

# Review on Using the "Time Constant" for Studying the Atrium Smoke-Filling Processes

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## ABSTRACT

A time constant has been proposed to characterize the time it takes to fill an atrium space with smoke for design purposes. This was defined through the use of the empirical equation expressing the mass entrainment rate to the 3/2 power of the clear height. However, the equation holds only when the flame tip touches the smoke layer, and the flame temperature was taken to be 1100 K (827°C 1521°F). Another time constant using the plume equation proposed by Zukoski is used and the concept is further evaluated in this paper using zone models developed at NIST and another model developed at the Building Research Institute, Ministry of Construction, Japan. A design fire of thermal power and area related to the volume of the atrium space is proposed in order to evaluate the time constant. Results of the zone modeling simulation supported the fact that the time required to fill 80% of the atrium space with smoke is related to its time constant. Full-scale experimental results on smoke-filling processes in atria available in the literature are compared. This quantity is recommended to specify the smoke-filling time for an atrium space for design purposes, and its use for smoke control design is also discussed.

## INTRODUCTION

In Hong Kong, whether a smoke extraction system must be installed in an atrium space is determined by the volume of the atrium space (FSD 1988, 1994; Chow 1989). The critical volumes are 7,000 m<sup>3</sup> (0.25 million ft<sup>3</sup>) for a basement, 7,000 m<sup>3</sup> (0.25 million ft<sup>3</sup>) for an atrium with high fire load density, and 28,000 m<sup>3</sup> (1 million ft<sup>3</sup>) for a normal atrium space. It appears that this specification is not good enough, as the geometry of the atrium space is not considered. Usually, the volume of atrium spaces in Hong Kong is very large, and most of them are of "open" design. More than 10,000 people might be passing through a shopping mall atrium on public holidays. A survey

(Chow and Wong 1993a, 1993b) showed that atrium buildings can be classified into three types: cubic, flat, and high. Simulations of the smoke-filling processes in those three types of atria using zone models (Mitler and Rockett 1987) have been reported and the smoke-filling times in different types of atria with the same volume would be very different. A time constant (Chow 1993, 1994a, 1994b, 1994c) was proposed to specify the smoke-filling time in the atrium space. Correlation of the time constant with the time required to fill 80% of an atrium with smoke was found for an atrium space with and without installing a smoke extraction system. Comparison with experimental data on full-size atria available in the literature was made and fairly good correlation was obtained (Chow 1993, 1994a). It is common in local design (FSD 1994) to assume the time of escape to be 2.5 minutes (150 seconds), and the time required to fill 80% of the atrium with smoke is suggested to be longer than this value.

However, in determining the time constant, the air entrainment rate (Morgan and Gardner 1990),  $\dot{m}$  (in kg/s), is assumed to be related to the 3/2 power of the clear height,  $y$  (in m), through the perimeter of the fire,  $p$  (in m):

$$\dot{m} = 0.188py^{3/2}. \quad (1)$$

This equation holds only when the flame tip touches the smoke layer, as proposed recently by Thomas (1992). Also, the empirical value of 0.188 was deduced by assuming the flame temperature to be 1100 K (827°C 1521°F) (Butcher and Parnell 1992). Another time constant using the plume equations by Zukoski et al. (1980/1981) was proposed (Chow 1994b). There, the expressions for vertical upward speed  $v$  (in m/s), plume radius  $r$  (in m), and central temperature rise  $DT_o$  (in °C) at height  $z$  (in m) above a fire for thermal plumes in free spaces are given by

$$v = C_v \left( \frac{g}{C_p \rho_\infty T_\infty} \right)^{1/3} \dot{Q}^{1/3} (z - z_o)^{-1/3} \quad (2)$$

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$$r = C_r \left( \frac{T_o}{T_\infty} \right)^{1/2} (z - z_o) \quad (3)$$

$$\Delta T_o = C_T \left( \frac{T_\infty}{g C_p \rho_\infty} \right)^{1/3} \dot{Q}^{2/3} (z - z_o)^{-5/3} \quad (4)$$

where  $\dot{Q}$  is the heat release rate in kW,  $C_v$  is 3.4,  $C_r$  is 0.12,  $C_T$  is 9.1,  $g$  is  $9.8 \text{ m/s}^2$  ( $32.2 \text{ ft/s}^2$ ),  $r_\infty$  is taken to be  $1.21108 \text{ kg/m}^3$  ( $0.07569 \text{ lb/ft}^3$ ),  $C_p$  is  $1,015 \text{ J/(kg}\cdot\text{K)}$  ( $0.24 \text{ Btu/(lb}\cdot\text{°F)}$ ),  $T_\infty$  is  $290 \text{ K}$  ( $17^\circ\text{C}$   $63^\circ\text{F}$ ) for SI units (following Klote [1994], the value of  $C_v$  is 0.0172,  $C_r$  is 0.0366, and  $C_T$  is 0.0067 for British units;  $\Delta T_o$  is in  $^\circ\text{R}$ ,  $r_\infty$  is in  $\text{lb/ft}^3$ , and  $\dot{Q}$  is in  $\text{Btu/s}$ ), and  $z_o$  is calculated from

$$z_o = -1.02D + 0.083(\dot{Q})^{2/3} \quad (5)$$

The parameter  $D$  (in m) is the effective diameter of the fire source and is calculated by equating the fire area to  $(\pi D^2)/4$ .

The concept of using a time constant derived from the above plume equations is further evaluated in this paper using three other zone models—CFAST (Peacock et al. 1993) and CCFM.VENTS (Cooper and Forney 1990) developed in the U.S. and BRI2 (Tanaka and Nakamura 1989) developed in Japan. All three models have been validated. Experimental results on full-scale atrium smoke-filling processes available in the literature (Yamana and Tanaka 1985; Hägglund et al. 1985; Bengtson and Hägglund 1981) are also used to assess the derived correlation relations with the time required to fill 80% of an atrium with smoke. Finally, the effects of a mechanical ventilation system are discussed.

It is understood that there are reservations in applying a zone model for simulating an atrium fire (e.g., Milke and Mowrer 1994), and a modification known as the FMD algorithm has been made. However, the model can be applied if the cooling effect of the upper layer is included (e.g., Law 1990), and a comparison of simulating smoke filling an atrium with different zone models is reported by Chow (1994c). Further experimental verification must be made to justify those arguments on using a zone model. A new full-scale burning hall for atrium fire studies was built (Chow et al. 1993) to verify the results on atrium fires.

## TIME CONSTANTS

A survey of the geometric shapes of atrium spaces was made and three main types of atrium space classified (Chow and Wong 1993a, 1993b). The type 1 (or cubic) atrium is cubic in shape, the type 2 (or flat) atrium has a large transverse dimension compared with the height, and type 3 (or high) has a height-to-width (or length) ratio of more than two. A time constant,  $t_1$ , describing how fast the atrium will be filled with smoke has been defined by Chow (1993, 1994a) using the air entrainment rate equation (Equation 1) for an atrium space of volume  $V$ , floor area  $A_f$ , height  $H$ , and smoke density  $r$ :

$$t_1 = \frac{2\rho V}{0.188\rho H^{3/2}} = \left( \frac{2\rho}{0.188\rho} \right) \left( \frac{\sqrt{V}}{\xi} \right) \quad (6)$$

The geometric aspect factor,  $\xi$ , of the atrium space is given by

$$\xi = \sqrt{\frac{H^2}{A_f}} \quad (7)$$

Note that there are two parts in the expression for the time constant. The first part is related to the properties of a fire and the second part to the geometry of the atrium space. However, the definition of this time constant comes from Equation 1, which has to be reviewed carefully as proposed by Thomas (1992).

Another time constant,  $t_2$ , is then defined by calculating the air entrainment rate using the plume expressions (Equations 2 through 5) to give a form similar to that of Equation 1 (Chow 1994b):

$$t_2 = \left( \frac{3\rho}{2K_2} \right) \left( \frac{A_f}{H^{2/3}} \right) \quad (8)$$

where

$$K_2 = \frac{6\pi}{5} \rho_\infty \alpha_p K_1 C_r \quad (9)$$

and

$$K_1 = C_v \left( \frac{g}{C_p \rho_\infty T_\infty} \right)^{1/3} \dot{Q}^{1/3} \quad (10)$$

The entrainment coefficient,  $\alpha_p$ , of the plume lies between 0.0980 and 0.1878. Putting in expressions  $K_1$  and  $K_2$  with the numerical figures of  $r_\infty$  ( $1.2111 \text{ kg/m}^3$  [ $0.0757 \text{ lb/ft}^3$ ]),  $g$  ( $9.81 \text{ m/s}^2$  [ $32 \text{ ft/s}^2$ ]),  $C_p$  ( $1,015 \text{ J/(kg}\cdot\text{K)}$  or  $0.24 \text{ Btu/(lb}\cdot\text{°F)}$ ),  $T_\infty$  ( $290 \text{ K}$  [ $20^\circ\text{C}$   $63^\circ\text{F}$ ]),  $T_o$  ( $1500 \text{ K}$  [ $1227^\circ\text{C}$   $2241^\circ\text{F}$ ]), and  $C_v$  (3.4) would give

$$t_2 = \frac{6.245}{\alpha_p \dot{Q}^{1/3}} \left( \frac{A_f}{H^{2/3}} \right) \text{ or } \frac{6.245}{\alpha_p \dot{Q}^{1/3}} \frac{V}{H^{5/3}} \quad (11)$$

Again, there are two parts in the expression for  $t_2$ , as seen from Equations 8 and 11. The first part is related to the fire itself, given by the plume entrainment coefficient ( $\alpha_p$ ) and the heat release rate ( $\dot{Q}$ ). The second part depends on the geometry of the atrium space. In this study, the entrainment coefficient,  $\alpha_p$ , is taken to be 0.15, which is arbitrarily defined for calculating the time constant  $t_2$ .

## DESIGN FIRE

The design fire in a building depends not only on the fire load but also on the statistical record of building fires. In previous simulations (Chow and Wong 1993a, 1993b), a pool fire measuring 3 m (9.84 ft) in diameter was used. However, as explained in the above references, some local engineers would like to see the effects on the building with a larger fire; a fire 3 m (9.84 ft) in diameter might be too small for a large atrium and too big for a small atrium. Therefore, a square design fire of thermal power  $0.5 \text{ MWm}^{-2}$  ( $0.1587 \times 10^8 \text{ Btu/(h}\cdot\text{ft}^2)$ ) with a

fixed perimeter of 12 m (39.37 ft) was proposed (Chow 1994a) for an atrium with a volume of less than 15,000 m<sup>3</sup> (530,035 ft<sup>3</sup>); the perimeter (*p*) will be increased linearly with the atrium volume if the volume is greater than 15,000 m<sup>3</sup>:

$$P = \begin{cases} 12, & V \leq 15,000 \text{ m}^3 \\ \left(\frac{12}{15,000}\right)V, & \text{otherwise} \end{cases} \quad (12)$$

This is only a proposal for satisfying the needs of using a larger design fire in a large atrium. The physical basis must be verified through field surveys (Chow 1994b).

### SIMULATION WITH LOCAL ATRIA

To illustrate the idea, atrium spaces of the three types with volumes varying from 2,500 to 35,000 m<sup>3</sup> (88,286 to 1,236,006 ft<sup>3</sup>) were considered. The type 2 atrium was further subdivided into 2A, 2B, and 2C with dimensions (length  $\times$  width  $\times$  height) of 2H  $\times$  H  $\times$  H, 3H  $\times$  H  $\times$  H, and 4H  $\times$  H  $\times$  H; type 3 was subdivided into 3A, 3B, and 3C as H/2  $\times$  H/2  $\times$  H, H/3  $\times$  H/3  $\times$  H, and H/4  $\times$  H/4  $\times$  H. The dimensions of the type 1 atrium are, therefore, H  $\times$  H  $\times$  H. The time constants calculated from this design fire for those atria are listed in Table 1. Simulations with the zone models CFAST, CCFM.VENTS, and BRI2 were performed on these three types of atria with volumes varying up to 35,000 m<sup>3</sup> (1,236,006 ft<sup>3</sup>), as shown in Table 1. The design fire described by Equation 12 was located at the center of the atrium floor. A smoke curtain is assumed to be operated so that all higher level spaces opened to the atrium are covered except for four openings measuring 3 m (9.84 ft) high at the bottom. The time, *t<sub>r</sub>*, required to fill the atrium with a smoke layer equal to 80% of the atrium height (NFPA 1995; Klote and Milke 1992) is taken as a reference. Results on *t<sub>r</sub>* predicted by CFAST, CCFM.VENTS, and BRI2 for the three types (or seven sets) of

atrium spaces are shown in Figures 1 through 3, respectively. For type 2 atria with small volumes, smoke might not be able to fill up 80% of the atrium. When using the CCFM.VENTS model, the heat loss parameter (*l<sub>r</sub>*) specifying the heat lost to the boundary rooms was set at 0.5.

### DISCUSSION

A crude model with mass transfer only has been proposed by Chow (1994b) to relate the time, *t<sub>r</sub>*, required to fill 80% of an atrium space with smoke to the second time constant, *t<sub>2</sub>*:

$$t_r = 1.924t_2 \quad (13)$$

It can be seen that it takes longer to fill the atrium building with smoke for an atrium of the same time constant *t<sub>2</sub>* if the thermal effect is neglected without considering thermal expansion of hot gases. A correlation of *t<sub>r</sub>* with the time constants predicted using another zone model (Mitler and Rockett 1987) was found (Chow 1994b):

$$t_r = 0.798t_2 \quad (14)$$

Those two curves are plotted and shown in Figures 1 through 3 as well, and fairly good agreement with the results simulated by the three zone models was observed.

Experimental data on the smoke-filling process available in the literature were used to evaluate the time constant. The full-scale experiment on smoke filling an atrium reported by Yamana and Tanaka [1985] was considered. The atrium is located at a Japanese research institute and measured 30 m (98.4 ft) long by 24 m (78.74 ft) wide by 26.3 m (86.29 ft) high. The size of the fire was 1.8 m (5.9 ft)  $\times$  1.8 m (5.9 ft), and it burned with methanol to give a heat release rate of 1.3 MW (4.43  $\times$  10<sup>6</sup> Btu/h). The time constant *t<sub>2</sub>* was 5.13 minutes, and the experimental value of *t<sub>r</sub>* was 5.40 minutes.

TABLE 1  
Atrium Spaces Simulated

Volume (m <sup>3</sup> )	Type 1 Atrium x = 1		Type 2A Atrium x = 0.7071		Type 2B Atrium x = 0.5774		Type 2C Atrium x = 0.5		Type 3A Atrium x = 2		Type 3B Atrium x = 3		Type 3C Atrium x = 4	
	<i>t<sub>1</sub></i> /s	<i>t<sub>2</sub></i> /s	<i>t<sub>1</sub></i> /s	<i>t<sub>2</sub></i> /s	<i>t<sub>1</sub></i> /s	<i>t<sub>2</sub></i> /s	<i>t<sub>1</sub></i> /s	<i>t<sub>2</sub></i> /s	<i>t<sub>1</sub></i> /s	<i>t<sub>2</sub></i> /s	<i>t<sub>1</sub></i> /s	<i>t<sub>2</sub></i> /s	<i>t<sub>1</sub></i> /s	<i>t<sub>2</sub></i> /s
2500	54.08	178.14	76.48	115.85	93.66	145.11	108.16	170.26	27.04	36.49	18.03	23.25	13.52	16.89
5000	76.48	106.25	108.16	157.64	132.45	197.47	152.96	231.69	38.24	49.65	25.49	31.64	19.12	22.99
10,000	108.16	144.47	152.96	214.52	187.32	268.71	216.31	315.28	54.08	67.57	36.05	43.06	27.04	31.28
15,000	132.46	172.92	187.33	256.88	229.41	321.78	264.93	377.54	66.23	80.91	44.15	51.56	33.16	37.46
20,000	114.72	178.48	162.24	265.22	198.68	332.23	229.43	389.80	57.36	83.54	38.24	53.24	28.68	38.67
25,000	102.61	182.92	145.11	271.88	177.70	340.57	205.21	399.59	51.30	85.64	34.20	54.58	25.65	39.64
28,000	96.95	185.21	137.11	275.32	167.91	344.88	193.91	404.65	48.48	86.72	32.32	55.27	24.24	40.15
30,000	93.67	186.62	132.46	277.44	162.22	347.54	187.33	407.77	46.83	87.39	31.22	55.69	23.42	40.46
35,000	86.72	189.81	122.64	282.24	150.19	353.54	173.44	414.81	43.36	88.90	28.91	56.65	21.68	41.15

(Taking density of air to be 1.22 kgm<sup>-3</sup>.)

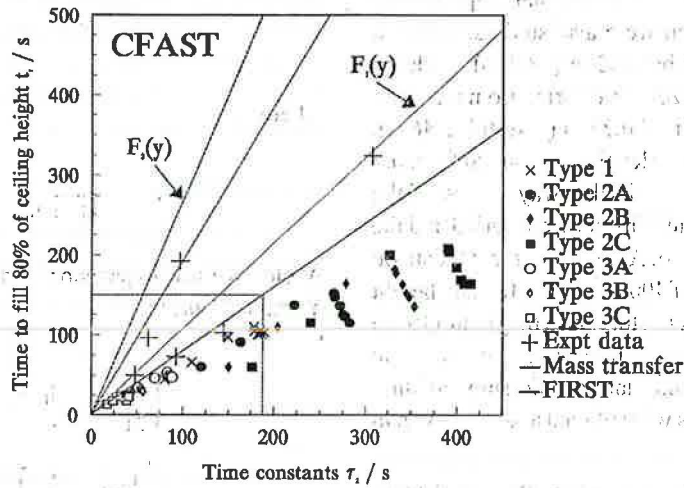


Figure 1 Results predicted by CFAST.

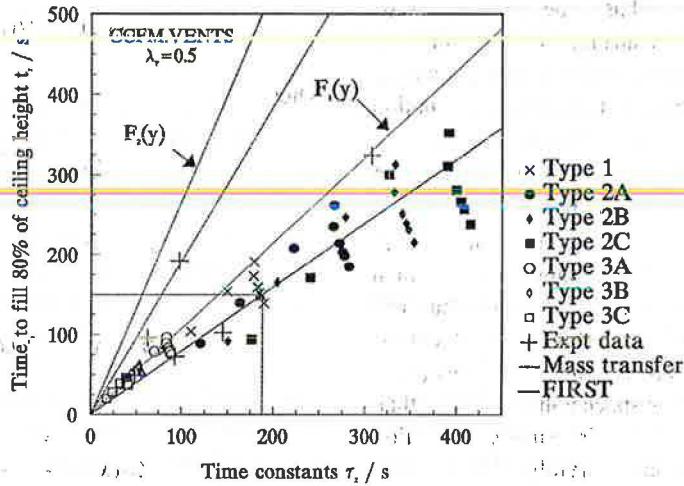


Figure 2 Results predicted by CCFM.VENTS ( $\lambda_T = 0.5$ ).

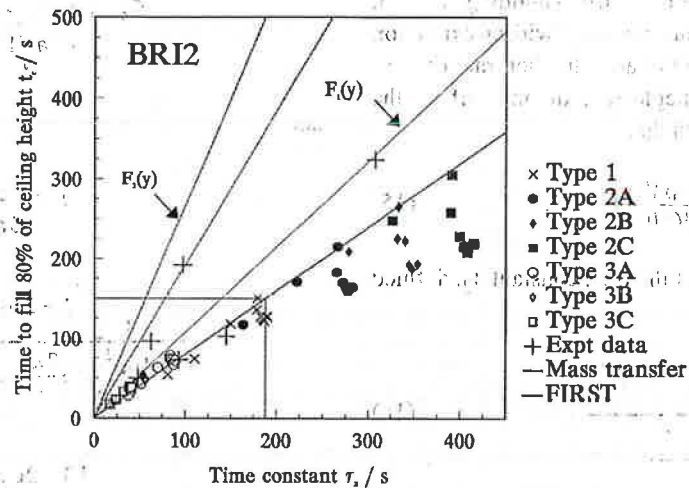


Figure 3 Results predicted by BRI2.

The full-scale experiments on a cubic hall reported by Hägglund et al. (1985) in Sweden were also studied. The hall measured 5.62 m (18.44 ft) long by 5.62 m (18.44 ft) wide by 6.15 m (20.18 ft) long. The fire sizes were varied from 0.25 m × 0.25 m (0.82 ft × 0.82 ft) to 0.75 m × 0.75 m (2.46 ft × 2.46 ft). The height of the fire above the floor level was changed from 0.2 m (0.66 ft) to 4.5 m (14.76 ft), and in this way values of the geometric aspect ratio and the time constant were varied and the smoke-filling time ( $t_f$ ) was changed. Also, experimental data on an assembly hall with floor area of 400 m<sup>2</sup> (4,306 ft<sup>2</sup>) and height of 10 m (32.8 ft) reported by Bengtson and Hägglund (1981) were also studied. The fire size was 16 m<sup>2</sup> (172 ft<sup>2</sup>) and the time constant was 2.41 minutes. All the data on the smoke-filling time,  $t_f$ , at different time constants were plotted and are shown in Figures 1 through 3.

It can be seen that Equation 13 (except  $t_2 > 100$  s), with mass transfer, and Equation 14, with both heat and mass transfer, are good for assessing the "smoke" hazard of atrium buildings. Whether a smoke extraction system has to be installed can be determined by checking whether the atrium with a certain time constant ( $t$ ) will give a time ( $t_f$ ) required to fill 80% of the atrium with smoke that is less than 2.5 minutes (150 s) using Equation (14).

To illustrate the use of the concept, a field survey of 40 atrium buildings was carried out. Their geometric configurations and the calculated time constants  $t_1$  and  $t_2$  are shown in Table 2. Most values of the time constants  $t_2$  in this sample are less than 188 seconds, giving an 80% smoke-filling time of less than 150 seconds. It is suggested that smoke extraction systems be installed. However, according to current fire regulations (FSD 1988, 1994), some of the atria having space volumes of less than 28,000 m<sup>3</sup> (1 million ft<sup>3</sup>) do not require smoke extraction systems. This point must be considered carefully.

### SMOKE EXTRACTION WITHOUT THERMAL EFFECTS

For the smoke-filling process in an atrium building of height  $H$ , floor area  $A_f$ , volume  $V$ , and time constant  $t$  with an extraction fan operating at the start of a fire at an extraction rate of a air changes per hour (ACH) and neglecting thermal effect, the decrease in clear height,  $y$ , is such that

$$-\frac{d}{dt}[\rho A_f(H-y)] = \frac{\alpha \rho V}{3600} K_2 y^{5/3} \quad (15)$$

This can be expressed in terms of the time constant,  $t_2$ , defined by Equation 8 as

$$\frac{dy}{dt} = \frac{\frac{2}{3} \frac{\alpha}{3600} \tau_2 H^{5/3} - y^{5/3}}{2 \tau_2 H^{2/3}} \quad (16)$$

The time required,  $t$ , to fill the atrium with smoke down to a clear height,  $y$ , is such that

$$\int_H^y \frac{dy}{y^{5/3} - a^5} = -\frac{3}{2 \tau_2 H^{2/3}} \int_0^t dt \quad (17)$$

where

$$a = \left[ \frac{2}{3} \frac{\alpha}{3600} \tau_2 H^{5/3} \right]^{1/5} \quad (17a)$$

A closed-form expression has been reported by Tanaka and Yamana (1985) as

$$\begin{aligned} \left( \frac{1}{2 \tau_2 H^{2/3}} \right) t = r \ln \left[ \frac{C_1(H)}{C_1(y)} \right] \\ + \frac{P_1}{2} \ln \left[ \frac{C_2(H)}{C_2(y)} \right] + \frac{P_1}{2} \ln \left[ \frac{C_3(H)}{C_3(y)} \right] \\ + P_3 \{ \tan^{-1} [C_4(H)] - \tan^{-1} [C_4(y)] \} \\ + P_4 \{ \tan^{-1} [C_5(H)] - \tan^{-1} [C_5(y)] \} \end{aligned} \quad (18)$$

where

$$C_1(x) = x^{1/3} - a \quad (18a)$$

$$C_2(x) = x^{2/3} - 2ax^{1/3} \cos \frac{2\pi}{5} + a^2 \quad (18b)$$

$$C_3(x) = x^{2/3} - 2ax^{1/3} \cos \frac{4\pi}{5} + a^2 \quad (18c)$$

$$C_4(x) = \frac{x^{1/3} - a \cos \frac{2\pi}{5}}{a \sqrt{1 - \cos^2 \frac{2\pi}{5}}} \quad (18d)$$

$$C_5(x) = \frac{x^{1/3} - a \cos \frac{4\pi}{5}}{a \sqrt{1 - \cos^2 \frac{4\pi}{5}}} \quad (18e)$$

and

$$r = \frac{1}{5a^2} \quad (18f)$$

$$P_1 = \frac{1}{10a^2} \left( \frac{3 + 2 \cos \frac{2\pi}{5}}{\cos \frac{2\pi}{5} \cos \frac{4\pi}{5}} \right) - \frac{0.3236}{a^2} \quad (18g)$$

$$P_2 = \frac{1}{10a^2} \left( \frac{3 + 2 \cos \frac{4\pi}{5}}{\cos \frac{2\pi}{5} \cos \frac{4\pi}{5}} \right) - \frac{0.4472}{a^2} \quad (18h)$$

**TABLE 2**  
**Survey on Atrium Buildings In Hong Kong**

Atrium Number	Length/m	Width/m	Height/m	Volume/m <sup>3</sup>	Time Constant t <sub>1</sub> /s	Time Constant t <sub>2</sub> /s
1	28	15	12	5040	131.13	202.07
2	29	20	24	13920	128.05	175.79
3	20	14	24	6720	61.82	84.86
4	28	20	24	13440	123.63	169.73
5	27	14	30	11340	74.64	98.73
6	27	25	24	16200	137.98	194.35
7	25	14	15	5250	97.74	145.12
8	36	23	16	13248	223.88	328.84
9	38	32	5.2	6323	576.74	1021.65
10	18	10	12	2160	56.2	86.60
11	25	20	15	7500	139.63	207.31
12	30	6.5	15	2925	54.46	80.85
13	36	17	45	27540	53.74	81.36
14	40	30	26	31200	122.37	211.61
15	12	12	24	3456	31.79	43.64
16	11	11	75	9075	15.11	17.16
17	15	15	40	9000	38.48	48.51
18	32	25	24	19200	137.98	205.67
19	60	14	14	11760	242.81	364.67
20	30	25	16	12000	202.79	297.86
21	14	14	16	3136	53	77.84
22	40	25	12	12000	312.22	481.12
23	32	25	24	19200	137.98	205.67
24	14	14	40	7840	33.52	42.26
25	10.5	9.5	13	1297	29.92	43.50
26	20	20	20	8000	96.74	136.90
27	21	21	7.8	3440	170.78	282.76
28	48	48	17	39168	231.49	463.44
29	30	20	15	9000	167.56	248.77
30	20	20	15	6000	111.7	165.85
31	40	35	20	28000	181.39	316.06
32	20	10	20	4000	48.37	68.45
33	28	25	20	14000	169.29	239.58
34	32	18	16	9216	155.75	228.76
35	28	14	12	4704	122.39	188.6
36	21	19	12	4788	124.58	191.97
37	28	12	24	8064	74.18	101.84
38	20	10	20	4000	48.37	68.45
39	17	14	20	4760	57.56	81.46
40	16	12	10	1920	65.67	104.31

$$P_3 = \frac{q_1 + P_1 a \cos \frac{2\pi}{5}}{a \sqrt{1 - \cos \frac{22\pi}{5}}} = \frac{0.2351}{a^2} \quad (18i)$$

$$P_4 = \frac{q_2 + P_2 a \cos \frac{4\pi}{5}}{a \sqrt{1 - \cos \frac{24\pi}{5}}} = \frac{-1.3764}{a^2} \quad (18j)$$

$$q_1 = \frac{1}{10a} \frac{(3 + 2 \cos \frac{2\pi}{5})}{(\cos \frac{2\pi}{5} - \cos \frac{4\pi}{5})} = \frac{0.3236}{a} \quad (18k)$$

$$q_2 = \frac{1}{10a} \frac{(3 + 2 \cos \frac{4\pi}{5})}{(\cos \frac{2\pi}{5} - \cos \frac{4\pi}{5})} = \frac{-0.4472}{a} \quad (18l)$$

This form is very complicated and perhaps a numerical solution of Equation 16 is better. The time required to fill 80% of the atrium space with smoke can be obtained by setting  $y = 0.2H$ . In this way,  $t_r$  is given by

$$t_r = \frac{2\tau_2}{5\beta^2} \ln \left[ \frac{1-\beta}{0.5848-\beta} \right] - \frac{0.3236\tau_2}{\beta^2} \ln \left[ \frac{1-0.618\beta+\beta^2}{0.342-0.3614\beta+\beta^2} \right] \\ - \frac{0.3236\tau_2}{\beta^2} \ln \left[ \frac{-1+1.618\beta+\beta^2}{0.342+0.9462\beta+\beta^2} \right] \\ + \frac{0.4702\tau_2}{\beta^2} \left\{ \tan^{-1} \left( \frac{1-0.309\beta}{0.9511\beta} \right) + \tan^{-1} \left( \frac{0.5848-0.309\beta}{-0.9511\beta} \right) \right\} \\ - \frac{2.7528\tau_2}{\beta^2} \left\{ \tan^{-1} \left( \frac{1+0.809\beta}{0.5878\beta} \right) - \tan^{-1} \left( \frac{0.5848+0.809\beta}{0.5878\beta} \right) \right\} \quad (19)$$

where

$$\beta = \left( \frac{2\alpha}{3(3600)\tau_2} \right)^{1/5} \quad (19a)$$

Results of  $t_r$  for air change rates of 6, 8, and 10 at different time constants are plotted in Figures 4 through 6. Note that for atrium spaces with small time constants, the time required to fill 80% of the atrium with smoke is undetermined, indicating that the air change rate is too great and the atrium space cannot be filled.

In view of the analytical expression of  $t_r$  with mass transfer only, a comparison of the relative magnitude of the different terms in Equation 19 indicated that the first term predominates. Therefore, the time required to fill 80% of the atrium with smoke,  $t_{r1}$  can be written as

$$t_r \sim \frac{2\tau_2}{5\beta^2} \ln \left( \frac{1-\beta}{0.5848-\beta} \right) \quad (20)$$

This expression is defined only in the range of  $b > 1$  and  $b < 0.5848$ , giving  $(at_2) > 5,400$  and  $(at_2) < 369.3$ , respectively. A plot of  $b$  against  $t_r$  at different rates of  $t_r$  is shown in Figure 7, and the curves can be used to determine the smoke extraction rate,  $a$ .

A simplified picture can be obtained by approximating the air entrainment equation. One can rewrite the time,  $t_r$ , required to fill 80% of the atrium with smoke as

$$t_r = C \int_H^{0.2H} F(y) dy \quad (21)$$

where

$$F(y) = \frac{1}{B-y^{5/3}} \quad (22a)$$

with

$$B = \frac{2}{3} \frac{\alpha}{(3600)} \tau_2 H^{5/3} \quad (22b)$$

and

$$C = \frac{2\tau_2 H^{2/3}}{3} \quad (22c)$$

The function  $F(y)$ , which depends on the air entrainment equation, is approximated between two extreme functions,  $F_1(y)$  and  $F_2(y)$ , defined to approximate  $y^{5/3}$  by  $f_1(y)$  and  $f_2(y)$ :

$$f_1(y) \sim H^{2/3} y \quad (23)$$

giving

$$F(y) \cong F_1(y) = \frac{1}{B-H^{2/3}y} \quad (24)$$

and

$$f_2(y) \sim \frac{y^2}{H^{1/3}} \quad (25)$$

giving

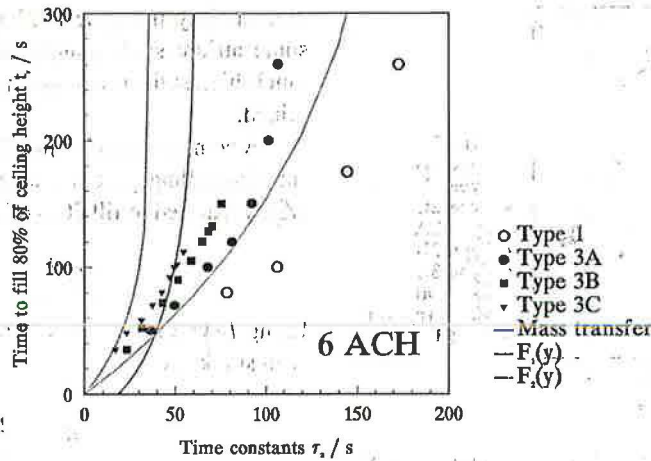
$$F(y) \cong F_2(y) = \frac{1}{B - \frac{y^2}{H^{1/3}}} \quad (26)$$

Using the first approximation,  $F_1(y)$ , the time  $t_{r1}$  required to fill 80% of the atrium with smoke is

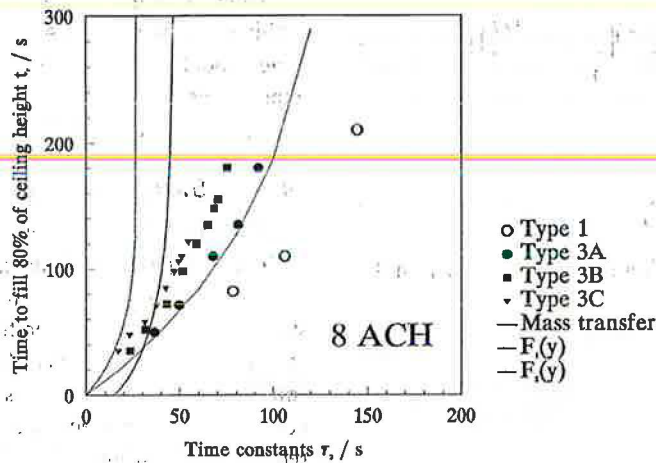
$$t_{r1} = \frac{2\tau_2}{3} \ln \left[ \frac{1-Z}{0.2-Z} \right] \quad (27)$$

where

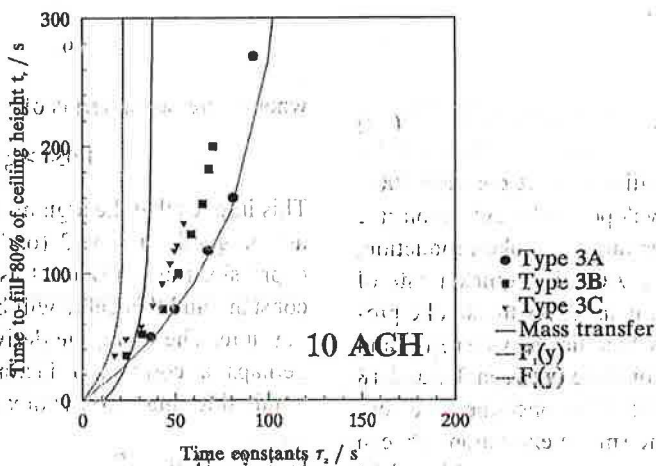
$$Z = \frac{2\alpha\tau_2}{3 \times 3600} \quad (28)$$



**Figure 4** Effect of smoke extraction at extraction rate of 6 air changes per hour.

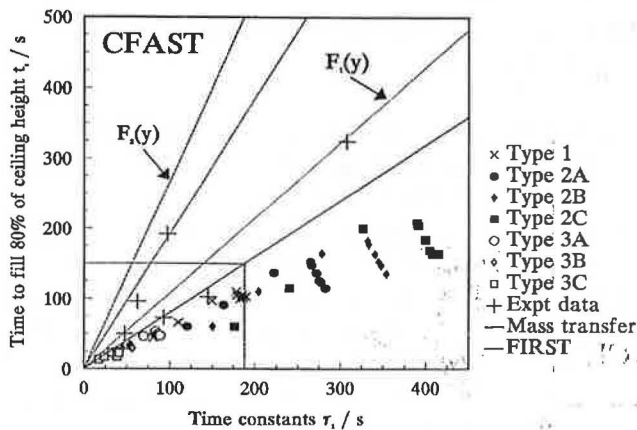


**Figure 5** Effect of smoke extraction at extraction rate of 8 air changes per hour.



**Figure 6** Effect of smoke extraction at extraction rate of 10 air changes per hour.





**Figure 7** Parameter  $\beta$  against time constants to get different smoke filling time,  $t_r$ .

Similarly, with the second approximation,  $F_2(y)$ , the time  $t_{r2}$  required to fill 80% of the atrium with smoke is

$$t_{r2} = \frac{2\tau_2^{1/2}}{9X^{1/2}} \ln \left[ \frac{0.2 - (x\tau)^{1/2}}{0.2 + (x\tau)^{1/2}} \left( \frac{1 + (x\tau)^{1/2}}{1 - (x\tau)^{1/2}} \right) \right] \quad (29)$$

where

$$X = \frac{2}{3} \frac{\alpha}{(3600)} \quad (30)$$

The first approximate expression for  $t_r$  implies that a smoke layer will be developed if

$$Z < 0.2$$

or

$$\alpha\tau_2 < 1080 \quad (31)$$

and the second approximation would give the following condition for developing a stable smoke layer:

$$(X\tau_2)^{1/2} < 1$$

or

$$\alpha\tau_2 < 5400 \quad (32)$$

This means that for an atrium building of time constant 300 s (5 min), a smoke layer will be developed if the extraction rate is less than 3.6 ACH for a slower rate of smoke production, given by  $f_1(y)$ , and is less than 18 ACH if a quicker rate of smoke production, given by  $f_2(y)$ , is used. As the smoke production rate is described by  $y^{5/3}$ , which lies between  $f_1(y)$  and  $f_2(y)$ , the smoke extraction rate would lie between 3.6 and 18 ACH for this particular atrium. Therefore, once the time constant of the atrium is specified, the smoke extraction rate can be estimated. The values of  $t_r$  given by Equations 27 and 29 for smoke extraction rates of 6, 8, and 10 ACH, are also plotted in Figures 4 through 6. These two curves can be treated as

the lower and upper limits for the smoke-filling time. Results predicted by the zone model FIRST are shown as well. For some atrium spaces such as type 2 atria, the clear height is much higher than 80% of the ceiling height, so  $t_r$  is undetermined.

A comparison of using  $f_1(y)$  and  $f_2(y)$  with  $y^{5/3}$  in the natural smoking-filling process can be derived. Using  $f_1(y)$ , the time ( $t_{r2}^{Nat}$ ) required to fill 80% of atrium with smoke is

$$t_{r2}^{Nat} = 1.073\tau_2 \quad (33)$$

Using  $f_2(y)$ , the time ( $t_{r2}^{Nat}$ ) required to fill 80% of the atrium with smoke is

$$t_{r2}^{Nat} = 2.667\tau_2 \quad (34)$$

These two expressions are also treated as the lower and upper limits of the time required to fill 80% of the atrium with smoke. The two curves are plotted against the atrium time constants from Figures 1 through 3 as well. The curve fitted by using mass transfer only (i.e., given by Equation 13) fell within these two limits. Again, the simulation results using the zone model are lower than the results given by these three expressions, as heat transfer is included in the zone models, giving a faster rate of smoke filling.

## DISCUSSION

Note that the time constant does not include other variables such as fuel package arrangement, nature of fuel, variation in plume radius and temperature with height, air movement in surroundings, etc. However, this is a much better parameter than the atrium space volume (FSD 1994) for determining whether a smoke extraction system has to be installed, and it should be considered in the specification of the fire protection systems required in an atrium.

An important note is that the ratio of the area to the square of the height in the atrium,  $A_f/H^2$ , would be important for the plume equations to hold (Klote and Milke 1992). The range of validity is

$$0.9 \leq A_f/H^2 \leq 14 \quad (35)$$

when expressed in terms of  $x$  or  $\sqrt{H^2/A_f}$ :

$$1.054 \geq \xi \geq 0.267 \quad (36)$$

This implies that the argument for using a time constant might not be good for type 3 (or high) atria. However, the plume expressions are only used as a vehicle for defining the time constant, and this value will be correlated with the smoke-filling time. Therefore, the derived expressions can be used and perhaps a correlation factor determined experimentally is required for high atria with  $x$  greater than 1.054.

## CONCLUSIONS

Use of the time constant  $t_2$  (defined by Equation 8) to correlate with the time required to fill 80% of an atrium with smoke

was evaluated. A correlation relation given by Equation 14 was derived using the zone model FIRST. The results simulated by three other zone models—CFAST, CCFM.VENTS, and BRI2—support this concept. Note that the approach is to define a parameter that can be derived from plume theory. With it, the smoke-filling time for atria of different shapes is fitted with fire zone models. It was proposed to get a simple form for design purposes; therefore, it was not surprising to see that values of  $t_1$  and  $t_2$  are different because different plume equations are being assessed.

The limits of validity for the plume expressions might be restricted within a certain range. However, as the smoke-filling time is simulated from fire models and correlated with the time constants, such deviation in the plume expression would not give serious errors. Full-scale experimental data did not show large deviation from the correlation relation given by Equation 4 for the natural smoke-filling process in some atria described here.

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## NOMENCLATURE

$A_f$	= floor area of atrium
$C_v$	
$C_p, C_T$	= parameters in plume equation
$g$	= acceleration due to gravity
$H$	= height of atrium
$\dot{m}$	= air entrainment rate to the plume
$p$	= perimeter of fire
$Q$	= heat release rate
$r$	= plume radius
$t_r$	= time required to fill 80% of atrium with smoke
$t_{r1}^{Nat}, t_{r2}^{Nat}$	= natural smoke-filling time
$DT_o$	= central temperature rise of plume
$V$	= volume of atrium
$v$	= vertical upward speed of air in the plume
$y$	= clear height of smoke layer
$z$	= height above the plume
$a$	= smoke extraction rate, ACH
$a_p$	= entrainment coefficient of plume
$b$	= constant for determining smoke extraction rate
$x$	= geometric aspect ratio of atrium
$\rho$	= density of air
$t_1$	= first time constant
$t_2$	= second time constant

## REFERENCES

- Butcher, E.G., and A.C. Parnell. 1992. The testing of smoke control systems. *Fire Engineers Journal*, September, pp. 23-25.
- Bengston, S., and B. Hägglund. 1981. A smoke filling simulation model and its engineering application. *Fire Technology-NFPA* 22(2): 92-103.
- Chow, W.K. 1989. FSD circular letter no. 13/88: A comment. *The Hong Kong Engineer*, November, p. 19.
- Chow, W.K. 1993. Smoke development and engineering aspects of smoke extraction systems for atria in Hong Kong. *Fire and Materials* 17(2): 71-77.
- Chow, W.K. 1994a. Smoke filling in atria: Time constant. *Building Services Engineering Research and Technology* 15(3): 165-170.
- Chow, W.K. 1994b. Fire aspects for atrium buildings in Hong Kong. Proceedings of the 7th International Research and Training Services, Regional Development Planning for Disaster Prevention: Improved Firesafety System in Development Countries, United Nations Centre for Regional Development, October 7, Tokyo, Japan.
- Chow, W.K. 1994c. A short note on the simulation of atrium smoke filling process using fire zone models. *Journal of Fire Sciences* 12(6): 516-528.
- Chow, W.K., and W.K. Wong. 1993a. On the simulation of atrium fire environment in Hong Kong using zone models. *Journal of Fire Sciences* 11(1): 3-51.
- Chow, W.K., and W.K. Wong. 1993b. Application of the zone model FIRST on the development of smoke layer and evaluation of smoke extraction design for atria in Hong Kong. *Journal of Fire Sciences* 11(4): 329-349.
- Chow, W.K., W. Fan, and L. Chen. 1993. Full-scale burning facilities for atrium fire research. *Fire Safety Science* 2(2): 60-63 (in Chinese).
- Cooper, L.Y., and G.P. Forney. 1990. The consolidated compartment fire model (CCFM) computer code application CCFM.VENTS—Part I: Physical basis. NISTIR 4342. Gaithersburg, Md.: National Institute of Standards and Technology.
- FSD. 1988. Smoke extraction system. Circular Letter No. 13/88. Hong Kong: Fire Services Department.
- FSD. 1994. *Code of practice for minimum fire services installations*. Hong Kong: Fire Services Department.
- Hägglund, B., R. Jansson, and K. Nirens. 1985. Smoke filling experiments in a 6 m x 6 m x 6 meter enclosure. FOA Report C20585. Sundbyberg, Sweden: National Defence Research Establishment.
- Klote, J.H. 1994. Method of predicting smoke movement on atria with application to smoke management. NISTIR 5516. Gaithersburg, Md.: National Institute of Standards and Technology.
- Klote, J.H., and J. Milke. 1992. *Design of smoke management systems*. Atlanta: American Society of Heating, Refrigerating and Air-Conditioning Engineers, Inc.

- Law, M. 1990. Fire and smoke models—Their use on the design of some large buildings. *ASHRAE Transactions* 90(2): 963-971.
- Milke, J.A., and F.W. Mowrer. 1994. Computer-aided design for smoke management in atria and covered malls. *ASHRAE Transactions* 100(2): 448-456.
- Mitler, H.E., and J.A. Rockett. 1987. *User's guide to FIRST: A comprehensive single-room fire model*. NBSIR 87-3595. Gaithersburg, Md.: Center for Fire Research, National Bureau of Standards, (NIST).
- Morgan, H.P., and J.P. Gardner. 1990. Design principles for smoke ventilation in enclosed shopping centres. Building Research Establishment Report. Borehamwood, U.K.: Fire Research Station.
- NFPA. 1995. *NFPA 92B, Guide for smoke management systems in malls, atria and large area*. Quincy, Mass.: National Fire Protection Association.
- Peacock, R.D., G.P. Forney, P. Reneke, R. Portier, and W.W. Jones. 1993. CFAST, the consolidated model of fire growth and smoke transport. *NIST Technical Note* 1299. Gaithersburg, Md.: National Institute of Standards and Technology.
- Tanaka, T., and I. Nakamura. 1989. A model for predicting smoke transport in buildings. Report of Building Research Institute, No. 123. Tokyo: Ministry of Construction.
- Tanaka, T., and T. Yamana. 1985. Smoke control in large spaces—Part 1. *Fire Science and Technology* 5(1): 31-40.
- Thomas, P.H. 1992. Smoke control system clarification. *Fire Engineers*, December, p. 6.
- Yamana, T., and T. Tanaka. 1985. Smoke control in large scale spaces—Part 2. *Fire Science and Technology* 5(1): 41-54.
- Zukoski, E.E., T. Kubota, and B. Cetegen. 1980/81. Entrainment in fire plumes. *Fire Safety Journal* 3(1): 107-121.