Dynamic Modelling of Moisture Absorption and Desorption in Buildings

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Moisture absorption and desorption by building materials in the hygroscopic range have been theoretically modelled. The Biot number which can be defined as the ratio between the material moisture resistance to the convective mass transfer resistance has a significant meaning in relating where the greater resistance to moisture transfer occurs. Using Biot number, the dynamic moisture behaviour of building materials within the indoor environment has been classified into three main categories. At high Biot number ($Bi \to \infty$), the material surface attains instantaneous moisture equilibrium with the surroundings. At low Biot number ($Bi \to 0$), the material moisture behaviour can be described through a lumped-parameter modelling. For most materials in buildings, moisture interaction with the surroundings occurs through a thin layer of material surface, and the amount of moisture absorption or desorption is mainly determined by the material surface moisture conditions. In order to evaluate material surface moisture conditions, the governing moisture transfer equation is solved via an approximate analytic technique (i.e. the moment method) in conjunction with numerical formulation. Both the dynamic and the alternating nature of the absorption/desorption processes can be modelled by this proposed analytic-numeric method. Comparison with experimental results and numerical solutions shows satisfactory agreement with the proposed model. Using this model the dynamic effect of moisture absorption and desorption by interior materials on indoor air humidity can be modelled.

NOMENCLATURE

A_e exposed area, m²
Bi Biot number (dimensionless)
C_p air specific heat, j/kg·C
C_m material moisture capacity, kg/kg·Pa
D_v material vapour diffusion coefficient, kg/m·Pa·s
h_c surface heat transfer coefficient, w/m²·C
h_m surface mass transfer coefficient, kg/m²·Pa·s
L_m moisture interaction depth, m
P_m vapour pressure within material, Pa
P_{mo} initial vapour pressure within material, Pa
P_{vr} indoor air vapour pressure, Pa
t time, s
V_m material volume, m³
w moisture content, kg/kg
ξ moisture sorption isotherm slope, kg/kg·rh
α_m material moisture diffusivity, m²/s
ρ_m material density, kg/m³
ρ_a air density, kg/m³.

INTRODUCTION

INDOOR air humidity is an important factor in determining both the short and long term building performance. Thermal performance of building components, energy consumption, occupants' health and comfort, surface condensation and durability of interior furnishings are all influenced by the level of indoor humidity. For accurate building performance assessment, both indoor humidity and its variational behaviour must be determined. Considerable efforts have been devoted to the prediction of indoor thermal behaviour, while little attention has been given to indoor humidity evaluation. Until recently, most indoor humidity evaluation models were strictly based on the mass balance between humidity generation rate and humidity dilution by air leakage, ignoring moisture absorption and desorption by interior surfaces. This could lead to considerable inaccuracy in the predicted humidity level since as much as one third of the moisture generated in a room could be absorbed by its surfaces [1]. Moisture absorption and desorption taking place in most residential and commercial buildings are dynamic and alternating processes depending on the level and the variational behaviour of indoor humidity. Therefore, a time-dependent model is required to predict the effect of moisture absorption and desorption on indoor humidity.

In recent years, some theoretical and experimental studies have dealt with the dynamic modelling of moisture absorption and desorption by building materials. As part of an indoor humidity calculation model [1], a method has been described to calculate the amount of moisture absorption by the room surfaces. In this method, moisture absorption is assumed to be limited to a thin film of material which attains instantaneous moisture equilibrium. Two parameters, the surface mass transfer coefficient and the room surface average moisture content, must be determined experimentally to fit the measured indoor humidity in a particular enclosure.

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These values may not be universally applicable for other cases. However, it has been found by Kusuda [1], that the mass transfer coefficient is not as sensitive to room characteristics as the other parameter, therefore, approximate theoretical relationships such as the well known Lewis relationship can be used to evaluate it. Evaluating moisture conditions and transfer dynamics at the room surfaces is the main part of an absorption/desorption model. Kusuda and Miki [2], have conducted an experimental program for evaluating surface moisture conditions of some building materials at certain ambient conditions using an infrared reflectance technique called the 'Quadra-Beam' method. These surface moisture condition results can be implemented in the moisture absorption model described by Kusuda [1], and hence, the amount of moisture absorbed or desorbed by building materials and its effect on indoor humidity can be calculated without resorting to the unknown values of the average surface moisture content. However, this requires an extensive experimental effort since the moisture conditions of each material must be evaluated at different ambient humidity levels. Moreover, the applicability of these experimental results are almost confined to these indoor environments with similar conditions to which the test specimens were subjected.

Moisture absorption and desorption within heated or cooled spaces are increasingly becoming an important part of the energy analysis techniques. A theoretical approach has been suggested by Franssen and Koppen [3] in which the unsteady state, one dimensional diffusion equation was solved by assuming instant equilibrium and that the slab is semi-infinite. Although these assumptions were found to be acceptable by experimental verification for some materials, these assumptions may not hold true for other materials at different conditions and times of exposure. Moreover, the alternation of the absorption and desorption processes has not been addressed. In an energy consumption model, Miller [4] has suggested the use of a simple resistor-capacitor electrical circuit to describe the dynamic moisture behaviour of materials. Two constants similar to what Kusuda [1] described must be determined experimentally. In an extensive experimental study Martin and Verschoor [5] have investigated the cyclical moisture absorption and desorption of fifteen materials in order to evaluate their latent heat storage effect on cooling energy consumption. The moisture response (i.e. variation in surface and bulk moisture contents) of most materials to changes in ambient humidity has been shown to follow an exponential decay behaviour. However, experimental evaluation of moisture absorption/desorption rates at all possible initial and boundary conditions is an extremely difficult task.

A detailed three dimensional finite element model has been developed [6] to evaluate moisture absorption and desorption rates of building materials. Assuming that the surface material comes into instant equilibrium with the environment, then by solving a system of differential heat and mass transfer equations, the amount of absorbed or desorbed moisture can be determined. However, solving these equations requires the knowledge of certain transfer coefficients (i.e. moisture transfer potential) which are available only for a few materials and are difficult to evaluate. Moreover, assuming instantaneous moisture equilibrium can only be justified for some materials at certain conditions. A more simplified moisture absorption and desorption evaluating procedure has been suggested [7, 8]. The objective of this procedure is to match the experimental or detailed theoretical moisture absorption and desorption rate through a simple lumped theoretical model by varying the material dry weight. An effective moisture penetration thickness with uniform moisture content can then be established to be used in the dynamic modelling of moisture absorption and desorption processes. This approach is not practical in terms of the effort needed to establish the effective material thickness, which in itself may not be a reliable parameter in modelling moisture absorption/desorption dynamics inside buildings. A major limitation of the effective thickness approach is that the cyclic integral of the total moisture absorption and desorption over a relatively short period of time must be zero. Consequently, this approach is not applicable for long term moisture storage. A more practical mathematical model for predicting water vapour sorption by interior building surfaces is discussed [9]. In this model, the governing heat and moisture transfer equations have been numerically solved by the implicit finite-difference method. The model solutions were compared with experimental results and good agreement was found. The accuracy of the numerical solution is dependent on the time step and the nodal spacing. Therefore, a reasonably accurate solution requires the use of a small time step and nodal spacing. This makes the solution method impractical in modelling long term moisture absorption/desorption problems in buildings.

A time-dependent model based on a sinusoidal boundary conditions for predicting indoor vapour content in a room and its corresponding latent heat has been described [10]. The model takes into account the moisture storage capacity of the walls, which has been shown to have a significant effect on the behaviour of indoor humidity variation. In most cases, however, a steady sinusoidal behaviour cannot be justified since many inter-related factors may contribute to the indoor environment behaviour. In addition, identifying such boundary conditions across partitions within the space or those separating two different zones may prove to be impossible, especially if inter-zonal airflow is taking place. In an effort to simulate the dynamic behaviour of indoor air humidity and temperature [11], moisture absorption by interior surfaces and furnishings was modelled by identifying a mass change time constant for each material. The rate change in the moisture content of the material in response to step changes in room humidity is assumed to follow an exponential decay. Although this exponential behaviour has been proven by many studies, however, for some interior surfaces a time constant is meaningless because of material moisture and geometrical characteristics. Moreover, when it is impossible to theoretically determine the time constant, considerable effort is needed to evaluate it for one particular material configuration either experimentally or through detailed theoretical analysis. As part of an analytical model for assessing the wall cavity long term moisture behaviour, Cunningham [12, 13] assumed that the moisture behaviour of the hygroscopic materials within the wall cavity follows an

exponential decay. In modelling moisture interaction between the material and the surrounding air, an approximate lumped model is used in which moisture transfer from and to the material is modelled by using a fictitious moisture transfer coefficient representing the surface and moisture transfer of moisture. This coefficient is a function of material moisture and geometrical characteristics, as well as the surface mass transfer coefficient and material surface moisture conditions. Some of these determining factors and consequently the moisture transfer coefficient are time-dependent. However, for the purposes of simplifying his model Cunningham [11] has assumed a mean fixed value for the moisture transfer coefficient. Such an assumption can probably be justified when modelling long term moisture behaviour of materials under certain boundary conditions, but it cannot be justified in modelling short term moisture transfer problems (e.g. moisture absorption/desorption as part of an indoor air humidity evaluation model).

Available theoretical models that deal with moisture absorption/desorption processes in buildings are either impractical to be implemented or incapable of modelling the actual dynamic moisture behaviour of such processes. For some models to be implemented certain parameters, which can only be evaluated through an extensive experimental program or detailed theoretical analysis, are required. When the lumped approach is used in modelling both the material moisture conditions and the moisture transfer processes from and to the material, material dynamic moisture behaviour, especially short term behaviour, cannot be accurately modelled. In addition, difficulty will arise when evaluating the model lumped parameters. The main objective of the present paper is to develop a practical theoretical model for evaluating moisture absorption and desorption by interior building materials. In order to evaluate the amount of moisture absorption and desorption by a given material, its surface moisture conditions must be known. The dynamic behaviour of surface moisture conditions differs from one material to another depending on its moisture and geometrical characteristics. Therefore, for accurate modelling of moisture absorption/desorption processes inside buildings, where many different materials may exist, the diversity of material moisture behaviour and hence, the diversity in the modelling approach must be considered. Since material moisture behaviour is mainly determined by its moisture and geometrical characteristics, the Biot number, given in equation (1), is used in this paper to determine the moisture behaviour of the material by relating its moisture diffusion coefficient D_v to its moisture interaction thickness L_m. Based on the value of the Biot number, the modelling approach is determined. Within the building indoor environment a very low value of Biot number would indicate that moisture conditions across the material thickness is constant and hence the lumped parameter approach can be used. On the other hand, a very high value of the Biot number would reveal that instant moisture equilibrium is reached between the material surface and the ambient air. In the building indoor environment, where the value of the surface mass transfer coefficient h_m is confined to a narrow range, a high value of Biot number will be associated with a very low moisture diffusion coefficient for the material in question. Consequently, its contribution to the moisture absorption/desorption processes in buildings can be neglected. Most interior materials, however, remain in a state of continuous moisture exchange with the indoor ambient air. These materials, which correspond to moderate values of Biot number, are the most difficult to model since their surface moisture conditions are dependent on the non-linear moisture distribution across the material thickness. In this paper, surface moisture conditions of such materials is determined by solving the governing differential equation with the corresponding boundary and non-linear initial conditions using an approximate analytic method called the Moment Method. Comparison with experimental results and numerical solutions has revealed good agreement with the solutions of the proposed model.

THEORETICAL APPROACH

Moisture absorption and desorption by interior materials in buildings are continuous and randomly alternating processes. Generally, for materials with large surface area to volume ratio, or materials having a very small equilibrium moisture capacity, the equilibrium moisture content corresponding to the ambient humidity change can immediately be attained. In this case, evaluation of the amount of moisture absorbed or desorbed requires only the knowledge of material weight and its moisture sorption isotherm. However, in most cases moisture absorption and desorption are time-dependent processes. The mathematical modelling approach of these processes is dependent on the moisture behaviour of the material (i.e. behaviour of moisture distribution across the material thickness) which is determined by the boundary conditions and the physical and geometrical characteristics of the material. Material moisture conditions in the building indoor environment are normally kept within the hygroscopic moisture range hence, it is unlikely that a continuous liquid phase can exist within interior materials. In this case, moisture transport within the material can be described by the vapour diffusion coefficient D_v. The one dimensionless parameter that can relate these factors is the Biot number, Bi,

$$Bi = \frac{h_m V_m / A_e}{D_v}.$$ (1)

The Biot number can be viewed as the ratio between the material resistance to moisture transfer, $(V_m / A_e)/D_v$, to the convective mass transfer resistance, $1/h_m$. The value of Bi has a significant physical meaning in relating where the greater resistance to moisture transfer occurs. In this paper, the Biot number is used to categorize the moisture behaviour of the different materials that would normally be available within buildings. Depending on the value of the Biot number, the modelling approach for moisture absorption/desorption processes will be determined. The surface mass transfer coefficient, h_m in equation (1) can be expressed in terms of the convective heat transfer coefficient, h_c using Lewis relationship given in equation (2).

In the building indoor environment, the value of the surface

$$h_m = \frac{h_c}{\rho_a C_p} \qquad (2)$$

convective transfer coefficient, h_c is either determined by the natural convective process or the airflow regime within the space. According to Kusuda [1], an average surface convective transfer coefficient of about 0.85 $w/m^2 \cdot C$ can be used in evaluating the surface mass transfer coefficient within the indoor environment. Although this value is relatively small compared to the design values suggested by ASHRAE [15], it must be noted that this value is an average value representing convective transfer over all interior surfaces rather than the interior surfaces of the exterior walls or roofs. In a natural convective environment, h_c will depend on the temperature difference between the material surface and the ambient air. On the other hand, in the presence of air movement within the space, h_c for a particular surface will be dependent on the air speed and the flow pattern over that surface. For both the natural and forced convection processes, an approximate average value of the surface convective transfer coefficient within a space can be determined using the appropriate model available in ASHRAE [15]. More accurate evaluation of the surface convective transfer coefficient can be obtained by utilizing the available detailed air movement simulation models. Once the surface convective transfer coefficient h_c is known, the surface mass transfer coefficient h_m can be evaluated by equation (2).

The dynamic moisture behaviour of building materials can be related to the value of Bi through the following three cases:

(i) *Building materials at high Biot number* $(Bi \to \infty)$

A large value of Bi indicates that the diffusive resistance controls the moisture transfer process, which means that there is more capacity for moisture transfer by convection than by diffusion through material. When the Biot number approaches infinity $(Bi \to \infty)$, the material surface immediately attains moisture equilibrium with the ambient conditions $(P_{ms} = P_{vr})$. In the building indoor environment, a very high value of Biot number can only be attributed to the moisture characteristics of the material in question. The contribution of such materials to moisture absorption and desorption processes is nil or very small such that it can be disregarded since a negligible amount of moisture is transported to or from the material due to its high moisture resistance. For most building materials, moisture equilibrium cannot be attained immediately. Hence, surface moisture conditions, which determine the rate of absorption or desorption, are usually different from the ambient conditions.

(ii) *Building materials at low Biot number* $(Bi \to 0)$

A small value of Bi represents the case where internal resistance to moisture transfer is negligibly small and there is more capacity for moisture transfer by diffusion than there is by convection. As Bi approaches zero (in practice $\leqslant 0.1$), the moisture content gradient within the medium gets extremely small, and hence a lumped-parameter analysis can be performed.

Assuming a constant temperature and ignoring the effect of hysteresis, the dynamics of the vapour pressure within the material can be described by equation 3.

$$P_m(t) = P_{vr} + (P_{mo} - P_{vr}) \exp\left(\frac{-h_m A_e}{C_m \rho_m V_m} t\right) \qquad (3)$$

where,

$$C_m = \frac{\xi}{P_{vs}} \quad (kg/kg \cdot Pa).$$

The amount of moisture absorbed or desorbed during a period of time can be determined by equation (4), using the change in the pressure evaluated from equation (3) and the moisture isotherms of the material

$$w_m = \rho_m V_m C_m \Delta P_m. \qquad (4)$$

(iii) *Building materials at moderate Biot number* $(0 \ll Bi \ll \infty)$

In most cases, moisture interaction between the ambient air and the materials within the space occurs through a thin layer at the material surface. The amount of moisture transfer is mainly determined by the surface moisture condition, which depends on the thickness and the moisture conditions of the moisture interaction layer, as well as the surface mass transfer coefficient.

The dynamic moisture transfer process in the material can be described by equation (5), based on the following assumptions:

—vapour pressure is the only driving force, and the process is a one-dimensional;

—the problem is isothermal (i.e. heat generated by moisture sorption is assumed negligible;

—hysteresis effect is negligible. The absorption isotherm is used to model material moisture capacity, since the moisture capacity of interest is within the hygroscopic range;

—the moisture diffusion coefficient is constant.

$$\frac{\partial w}{\partial t} = \alpha_m \cdot \frac{\partial^2 w}{\partial x^2} \qquad (5)$$

where,

$$\alpha_m = \frac{D_v}{\rho_m C_m} \quad (m^2/s).$$

However, equation (5) cannot describe the moisture interaction between the material surface and the surroundings, because of the discontinuity of the moisture content as a driving force at the boundaries. Instead, moisture transfer through the material may be expressed in terms of humidity ratio or vapour pressure as given in equation (6)

$$\frac{\partial P_m(x, t)}{\partial t} = \alpha_m \cdot \frac{\partial^2 P_m(x, t)}{\partial x^2}. \qquad (6)$$

In order to solve for vapour pressure at the material surface $P_m(0, t)$, it is essential to identify the boundary conditions, the initial conditions and the interactive thickness of the material. In the present modelling, the ratio of the material volume, V_m to its exposed area, A_e is identified as the moisture interaction depth, L_m

$$L_m = \frac{V_m}{A_e}. \qquad (7)$$

The material volume, V_m which determines the moisture capacity of a particular material, is dependent on the ambient air humidity behaviour, as well as, the moisture characteristics of the material. In the building indoor environment where air humidity normally fluctuates around a daily average value, a sinusoidal air humidity behaviour can be assumed to represent daily indoor humidity variations. For a given material, equation 6 can be solved numerically for the assumed indoor air humidity behaviour to determine the material interaction depth L_m. At L_m, the material is assumed to be impermeable to vapour flow which leads to identifying the first boundary condition. The second boundary condition is taken at the material surface where moisture transfer takes place by convection. Since moisture absorption and desorption in buildings are alternating processes, and because of the relatively large moisture interaction thickness involved in modelling these processes, equation (6) must be solved for a nonuniform initial condition. The governing boundary and initial conditions are given by equations (8a–c).

The exact solution of the above non-homogeneous mathematical

$$\frac{\partial P_m}{\partial x} = 0 \quad \text{at} \quad x = L_m \qquad (8a)$$

$$-D_v \frac{\partial P_m}{\partial x} = h_m(P_{vr} - P_m(0, t)) \quad \text{at} \quad x = 0 \qquad (8b)$$

$$P_m = F(x) \quad \text{at} \quad t = 0 \qquad (8c)$$

problem can be obtained by splitting it up into a set of simpler problems that may be solved by the separation of variables method outlined in [14]. However, the resulting solution involves an infinite series which requires the knowledge of the corresponding eigenvalues in order to evaluate. These eigenvalues are the roots of a transcendental equation which must be solved by iteration. Such a procedure is impractical and time consuming to be implemented as a part of an indoor moisture absorption/desorption model considering the dynamic nature of the problem and the number of materials involved. An alternative approach for solving equation (6) is the numerical analysis, however, by using such an approach, the moisture interaction between the material and the ambient air cannot be accurately modelled, especially in a dynamic environment such as the building indoor environment. Moreover, considering the number of materials and the nature of the moisture transfer problem a large amount of computations would be needed. For example, when the finite-difference technique is used, hundreds of nodes could be required to formulate the moisture transfer process through the material. Consequently, hundreds of equations may have to be solved simultaneously at each time interval.

When the exact analytical solutions are difficult to obtain or when the numerical solution cannot be justified, approximate analytic solutions are preferable. Considering the assumptions and the uncertainties involved in modelling moisture absorption and desorption in buildings, the use of the approximate analytic methods can be justified. The approximate analytic method that will be used to solve equation (6) is called the moment method [14]. This method is based on taking various moments of the differential equation of the moisture transfer with respect to a suitable weight function over the finite region of the problem.

The mathematical formulation of the present problem can be put in terms of dimensionless variables as:

$$\frac{\partial P_m(\eta, \tau)}{\partial \tau} = \frac{\partial^2 P_m(\eta, \tau)}{\partial \eta^2}. \qquad (9)$$

$$\frac{\partial P_m}{\partial \eta} = 0 \quad \text{at} \quad \eta = 1, \tau > 0 \qquad (9a)$$

$$-D_v \frac{\partial P_m}{\partial \eta} = h_m L_m(P_{vr} - P_m(0, \tau)) \quad \text{at} \quad \eta = 0, \tau > 0 \qquad (9b)$$

$$P_m(\eta, 0) = F(\eta) \quad \text{at} \quad \tau = 0. \qquad (9c)$$

The dimensionless quantities are defined as:

$$\eta = \frac{x}{L_m} \quad \text{and} \quad \tau = \frac{\alpha_m t}{L_m^2}.$$

To solve this problem by the moment method the pressure distribution $P_m(\eta, \tau)$ is expressed by a polynomial in the form;

$$P_m(\eta, \tau) = \sum_{k=0}^{n} a_k(\tau)\eta^k \qquad 0 < \eta < 1. \qquad (10)$$

In order to determine $(n+1)$ coefficients $(a_k(\tau)$, $k = 0, 1, \ldots, n)$, $(n+1)$ relations are required. Two relations can be obtained by utilizing the two boundary conditions (9a) and (9b). The remaining $(n-1)$ relations may be obtained by taking $(n-1)$ moments of the differential equation (9). By operating the weight functions $W_i(\eta)$, $i = 1, 2, \ldots, (n-1)$, on both sides of equation (9) and substituting in equation (10) we obtain the following equation [14]:

$$\sum_{k=2}^{n} \left[k(k-1) \int_{\eta=0}^{1} w_i(\eta)\eta^{k-2} \, d\eta \right] a_k(\tau)$$

$$= \sum_{k=0}^{n} \left(\int_{\eta=0}^{1} w_i(\eta)\eta^k \, d\eta \right) \frac{da_k(\tau)}{d\tau}$$

$$i = 1, 2, 3, \ldots, (n-1). \qquad (11)$$

Equation (11) provides a system of $(n-1)$ ordinary differential equations that could be solved to determine the remaining $(n-1)$ coefficients. Solving these equations, however, requires $(n-1)$ initial conditions. These initial conditions can be determined by equation (12) [14]

$$\sum_{k=0}^{n} \left[\int_{\eta=0}^{1} w_i(\eta)\eta^k \, d\eta \right] a_k(0) = \int_{\eta=0}^{1} w_i(\eta)F(\eta) \, d\eta$$

$$i = 1, 2, \ldots, (n-1). \qquad (12)$$

MATHEMATICAL FORMULATIONS

For the present problem a fourth degree polynomial given by equation (10) is assumed.

Moisture absorption and desorption in buildings are dynamic and

$$P_m(\eta,\tau) = a_0(\tau) + a_1(\tau)\eta + a_2(\tau)\eta^2 + a_3(\tau)\eta^3 + a_4(\tau)\eta^4 \tag{13}$$

randomly alternating processes, consequently they cannot be modelled by the analytic solution of equation 9. Instead, a numerical technique in conjunction with the analytic solution of equation (9) which describes these processes separately is the only feasible alternative. Through this approach, equation (9) with the corresponding boundary and initial conditions has to be solved at each time interval. Hence, the boundary and initial conditions have to be updated continuously.

Moisture absorption and desorption in buildings are long term processes, hence any pressure distribution across the moisture interaction thickness can be assumed as an initial condition for the first time interval of the modelling process. A uniform pressure distribution, as given by equation (14a), is assumed to represent the initial moisture status of the material. To ensure continuity of the solution, the end conditions of a particular interval, given by equation (14b), are taken as the initial conditions for the subsequent interval

$$P_m(\eta, 0) = P_{mo} \tag{14a}$$

$$P_m(\eta, 0) = a_0 + a_1\eta + a_2\eta^2 + a_3\eta^3 + a_4\eta^4. \tag{14b}$$

In order to solve for the pressure distribution by equation (13), the surface pressure as a function of time $P_m(0, \tau)$ must be known. Unfortunately, this function cannot be determined directly because of the integral operations involved in the coefficient $a_0(\tau)$ in equation (13). However, since a numerical technique with relatively short time intervals is needed to model the absorption/desorption processes, it can be safely assumed that the surface vapour pressure $P_m(0, \tau)$ is linearly related with time during that time interval, $t1$

$$P_m(0, \tau) = P_m(0, 0) + \beta_p\tau \tag{15}$$

where,

$$\beta_p = \frac{P_m(0, \tau_1) - P_m(0, 0)}{\tau_1}$$

$$\tau_1 = \frac{\alpha_m t1}{L_m^2}.$$

Utilizing the initial conditions given by equations (14a,b) and the surface vapour pressure function given by equation (15), the time-dependent coefficients in equation (13) can be determined by the moment method as:

$$a_1(\tau) = -Bi[P_d - \beta_p\tau] \tag{16}$$

$$a_0(\tau) = \left[168b1 - 1008b2 + 1008b3 - \frac{14}{5}BiP_d \right.$$
$$\left. - \frac{1}{15}Bi\beta_p \right]\exp(-52\tau) - \left[217b1 - 1302b2 + 1302b3 \right.$$
$$\left. - \frac{1519}{420}BiP_d - \frac{217}{2520}Bi\beta_p \right]\exp(-42\tau)$$
$$+ \left[55b1 - 320b2 + 315b3 - \frac{13}{12}BiP_d - \frac{Bi}{24}\beta_p \right]$$

$$\times \exp(-10\tau) + b1 + \frac{Bi}{3}P_d + \frac{Bi}{45}\beta_p + BiP_d\tau$$
$$- \frac{Bi}{2}\beta_p\tau^2 - \frac{Bi}{3}\beta_p\tau \tag{17}$$

$$a_2(\tau) = \left[-1008b1 + 6048b2 - 6048b3 + \frac{84}{5}BiP_d \right.$$
$$\left. + \frac{2}{5}Bi\beta_p \right]\exp(-52\tau) + \left[1218b1 - 7308b2 + 7308b3 \right.$$
$$\left. - \frac{203}{10}BiP_d - \frac{29}{60}Bi\beta_p \right]\exp(-42\tau)$$
$$- \left[330b1 - 1920b2 + 1890b3 - \frac{13}{2}BiP_d - \frac{Bi}{4}\beta_p \right]$$
$$\times \exp(-10\tau) - \frac{1}{6}Bi\beta_p + \frac{Bi}{2}P_d - \frac{Bi}{2}\beta_p\tau \tag{18}$$

$$a_3(\tau) = \frac{1}{6}Bi\beta_p + \frac{32}{5}[1 - \exp(-10\tau)]$$
$$\times \left[-105b1 + 630b2 - 630b3 + \frac{7}{4}BiP_d + \frac{Bi}{24}\beta_p \right]$$
$$\times \exp(-42\tau) + \left[220b1 - 1280b2 + 1260b3 - \frac{13}{3}BiP_d \right.$$
$$\left. - \frac{Bi}{6}\beta_p \right]\exp(-10\tau) \tag{19}$$

$$a_4(\tau) = \left[-105b1 + 630b2 - 630b3 + \frac{7}{4}BiP_d \right.$$
$$\left. + \frac{Bi}{24}\beta_p \right]\exp(-42\tau) - \frac{Bi}{24}\beta_p \tag{20}$$

where,

$$P_d = P_{vr} - P_m(0, 0)$$

$b1, b2, b3$ are expressed for the initial interval as:

$$b1 = P_{mo}, \quad b2 = \frac{P_{mo}}{2}, \quad b3 = \frac{P_{mo}}{3} \tag{21a}$$

and for the intermediate intervals as:

$$b1 = a_0(\tau_1) + \frac{a_1(\tau_1)}{2} + \frac{a_2(\tau_1)}{3} + \frac{a_3(\tau_1)}{4} + \frac{a_4(\tau_1)}{5}$$

$$b2 = \frac{a_0(\tau_1)}{2} + \frac{a_1(\tau_1)}{3} + \frac{a_2(\tau_1)}{4} + \frac{a_3(\tau_1)}{5} + \frac{a_4(\tau_1)}{6}$$

$$b3 = \frac{a_0(\tau_1)}{3} + \frac{a_1(\tau_1)}{4} + \frac{a_2(\tau_1)}{5} + \frac{a_3(\tau_1)}{6} + \frac{a_4(\tau_1)}{7}. \tag{21b}$$

When the moment method is used to solve equation (9), difficulty may be experienced in satisfying the initial conditions. Consequently, solutions by this method may not be accurate for short times. The accuracy of the solution depends on the initial conditions, the type of boundary conditions and the degree of polynomial representation used. For uniform or linear initial pressure distribution, a third degree polynomial representation is

sufficient to obtain a reasonably accurate solution provided that no convective transfer occurs at the boundaries. Higher polynomial representation is needed to solve for problems with more complicated initial conditions. However, with convective transfer at the boundaries, the accuracy of the solution becomes dependent on the ambient conditions, material surface conditions and the surface mass transfer coefficient. In this case, higher order polynomial representation will not enhance the accuracy in a decisive manner because of the variability of the ambient conditions. Instead, solution oscillation may occur for short time intervals at the beginning of the solution process with every change in the ambient conditions. Although, the solution normally stabilizes with time, this behaviour is not acceptable when modelling absorption and desorption processes in buildings because ambient (i.e. indoor) conditions are continuously changing even within a short period of time.

In order to avoid solution oscillation at the beginning of every absorption and desorption process, the initial conditions are relaxed whenever there is a change in the ambient conditions. The relaxation process is performed by using the coefficients given by equations (16–20) to modify the values of $b1$, $b2$ and $b3$. These coefficients are recalculated by setting the coefficient a_0 and the surface vapour pressure $P_m(0, \tau_1)$ equal to the surface vapour pressure at the end of the previous interval, while the ambient conditions, P_{vr}, are set to its new value. New values of $b1$, $b2$ and $b3$ are then evaluated using equation (21b). These values will constitute the initial condition of the new moisture absorption or desorption problem.

The vapour pressure on the material surface at the end of each time interval $t1$ can be given by:

$$P_m(0, \tau) = \frac{A}{B} \qquad (22)$$

$$A = \left[-217b1 + 1302b2 - 1302b3 + \frac{1519}{420} BiP_d \right.$$

$$\left. - \frac{217}{2520} Bi \frac{P_m(0,0)}{\tau_1} \right] \exp(-42\tau_1)$$

$$+ \left[168b1 - 1008b2 + 1008b3 - \frac{14}{5} BiP_d + \frac{Bi}{15} \frac{P_m(0,0)}{\tau_1} \right]$$

$$\times \exp(-52\tau_1) + \left[55b1 - 320b2 + 315b3 - \frac{13}{12} BiP_d \right.$$

$$\left. + \frac{Bi}{24} \frac{P_m(0,0)}{\tau_1} \right] \exp(-10\tau_1) + BiP_d\tau_1 + \frac{Bi}{2} P_m(0,0)\tau_1$$

$$+ \frac{Bi}{3} P_m(0,0) + \frac{Bi}{3} P_d - \frac{Bi}{45} \frac{P_m(0,0)}{\tau_1} + b1 \qquad (23)$$

$$B = 1 + \frac{Bi}{2} \tau_1 + \frac{Bi}{3} + \frac{Bi}{45\tau_1} - \frac{217}{2520} \frac{Bi}{\tau_1} \exp(-42\tau_1)$$

$$+ \frac{Bi}{15\tau_1} \exp(-52\tau_1) + \frac{Bi}{24\tau_1} \exp(-10\tau_1). \qquad (24)$$

For constant ambient conditions, the amount of moisture absorption or desorption during a time interval, $t1$ can be approximately given by:

$$w_m = h_m A_e \left(P_{vr} - \frac{P_m(0, \tau1) + P_m(0, 0)}{2} \right) \cdot t1 \qquad (25)$$

and the total moisture lost or gained within a room is.

$$w_{mt} = h_{ma} \sum_{i=1}^{n} A_{ei} \left(P_{vr} - \frac{P_{mi}(0, \tau1) + P_{mi}(0, 0)}{2} \right) \cdot t1 \qquad (26)$$

where,

h_{ma} = average surface mass transfer coefficient, kg/m$^2 \cdot$Pa$\cdot$s

n = number of relevant materials in the room.

The average surface mass transfer coefficient h_{ma} can be evaluated by Lewis relationship given in equation (2) using the average surface convective transfer coefficient.

COMPARISONS AND DISCUSSIONS

The proposed theoretical model for evaluating the surface moisture conditions of the material is compared with Kusuda's and Miki's experimental results of an absorption test on a 3 mm thick balsa wood specimen [2]. The specimen was exposed to constant ambient conditions of 24°C and 70% relative humidity from both sides, thus half the specimen thickness (1.5 mm) is theoretically modelled. The material properties used include: material density, moisture diffusivity, and the moisture equilibrium curve. For balsa wood, a density of 160 kg/m^3, and a moisture diffusivity of 1×10^{-10} m^2/s are used. Since the experiment is conducted within the moisture hygroscopic range of the material, the moisture sorption isotherm for wood, shown in Fig. 1, was used in modelling material moisture capacity. Although it is an important parameter in the modelling process, the value of surface mass transfer coefficient, h_m during the experiment is not known and cannot be evaluated using Lewis relationship. However, the most likely value of the surface mass transfer coefficient in that particular environment (i.e. environmental chamber) can be determined. According to Kusuda [1], the average mass transfer coefficient in a room is estimated to be 5×10^{-9} kg/Pa$\cdot$m$^2 \cdot$s. However, in a test environmental chamber, the surface mass transfer coefficient will be dependent on the airflow regime within the space and the degree of exposure of the test specimen. The airflow regime within the test chamber is mainly determined by the method used for maintaining the air temperature and humidity. In an experimental validation program, Thomas and Burch [9], have evaluated the surface mass transfer coefficient within an environmental chamber using the cup method. A value of about 5×10^{-9} kg/m$^2 \cdot$Pa$\cdot$s was found to represent the mass transfer coefficient at the specimen surface. Since this value corresponds to an ambient environment similar to Kusuda's test environment, it can be used in modelling the moisture behaviour of the balsa wood specimen. Figure 2 compares the experimental results and the theoretical evaluation of the transient behaviour of the surface moisture conditions of the balsa wood specimen. It can be seen that there is a good agreement between the experimental and the theoretical results. However, a better agreement can be obtained
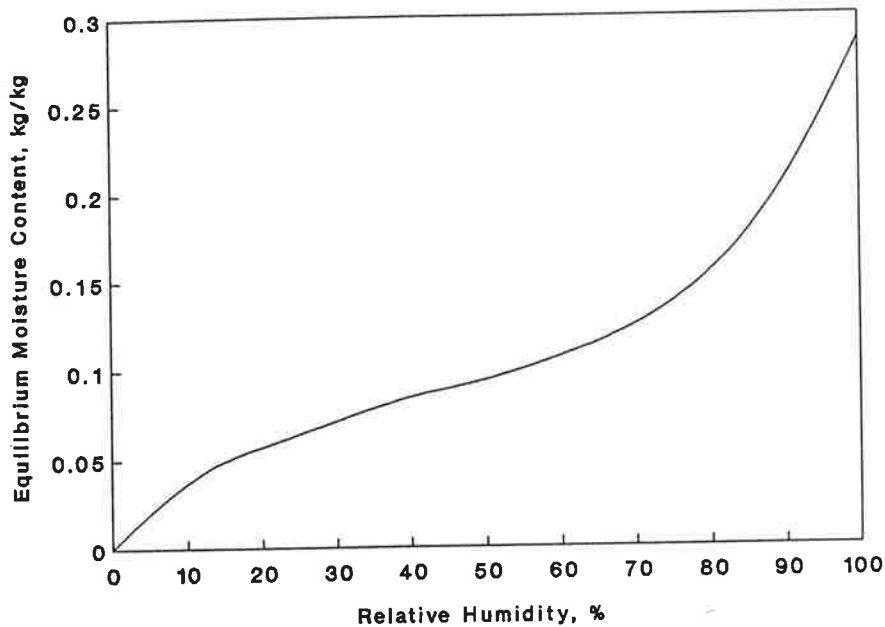
Fig. 1. Wood moisture absorption isotherm [8].

when smaller surface mass transfer coefficient is used, since the theoretical curve will be shifted downward.

The solution of the proposed model is also compared with the experimental results of a desorption test on a gypsum board specimen [9]. In this experimental work, material moisture and physical characteristics, initial conditions, as well as, the surface mass transfer coefficient are clearly defined. The material moisture capacity is evaluated from its absorption isotherm given in Fig. 3. The density of the gypsum board is 670 kg/m³, and its moisture diffusivity is about 1.8×10^{-8} m²/s. For this particular desorption test, a surface mass transfer coefficient of 0.0044 m/s, which corresponds to a value of 3.2×10^{-8} kg/m²·Pa·s, is used. The test specimen has an initial moisture content of 0.0079 kg/kg, which corresponds to about 74% relative humidity as can be found from Fig. 3. By exposing one side of the specimen (0.18 m in diameter), to a new ambient relative humidity of about 26% and regularly weighing the specimen, the desorption rates were determined. Experimental and theoretical evaluations of moisture desorption rates of the gypsum board specimen are shown in Fig. 4. It can be seen that the theoretical solution is in satisfactory agreement overall except at the beginning of the process.
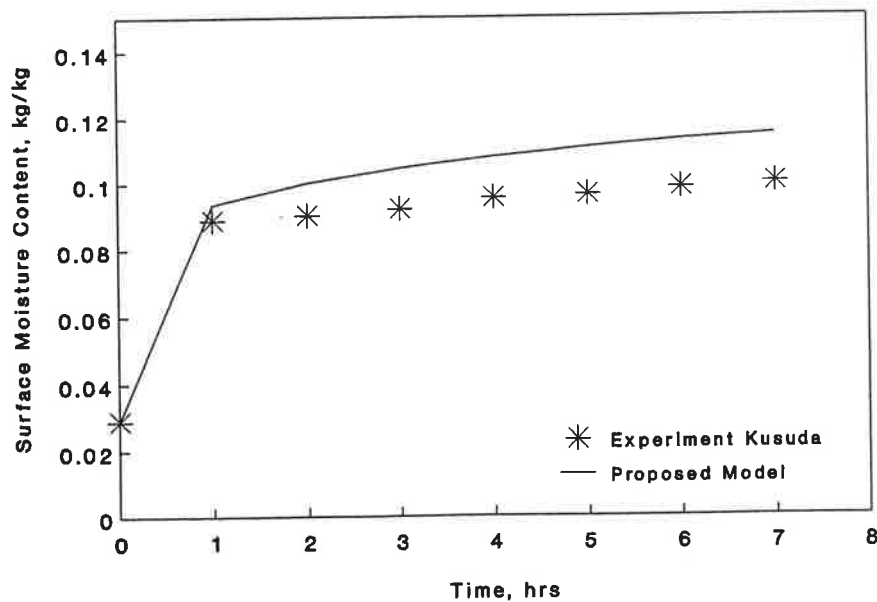

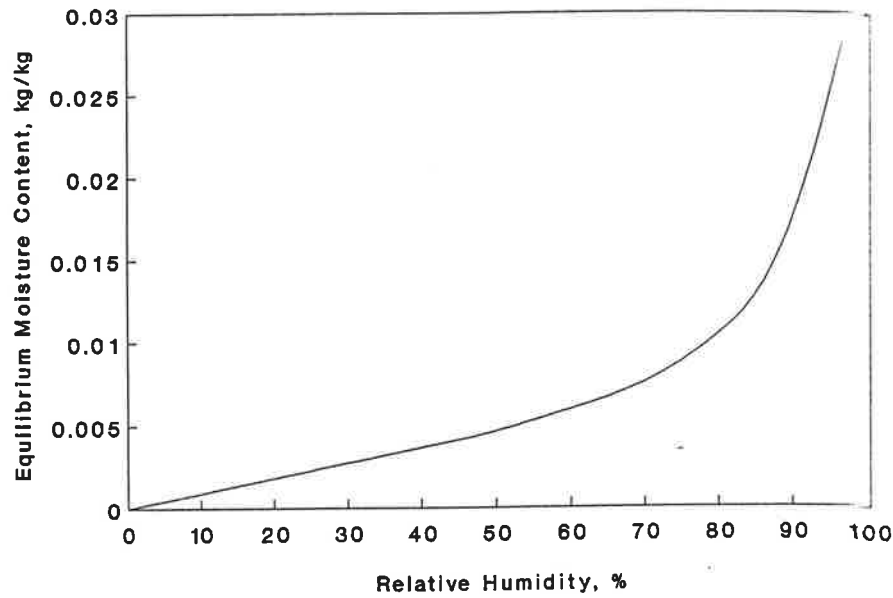Fig. 2. Comparison between experimental and theoretical surface moisture content.

Fig. 3. Gypsum board absorption isotherm.

This can be attributed to measurement uncertainties at the beginning of the experiment and the sensitivity of the mass balance scale used [9].

In the above theoretical modelling of material moisture behaviour, constant values are used to describe material moisture capacity C_m, and material moisture diffusivity α_m. In reality, both vary with the moisture content. The material moisture capacity (which is equivalent to the specific heat in thermal analysis) varies with equilibrium relative humidity according to the slope of the moisture sorption isotherm. In practice, when the relative humidity range in interest is relatively small, or when the slope of the sorption isotherm does not exhibit considerable variations within the range of interest, then a constant moisture capacity can be used in modelling dynamic moisture behaviour of materials. Similarly, when the material moisture content varies within a limited range, such as materials within the building indoor environment, a constant moisture diffusion coefficient can be used to describe moisture transfer through the material. However, for more accurate dynamic moisture modelling, the variability of the material moisture capacity and its moisture diffusion coefficient has to be considered whenever it is practically possible.
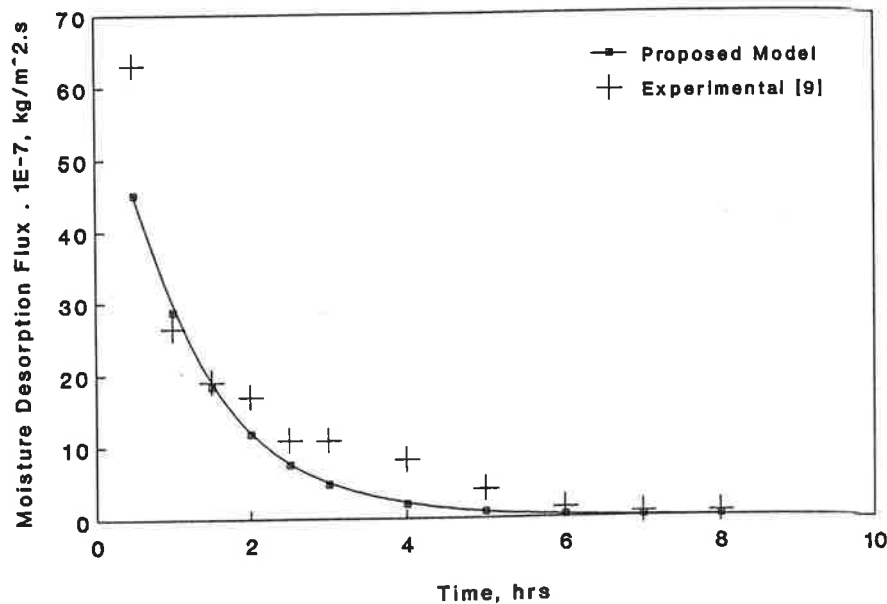

Fig. 4. Comparison between experimental and theoretical moisture desorption rate.
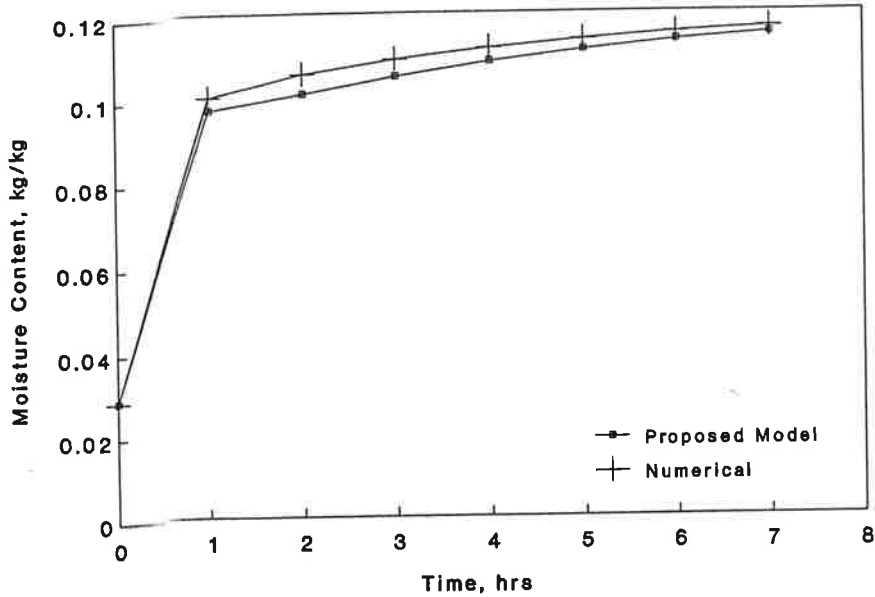
Fig. 5. Comparison between proposed and numerical evaluation of surface moisture content for balsa wood.

Numerical solutions using the finite-difference implicit formulations, have been compared with the solution of the proposed model in evaluating surface moisture conditions for some building materials. Figure 5 shows the predicted surface moisture content of the balsa wood sample using the numerical and the proposed approaches. Prediction of the surface vapour pressure of a 10 mm thick gypsum board by both techniques when the ambient relative humidity is changed from 50% to 60% is shown in Fig. 6. In both cases, good agreement is obtained between the two approaches. In the building indoor environment, air humidity is continuously fluctuating, therefore, for any model to be applicable for describing indoor moisture absorption and desorption,

it must handle the short term fluctuations in ambient conditions. The approximate analytic solution by the Moment Method cannot handle such behaviour with reasonable accuracy especially when large fluctuations occur. This shortcoming has been eliminated by relaxing the initial conditions of the moisture transfer problem whenever there is a change in ambient conditions. Figure 7 shows the solution of the numerical and the proposed techniques when there is a step change of 10% in the ambient relative humidity. Considering the high fluctuation pattern used for this particular case, which is unlikely to be encountered in practice within the building indoor environment, the proposed model can be judged to be an applicable and a reliable tool for describing
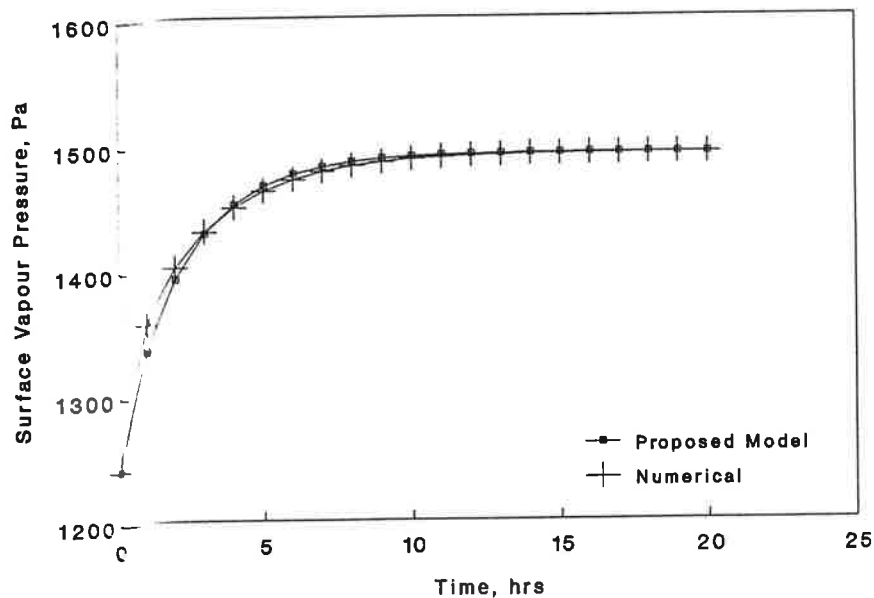

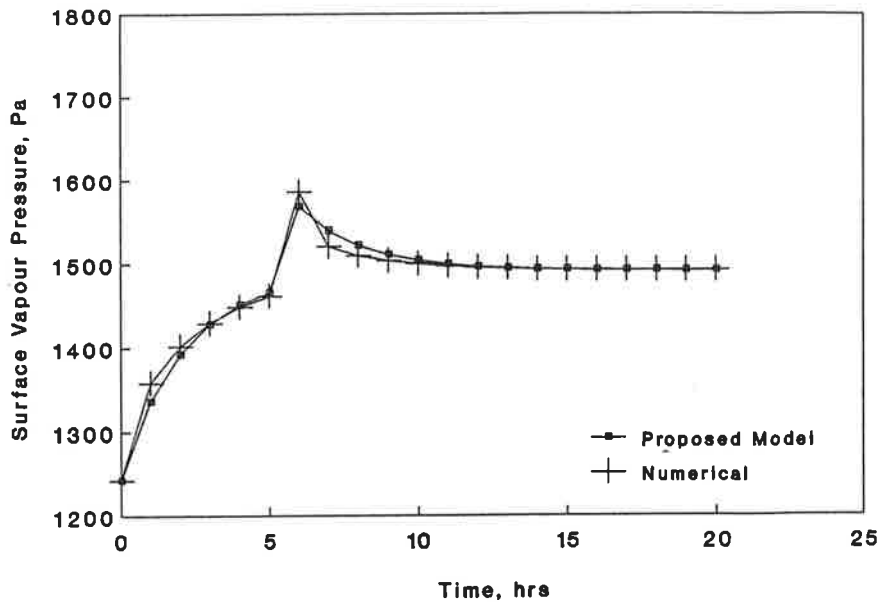Fig. 6. Surface vapour pressure using proposed and numerical techniques.

Fig. 7. Surface vapour pressure using proposed and numerical techniques.

moisture absorption and desorption processes within buildings.

CONCLUSIONS

In buildings, there are normally different materials with different moisture characteristics, consequently, each will respond differently when exposed to dynamic ambient conditions. Material moisture behaviour depends on its geometrical configuration and moisture characteristics, as well as, the boundary conditions. Some materials attain instant moisture equilibrium with the surroundings, others will not respond at all. The Biot number, Bi, was used to classify materials according to their moisture and geometrical properties. Hence, the material moisture behaviour can be identified and appropriately modelled. As Bi approaches zero, a lumped-parameter analysis can be used to model material moisture behaviour, in this case a resistor-capacitor analogy can be used. When Bi is very large, the material can be assumed passive since negligible amount of moisture is transported to or from the material. In most cases, however, moisture interaction with the material occurs near the surface ($0 \ll Bi \ll \infty$), resulting in a nonuniform moisture content distribution across the material moisture interaction thickness, L_m. For this case, determining the material surface moisture conditions is the main task in modelling moisture absorption and desorption processes. In this study, a practical and an efficient analytic-numeric technique for evaluating the material surface

moisture conditions is proposed and validated against experimental results. The required parameters in the proposed model are mainly the material moisture properties which are normally available in literature for all common building materials, giving it an advantage over many existing models which requires either an extensive experimental work or unavailable material moisture properties. In addition, the proposed model has some advantages over numerical techniques by requiring less computations and being able to more accurately describe the moisture interaction between the ambient conditions and the material surface.

As part of a numerical formulation, this analytic-numeric method can be used to model moisture absorption and desorption by construction and furnishing materials in building. Both the dynamic and the alternating nature of these processes can be modelled. Although the proposed method cannot accurately model material response to sudden changes in ambient conditions for short times, which is the case in many buildings, it has been modified to tolerate such dynamic behaviour by relaxing the initial moisture conditions of the material whenever changes in the ambient conditions occur. Comparisons with experimental results and numerical technique solutions show good agreement with the proposed method. Using this method in conjunction with a numerical technique for modelling air humidity behaviour within buildings, the dynamic effect of moisture absorption and desorption by interior materials on indoor air humidity can be modelled.

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