2}{3}$

We empirically developed an **exponential** form for Φ that worked well to reduce errors. The resulting forward and inverse models are not equivalent but have the same limits and trends. We optimized the coefficients k_{fw} and k_{inv} to best approximate the simulation results and we found different values for the annual and hourly data, as shown in Table 3.

We developed a fifth model, SISA, in order to avoid the complications of the exponential function. For the SISA model, Φ is the ratio of Q_{inf} to Q_t for the inverse approach. This ratio is referred to as α, as defined in Equation 3. α is a useful normalization to use for all the models when examining the trends and limits of the modelling errors. The forward version of SISA uses a semi-quadratic formulation:

$$\Phi_{\mathrm{inv}} = \frac{Q_{inf}}{Q_t} = \alpha \tag{3}$$

4 RESULTS

We evaluated the models by comparing the air flow prediction to the one obtained with the simulation. The forward model aims at predicting the total airflow so the error, E_{fw}, is given by:

$$E_{fw} = \frac{Q_{t,model} - Q_{t,sim}}{Q_{t,sim}} \tag{4}$$

In the same way, the error for the inverse model, E_{inv}, is the difference between the predicted and simulated fan flows. It is still divided by the total airflow since a division by a fan flow close to zero would give a significant error but the impact on the actual ventilation rates would be very small.

$$E_{inv} = \frac{Q_{f,model} - Q_{f,sim}}{Q_{t,sim}} \quad (5)$$

4.1 Hourly results

For the hourly data, the high number of points (over six million) requires the use of summary statistics, represented by box-and-whisker plots. We sorted the data into 20 bins by infiltration fraction (α) and each bin is represented by a box. The box widths are proportional to the square-root of the number of observations in the bin. The bottom and top of the box are the first and third quartiles, and the black band inside is the median. The ends of the whiskers represent the minimum and maximum of the data. In our case a box represents in average more than 300 000 points which explains why these values can be quite far from the median. When several parameters are plotted, each of them is identified by a color and the horizontal offset in αbetween them is only for the sake of clarity.

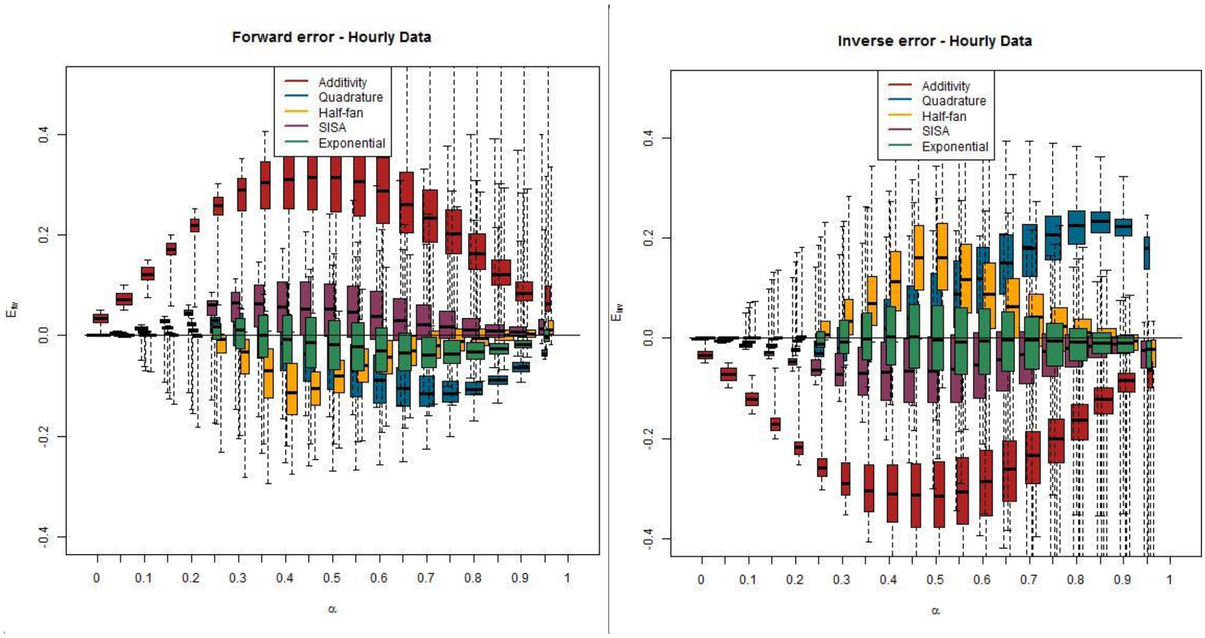

Figure 1: Forward and inverse hourly errors for every model

As shown in Figure 1, an over-prediction of the total ventilation (E_{fw}>0) results in an under-prediction of the fan flow with the equivalent inverse model (E_{inv}<0). We don't observe this for the exponential model since the forward and inverse models are not equivalent. For the additivity model the two errors have simply opposing values, but there is no such symmetry for the other models. The inverse error for the quadrature model reaches higher values than the forward error for high infiltration fractions. In the same way the half-fan model gives a higher peak in the inverse error than the forward one.

4.2 Annual results

For the annual analysis, there is a single result for each of the parameter combinations in Table 1. This reduced number of points (720) allows all the individual results to be shown. Compared to the hourly data, there are less extreme values and no point with α above 0.9. We can observe a gap around α = 0.15, which can be explained by the lack of an intermediate value between 0.6 ACH and 3 ACH in the airtightness levels of the simulated houses.

As shown in Figure 2, the trends are similar to those of the hourly errors. However since they are effective values, Q_t and Q_{inf} are smaller than a simple annual average, unlike Q_f that is constant over the year. As a result we can observe smaller over-predictions but greater under-predictions for the forward models, and the opposite for the inverse models.

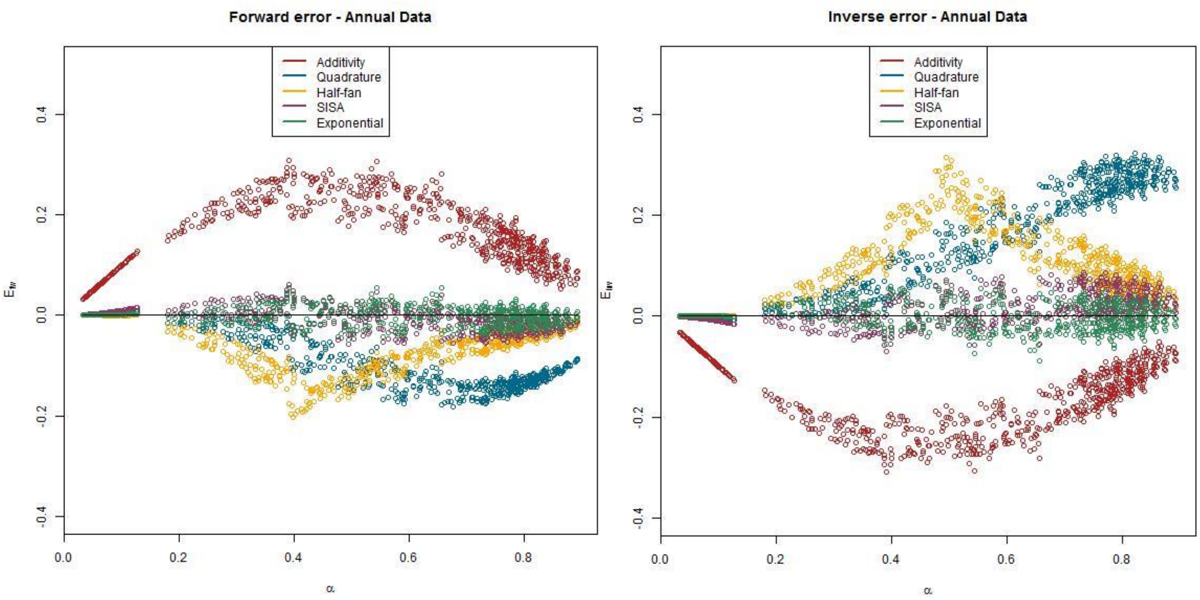

Figure 2: Forward and inverse annual errors for every model

4.3 Discussion

The characteristics of the hourly and annual errors are presented respectively in Tables 4 and 5. The various simulations give results covering a wide range of infiltration ratio (α) but not evenly dispersed, with for example fewer points around α=0.15. In order to compensate for this disparity, we calculated the bias and RMS of the errors for 20 bins of α values, and then we equally weighted them. The bias is the error over the full range of house and weather parameters exercised in this study. The RMS is representative of the error for an individual home and is most useful for most applications – such as sizing fans for an individual home to meet a ventilation standard, such as ASHRAE 62.2. Because of the high number of points, the maximum error is not meaningful for the hourly error. We use instead the maximum median among the 20 groups of data, and the maximum of 90% of the data.

The exponential models always give the best predictions with biases around or below 1%, RMS ranging from 1.5% to 5.5% and maximums around or under 10%. The additivity model is always the worst, with bias and RMS errors of around 20% and maximums above 30%. The quadrature and the half-fan are much better than the additivity, especially for the forward models applied to the hourly data, but with a significant difference compared to the exponential models.

The SISA model is the second best model, and is almost as good as the exponential for the annual data with biases below 1%, RMS around 2% and maximums at 6% and 8.7%. There is no reason to prefer this model to the exponential one for the forward prediction but it has a simpler expression for the inverse prediction. It also has the advantage of not having Q_t as a denominator, which, unlike Q_{inf}, can never equal to zero and may therefore be a good option for calculations determining fan size requirements to meet total ventilation rates.

Table 4: Error on the model predictions for the hourly data

Model	Forward error				Inverse error			
	Bias	RMS	Max. Med.	Max. 90%	Bias	RMS	Max. Med.	Max. 90%
Additivity	20.9%	21.8%	31.5%	34.1%	-20.9%	21.8%	31.5%	34.1%
Quadrature	-3.95%	6.65%	11.6%	13.1%	7.99%	11.0%	23.3%	25.2%
Half-fan	-2.85%	4.78%	11.3%	9.22%	4.20%	7.65%	16.1%	15.8%
Exponential	-1.15%	4.01%	3.78%	8.03%	-0.61%	5.57%	2.31%	11.3%
SISA	3.29%	5.39%	6.57%	9.43%	-4.12%	7.02%	7.21%	12.7%

Table 5: Error on the model predictions for the annual data

Model	Forward error			Inverse error		
	Bias	RMS	Max.	Bias	RMS	Max.
Additivity	17.4%	17.5%	30.9%	-17.4%	17.5%	30.9%
Quadrature	-7.51%	7.82%	18.1%	11.72%	12.1%	32.2%
Half-fan	-6.43%	6.58%	20.3%	9.86%	10.1%	31.6%
Exponential	-0.15%	1.57%	5.55%	0.18%	2.22%	8.85%
SISA	0.32%	1.95%	6.18%	0.68%	2.60%	8.65%

Each of the models have the same physical limits with Q_t equals to Q_f when α tends towards 0 (no infiltration) and Q_t equals to Q_{inf} when α tends towards 1 (no mechanical ventilation). That is the reason why the errors tend to 0 at the extreme values of α. One can notice that the additivity and the half-fan models have their maximum errors for α close to 0.5 whereas the quadrature model has its maximum error around 0.7 for the forward calculation and 0.8 for the inverse one. It means that depending on the airtightness of the building, the ranking of the best models from the literature is different, and could be one of the reasons why the previous studies did not agree on which model to recommend.

5 CONCLUSIONS

A superposition model with an exponential sub-additivity coefficient gives very satisfying results, always better than the models used in the literature. It takes different forms for the forward and inverse calculations, and has different optimized coefficient for the hourly and annual ones:

Table 6: Exponential sub-additivity model

	Forward	Inverse
Hourly	$Q_t = Q_f + \exp\left(-\frac{2}{3}\frac{Q_f}{Q_{inf}}\right) Q_{inf}$	$Q_f = Q_t - \exp\left(-\left(\frac{Q_t}{Q_{inf}} - 1\right)\right) Q_{inf}$
Annual	$Q_t = Q_f + \exp\left(-\frac{4}{9}\frac{Q_f}{Q_{inf}}\right) Q_{inf}$	$Q_f = Q_t - \exp\left(-\frac{2}{3}\left(\frac{Q_t}{Q_{inf}} - 1\right)\right) Q_{inf}$

In the case of inverse annual calculations, the SISA model is almost as good as the exponential one. Since it has a simpler expression, it is a good alternative and could be used to improve ventilation standards such as the ASHRAE 62.2.

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Appendix: Summary of previous superposition models and the results of the simulation/experimental comparison studies carried out on them

Name/Ref	Model	Range	Comparison		
			Ref.	**Sim/Exp**	**Results**
Additivity	$Q_t = Q_f + Q_{inf}$	All	Kiel & Wilson	Exp.	best agreement
			Wilson & Walker	Exp.	overpredicts Qt by 7%
			Li	Sim.	average error: 33%; maximum error: 64%
Quadrature	$Q_t = \sqrt{{Q_f}^2 + {Q_{inf}}^2}$	All	Modera & Peterson	Sim.	good agreement: error on Qt < 10%
			Kiel & Wilson	Exp.	underpredicts Qt by 15-30%
			Wilson & Walker	Exp.	underpredicts Qt by 20%
			Li	Sim.	good agreement: average error: 5%; maximum error: 17%
			Palmiter & Bond	Exp.	underpredicts the fan efficiency for Qinf<Qf; overpredicts the fan efficiency for Qinf>Qf
Levins (Levins, 1982)	$Q_t = Q_{inf} + Q_f . \exp\left(-\frac{Q_{inf}}{Q_f}\right)$	All	Kiel & Wilson	Exp.	underpredicts Qt by 15-30%
			Li	Sim.	good agreement: average error: 5%; maximum error: 20%
	$Q_t = \left({Q_f}^{\frac{1}{n}} + {Q_{inf}}^{\frac{1}{n}}\right)^n$	All	Modera & Peterson	Sim.	bigger errors on Qt than the quadrature model
			Kiel & Wilson	Exp.	underpredicts Qt by 10-25%
			Li	Sim.	average error: 11%; maximum error: 30%
Shaw (Shaw, 1985)	$Q_t = \begin{cases} Q_f & \text{for } h_0 > H \\ F\left({Q_{w-f}}^{\frac{1}{n}} + {Q_w}^{\frac{1}{n}}\right)^n & \text{for } h_0 < H \end{cases}$		Shaw	Exp.	in general within 25% of the measured values
			Kiel & Wilson	Exp.	underpredicts Qt by 15-30%
	$Q_t = \sqrt{{Q_f}^2 + (2Q_{inf})^2}$	$Q_f \gg Q_{inf}$	Kiel & Wilson	Exp.	very spread data; overpredicts Qt when Qf<0.7Qt ; mostly underpredicts Qt when Qf>0.7Qt
			Li	Sim.	average error: 56%; maximum error: 100%
Li	$Q_t = \left({Q_f}^{\frac{1}{n}} + (2Q_{inf})^{\frac{1}{n}}\right)^n$	$Q_f \gg Q_{inf}$	Li	Sim.	average error: 98%; maximum error: 160%
Kiel & Wilson	$Q_t = \sqrt{\left(\frac{Q_f}{2}\right)^2 + {Q_{inf}}^2} + \frac{Q_f}{2}$	All (Exhaust fan)	Kiel & Wilson	Exp.	underpredicts Qt by 10-30%
			Li	Sim.	average error: 12%; maximum error: 35%
			Palmiter & Bond	Exp.	overpredicts the fan efficiency
Wilson & Walker	$Q_t = \left(\left(\frac{Q_f}{2}\right)^{\frac{1}{n}} + {Q_{inf}}^{\frac{1}{n}}\right)^n + \frac{Q_f}{2}$	All (Exhaust fan)	Wilson & Walker	Exp.	underpredicts Qt by 7%
			Li	Sim.	average error: 18%; maximum error: 42%
Li	$Q_t = \frac{1}{2}\sqrt{{Q_f}^2 + \left(2Q_{inf}\right)^2}$	$Q_f < Q_{inf}$	Li	Sim.	average error: 22%; maximum error: 50%
Li	$Q_t = \frac{1}{2}\left({Q_f}^{\frac{1}{n}} + \left(2Q_{inf}\right)^{\frac{1}{n}}\right)^n$	$Q_f < Q_{inf}$	Li	Sim.	average error: 21%; maximum error: 50%
Half-fan - Palmiter & Bond	$Q_t = \begin{cases} \frac{Q_f}{2} + Q_{inf} & \text{for } Q_f < 2Q_{inf} \\ Q_f & \text{for } Q_f \geq 2Q_{inf} \end{cases}$		Palmiter & Bond	Exp.	good agreement