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Energy Use for Ventilation Systems in Underground Car Parks

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The use of energy for operating ventilation systems in underground car parks in Hong Kong is considered. A site survey has been carried out in 22 underground car parks. The physical size, configurations, and electrical rating of ventilation fans were the main matters of interest. The indoor thermal environments, carbon monoxide concentrations, and operation of ventilation systems were examined. Mathematical expressions are developed and the predicted results examined for another 54 underground car parks. The results show that significant savings can be achieved if a two-level ventilation strategy is implemented. © 1998 *Elsevier Science Ltd. All rights reserved.*

NOMENCLATURE

A_f	floor area of car park (m^2)
C	average instantaneous carbon monoxide concentration (m^3/m^3)
C_i	initial concentration of carbon monoxide before emission (m^3/m^3)
C_{asy}	asymptotic concentration of carbon monoxide (m^3/m^3)
C_{CON}	control level of carbon monoxide (50 ppm)
C_{1min}	1 min time-weighted average of carbon monoxide concentration (m^3/m^3)
C_{5min}	5 min time-weighted average of carbon monoxide concentration (m^3/m^3)
C_{15min}	15 min time-weighted average of carbon monoxide concentration (m^3/m^3)
C_{30min}	30 min time-weighted average of carbon monoxide concentration (m^3/m^3)
C_{1h}	1 h time-weighted average of carbon monoxide concentration (m^3/m^3)
C_T	transient value of carbon monoxide concentration detected by gas analyser (m^3/m^3)
E_T	total energy consumed by fan (J)
G	specific ratio of carbon monoxide generated to control level (dimensionless)
g_c	rate of carbon monoxide generation (m^3/h)
κ	power consumption per hour (W h)
L	constant of ventilation system resistance (kg/m^7)
n	ventilation rate in terms of air changes per hour (h^{-1})
n_1	ventilation rate at the period 0–t_1 in terms of air changes per hour (h^{-1})
n_2	ventilation rate at the period $(t_2 - t_1)$ in terms of air changes per hour (h^{-1})
n_c	ventilation rate obtained from classical sizing, i.e. $N = 1$ in terms of air changes per hour (h^{-1})
η_f	overall efficiency of ventilation fan (%)
n_p	specific rate of carbon monoxide generation in terms of air changes per hour (h^{-1})
N	$nC_{CON}/n_p (0 < N \leqslant 1)$
N_1	$n_1 C_{CON}/n_p (0 < N_1 \leqslant 1)$
N_s	number of parking stalls
Δp	pressure developed by ventilation fan (N/m^2)
$\dot{Q}_a$	outdoor air supply rate (m^3/s)
R_t	time ratio $(0 < R_t \leqslant 1)$
r^2	linear regression correlation coefficient
τ	nominal time constant (min)
0–t_1	carbon monoxide emission period (h)
t_2	a complete time period (h)
t_1–t_2	period carbon monoxide emission ceased (h)
λ	constant (h)
V	volume of the car park (m^3)
W_c	power consumed by combined system (kWh)
W_e	power consumed by exhaust-only system (kWh)
W_{f1}	power consumed by ventilation fan at the period 0–t_1 (kWh)
W_{f2}	power consumed by ventilation fan at the period t_1–t_2 (kWh)
W_f	power consumed by fan at a specific period (kWh)

1. INTRODUCTION

A car park is a place where motor vehicles are kept [1]. An underground car park means any car park which has a significant part of its floor more than 1.2 m below the level of the surrounding ground. Where the car park is built on sloping ground or considerable areas of excavated ground is left around its perimeter, it is regarded as underground only if ventilation to open air is difficult to obtain. Mechanical ventilation is essential for underground car parks. With the ventilation system exhaust from vehicles are diluted and extracted from the occupied zone in order to maintain an acceptable air quality.

Statutory controls over the installation of ventilation systems in Hong Kong are included in the Building Regulations of Hong Kong [2]. The regulations apply to any ventilating system that embodies the use of ducting or trunking, which passes through any wall, floor or ceiling of a building from one fire compartment to another. The regulations do not specify the ventilation rate, but do specify the technical requirements of a ventilating system and the duty of the owners. Internationally recognised ventilation codes [3, 4] have been adopted by ventilation engineers in designing ventilation systems for underground car parks in Hong Kong.

Most car parks in shopping centres and commercial

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areas in Hong Kong are open 24 h and for 18 h in residential areas. This gives rise to significant energy use. Given that the recommended illumination levels in underground car parks is comparatively low [5], the energy requirement for lighting is also low. Very few car parks have any heating or cooling systems installed. The major proportion of the energy consumed in an underground car park is for running the ventilation system. Consequently, optimising the use of energy for ventilation is important.

A questionnaire survey was carried out during July–September 1995 on the ventilation system provisions in underground car parks. Of the 53 questionnaires returned 22 were suitable for full analysis with respect to their energy consumption. The systems employed are either "exhaust-only system" or "combined supply and exhaust system", with exhaust volume greater than supply volume. None of the systems used a "supply-only system" because of the possibility of uncontrolled dispersion or leakage of polluted air. When an underground car park is part of a building, a "supply-only system" has the potential for pollutants to migrate up stairwells, elevator shafts or plumbing and wiring ducts into occupied areas. By the same principle an "exhaust-only system" would likely draw air from the occupied areas. However, as the air is cleaner than that in the car park, such infiltration is a benefit rather than a problem. The air-conditioning loading in the adjacent area will not be increased as only a small part of those space air would be used as return air for most systems.

A "combined system" can provide control over both supply and exhaust functions. Such a system allows more control of the distribution of intake (outside) air, and by adjustment and balancing, e.g. exhaust volume greater than supply volume, can prevent uncontrolled migration of polluted air to other parts of a building. Supply points should be located adjacent to main pedestrian routes. It is inevitably that this requires more energy to run than any other system.

Carbon monoxide (CO) is generally regarded as the major indicator of air pollution in an underground car park, and as an index of satisfactory ventilation [6, 7]. Of the pollutants considered harmful to human health, CO is the most likely to reach levels of concern. In addition, CO has been found to be a good index of the haze caused by oil vapour and particulate matter. CO is relatively easy to detect and lends itself to continuous monitoring. Clearly, any unwarranted increase in health risk is not good practice, but excessive energy use to reduce CO levels carries other penalties. Consequently, a comparison is made between health criteria and ventilation energy based on measurements of CO levels.

2. ENERGY CONSUMPTION OF THE VENTILATION SYSTEMS

As required by the Hong Kong Fire Services Department car park ventilation systems should provide a ventilation rate of eight air changes per hour when a fire breaks out [8]. For the sake of energy saving, dual speed fans are a solution to this requirement. In some instances, CO sensors are installed as a control device for the two-level ventilation strategy. The underlying concept is to employ the high ventilation rate for periods of congestion (high-level mode) and a lower ventilation rate during off peak periods (low-level mode). The mode of control can be automatic or by an operation schedule. An operation schedule is formulated according to the anticipated or observed utilisation rate.

The CO sensors are usually installed at return grilles at a level compatible with the human breathing zone. This is defined as between the horizontal planes at 100 mm and 1700 mm above the floor, and more than 600 mm from the walls or fixed equipment [9]. The control level for CO follows health guidelines, such as the Health and Safety Executive [10] 8-h time-weighted average 50 ppm or the Occupational Safety and Health Administration [11] 8-h time-weighted average 35 ppm. In this study 50 ppm is chosen as the control level. When the prevailing CO concentration within a car park is under 50 ppm, the low-level ventilation rate remains in operation. The increase of CO concentration above 50 ppm triggers the switch to high-level mode. When high levels of CO prevail, the system operates at high-level mode until the CO concentration falls below the control level.

3. QUESTIONNAIRE SURVEY

A questionnaire (Appendix A) was sent to car park management companies for the purpose of collecting data related to energy consumption. There are three parts to the survey form. Part A covers the general description of the car park, such as capacity, physical layout and types of vehicles accommodated. This enabled an estimation of pollutant load in the car park. The configuration of a car park enables a determination of the flow pattern of exhaust and fresh air. The types of vehicles helped determine the total quantity and chemical species, including CO from petrol engines, or oxides of nitrogen from diesel engines. Part B of the questionnaire is on the mechanical aspects of the ventilation system. This part is directly related to the efficiency of pollutant removal. The last part is the equipment schedule of types of fans and motors, and their power rating. This enabled an estimate of power and energy consumed by the ventilation system.

4. FIELD MEASUREMENT

Following the questionnaire survey, CO measurements were carried out at the 22 sites for which completed questionnaires were available. The instrument used for monitoring CO was a passive type Metrosonics Personal Gas Monitor pm-7700, placed in the breathing zone [9] at 1.5 m above floor level. The differential concentration caused the air sample to flow into the sensor without the driving force of a pump. Zero setting was checked before each measurement in accordance with manufacturer's recommendations, and repeated again every 2 or 3 h immediately between measurements.

The monitor was used in conjunction with a data logger for continuous monitoring. The data logger was programmed to store 1 min averaged data. The full scale range is 0–1003 ppm with a display resolution of 1 ppm and response time for 90% of 35 s. Repeatability of the sensor, as claimed by manufacturer, is 2%. The logged data was down loaded onto the logger's storage unit via the Metrologger interface. The data was subsequently transferred to a personal computer for processing.

The sampling rate was four readings per second in which each reading was the transient value of CO (C_T). The 240 readings recorded per minute were averaged to give the mean value for 1 min ($C_{1\text{min}}$). The reading was then averaged to 5 min ($C_{5\text{min}}$), 15 min ($C_{15\text{min}}$), 30 min ($C_{30\text{min}}$) and 1 h ($C_{1\text{h}}$) time-weighted average which are calculated as follows [12]:

$$C_{1\text{min}} = \frac{1}{240}\int_0^{240} C_T(t)\,dt \tag{1}$$

$$C_{5\text{min}} = \frac{1}{1200}\int_0^{1200} C_T(t)\,dt \tag{2}$$

$$C_{15\text{min}} = \frac{1}{3600}\int_0^{3600} C_T(t)\,dt \tag{3}$$

$$C_{30\text{min}} = \frac{1}{7200}\int_0^{7200} C_T(t)\,dt \tag{4}$$

$$C_{1\text{h}} = \frac{1}{14.400}\int_0^{14.400} C_T(t)\,dt \tag{5}$$

5. RESULTS

The resulting 1 min ($C_{1\text{min}}$), 5 min ($C_{5\text{min}}$), 15 min ($C_{15\text{min}}$), 30 min ($C_{30\text{min}}$) and 1 h ($C_{1\text{h}}$) time-weighted average are compared against applicable health risk criteria such as those of the WHO, OSHA, etc (Table 1). A grade, Good, Acceptable, or Poor was given to each car park describing the acceptability of air quality.

The measurement of energy consumed by ventilation fans at 22 sites was by use of MICROVIP portable energy analyser model MK1. calibrated before each measurement. The period of monitoring was at congested hours during full load operation typically (18:00–21:00). The scope of data collection included actual power consumed, root mean square value of current and voltage, and power factor of motors. The results are presented in Tables 2 and 3. A comparison of energy consumed by different systems is shown in Table 4.

It is apparent that the "combined system" consumed more energy but the average CO levels at these sites were well below those employing the "exhaust-only system", as shown in Tables 2, 3, and 4. The concentrations of CO obviously decreases with increasing fresh air supply.

Although there is no observed correlation between the CO level and the macroscopic parameters of the car park, such as floor area (A_f m^2), space volume (V m^3), number of parking stalls (N_s) nor fruitful results for optimisation, it does give a rough estimate of energy consumption at the inception stage when detailed information is not available. It is not appropriate to correlate the CO measurement results directly with fresh air supply rate because of the wide variance in vehicle traffic, car park size, nature of car park, and diurnal variations.

Correlation equations between the energy consumed and the parameters A_f, V and N_s were derived. From the results, the possible energy consumption by a car park can be estimated if the volume, area or number of stalls is given. Ventilation engineers could gain a rough idea of energy use from this regression analysis.

The following regression equations of energy W_e (kWh) consumed by an "exhaust-only system" with A_f, V and N_s are found with correlation coefficients (r^2) of 0.368, 0.676 and 0.468, respectively:

$$W_e = -7 + 0.014A_f \tag{6}$$

$$W_e = 0.98 + 0.0027V \tag{7}$$

$$W_e = -6.58 + 0.4N_s \tag{8}$$

Regression equations W_c (kWh) consumed by a "combined system" with the parameters A_f, V and N_s are found with correlation coefficients (r^2) of 0.19, 0.228 and 0.238, respectively:

$$W_c = 47.13 + 0.048A_f \tag{9}$$

$$W_c = 43.11 + 0.0019V \tag{10}$$

$$W_c = 24.45 + 0.253N_s \tag{11}$$

Table 1. Air quality guidelines for carbon monoxide. Carbon monoxide limits in ppm for various time weighted intervals

Authority	Ceiling limits	15 min	1 h	8 h	Comments
ACGIH [15]		[a]		25[b]	TLV-TWA. This is a US guideline for industrial workplaces. Not directly applicable to car parks.
USEPA [16]			35	9	
HKAQO [17]			27	9	Aligns with EPA U.S.A. primary air standards. Applies as a clean air target. Health effects of pollutant at elevated ambient levels: impairment of co-ordinated. deleterious to pregnant women and those with heart and circulatory conditions.
HSE [10]		300		50	8 h TWA is revised to 15-min in 1994 issue.
NIOSH [14]	200			35	Health effects: cardiovascular effects.
OSHA [11]	200			35[c]	This is a U.S. Labor Dept Occupational Safety and Health Standard
WHO [18]		87[d]	25	9	Applies to the whole population.

[a] Replaced by Biological Exposure Index.

[b] Reduced from 50 ppm in 1992–93.

[c] Recently introduced. Transitional limit is 50 ppm.

[d] Exposure at these concentrations should be for no longer than the indicated times and should not be repeated within 8 h.

Table 2. Surveyed car parks in Hong Kong

Car park No.	Ventilation system	Floor area in m^2	Volume in m^3 (V)	No. of parking stalls	Surveyed No. of cars half an hour	Design ventilation rate in m^3/s ($\dot{Q}_a$)	Air changes per hour (n)	Time constant in minutes (τ)	Indoor temp. in °C	Outdoor temp. in °C
1	Exhaust	423	1358	42	10	1.69	4.48	13.4	29.8	29.0
2	Combined	17,040	34,081	395	122	56.80	6.00	10.0	31.0	29.5
3	Combined	17,000	34,000	519	116	60.30	6.38	9.4	31.0	29.5
4	Combined	33,000	79,200	644	54	206.80	9.40	6.4	29.8	28.0
5	Combined	7848	19,620	282	54	38.75	7.11	8.4	32.8	31.2
6	Exhaust	1449	4057	67	10	0.62	0.55	109.1	22.0	20.1
7	Combined	2500	5000	200	40	30.00	21.60	2.8	30.2	29.8
8	Exhaust	19,166	138,000	780	19	460.00	12.00	5.0	32.5	32.0
9	Combined	3246	7142	132	23	12.40	6.25	9.6	24.4	21.2
10	Combined	3325	7314	144	25	12.70	6.25	9.6	25.0	21.2
11	Combined	3325	7647	137	16	12.41	5.84	10.3	24.0	21.2
12	Exhaust	576	1843	11	20	3.00	5.86	10.2	31.0	31.0
13	Exhaust	4000	14,000	150	27	25.98	6.68	9.0	33.0	31.8
14	Combined	5520	17,664	178	16	29.78	6.07	9.9	31.6	32.0
15	Combined	2000	9500	35	13	10.56	4.00	15.0	30.0	28.6
16	Exhaust	4000	16,000	60	5	26.67	6.00	10.0	31.0	31.0
17	Exhaust	12,960	34,992	426	54	28.19	2.90	20.7	30.5	28.0
18	Exhaust	5455	42,000	450	12	70.00	6.00	10.0	29.1	28.9
19	Combined	28,764	65,260	542	127	147.92	8.16	7.4	32.8	31.0
20	Exhaust	2654	6900	60	6	15.33	8.00	7.5	30.5	29.5
21	Combined	6500	30,550	100	76	43.96	5.18	11.6	32.0	30.6
22	Combined	3113	10,428	140	17	22.80	7.87	7.6	30.5	28.6

Table 3. *In-situ* measurement results of carbon monoxide levels

Car park No.	$C_{1\ \mathrm{min}}$ in ppm	$C_{5\ \mathrm{min}}$ in ppm	$C_{15\ \mathrm{min}}$ in ppm	$C_{30\ \mathrm{min}}$ in ppm	$C_{1\ \mathrm{hour}}$ in ppm	Power (W) consumed per volume unit m^3	Power (W) consumed per unit area m^2	Power (W) consumed per stall	Comments on air quality	Remarks
1	348	156	92	70	40	0.12	0.38	3.81	Poor	1, 2, 4, 5,6
2	353	257	177	166	129	3.70	7.39	318.99	Poor	1, 2, 4, 5, 6
3	225	142	107	90	73	3.35	6.71	219.65	Poor	1, 2, 4, 5, 6
4	54	30	20	15	14	1.14	2.73	139.75	Acceptable	
5	20	15	15	14	14	13.76	34.40	957.45	Good	
6	87	13	8	7	6	0.49	1.38	29.85	Acceptable	
7	42	26	24	20	18	23.20	46.40	580.00	Good	
8	27	6	5	5	5	3.33	23.59	588.75	Good	
9	67	42	34	28	27	1.26	2.77	69.18	Poor	4
10	109	40	34	30	24	1.23	2.71	62.50	Good	
11	31	22	20	17	14	1.18	2.71	65.69	Good	
12	151	47	36	29	28	4.39	14.06	736.36	Poor	4, 5
13	234	173	84	46	37	2.00	7.00	186.68	Poor	1, 4, 5, 6
14	45	20	17	11	7	2.97	9.51	294.94	Good	
15	40	35	32	28	24	1.37	6.52	372.57	Good	
16	47	8	8	7	6	1.25	5.00	333.33	Good	
17	618	507	362	255	154	0.34	0.93	29.00	Poor	1, 2, 3, 4, 5, 6
18	9	8	7	7	6	1.50	11.55	140.00	Good	
19	12	10	9	8	6	8.16	20.39	606.06	Good	
20	101	47	32	24	20	7.04	18.30	810.00	Good	
21	667	74	70	45	28	3.11	14.62	950.00	Poor	1, 4, 5
22	19	15	15	12	11	6.68	22.70	720.72	Good	

Remarks: 1: Exceeded NIOSH [14] and OSHA [11] Ceiling limits (200 ppm); 2: Exceeded WHO [18] 15 min TWA (87 ppm); 3: Exceeded HSE [10] 15 min TWA (300 ppm); 4: Exceeded WHO [18] 1 h TWA (25 ppm); 5: Exceeded HKAQO [17] 1 h TWA (27 ppm); 6: Exceeded USEPA [16] 1 h TWA (35 ppm).

Comments: Good: Meet all recommended standards; Acceptable: Meet all recommended standards, but the maximum value lies in between 50–200 ppm; Poor: Do not meet any ONE of the recommended standards.

6. FUNDAMENTAL CONCEPTS

Studying the CO level under steady state condition is not a satisfactory approach for evaluating the performance of car park ventilation systems. The pollutant loading is a dynamic variable. The diurnal variation of vehicle flow presents an uncertain design factor. One of the solutions is to treat the whole space as a single well

Table 4. Comparison of combined and exhaust system

	"Combined system" (13 sites)	"Exhaust-only system" (9 sites)	Difference in % with respect to exhaust-only system
Average energy consumed per stall	412 W/stall	318 W/stall	30
Average energy consumed per unit volume	5.5 W/m^3	2.3 W/m^3	139
Average energy consumed per unit area	13.8 W/m^2	9.1 W/m^2	52
Average maximum carbon monoxide levels	130 ppm	172 ppm	−24
Average 5 min carbon monoxide levels	56 ppm	107 ppm	−48
Average 15 min carbon monoxide levels	44 ppm	70 ppm	−37
Average 1 h carbon monoxide levels	30 ppm	34 ppm	−9

mixed zone. The emission of CO is assumed to follow a periodic function with step input. This assumption is closer to the real situation than that represented by a steady state equation.

If C is the instantaneous CO concentration, the mass balance for complete mixing of CO and air will give:

$$\frac{\mathrm{d}C}{\mathrm{d}t} = n_p - nC \qquad (12)$$

In the above equation, n is the ventilation rate in terms of air changes per hour of the car park (h^{-1}) expressed in terms of the rate of outdoor air supplied $\dot{Q}_a$ (m^3 s) and the volume of car park V (m^3):

$$n = \frac{3600 \cdot \dot{Q}_a}{V} \qquad (13)$$

n_p is the specific rate of CO generation expressed in terms of air changes per hour, which is the CO generation rate (g_c) divided by the volume (m^3).

$$n_p = \frac{g_c}{V} \qquad (14)$$

An asymptotic concentration C_{asy} may be expressed as the specific rate of CO generation divided by the ventilation rate of the car park:

$$C_{asy} = \frac{n_p}{n} \qquad (15)$$

If C_{CON} is the control level for CO (as shown at Fig. 1), a new parameter N can be defined as the ratio of C_{CON} to C_{asy}:

$$N = \frac{C_{CON}}{C_{asy}} (0 < N \leqslant 1) \qquad (16)$$

N is a macroscopic number having a value between zero and unity. This number N is not expected to be higher than unity because the controlled CO level is less than the asymptotic value of CO in the car park. No optimisation is required if the CO level is controlled within the healthy standard.

Another useful design parameter the nominal time constant τ (in minutes) of the ventilation system is expressed as:

$$\tau = \frac{V}{60 \cdot \dot{Q}_a} \qquad (17)$$

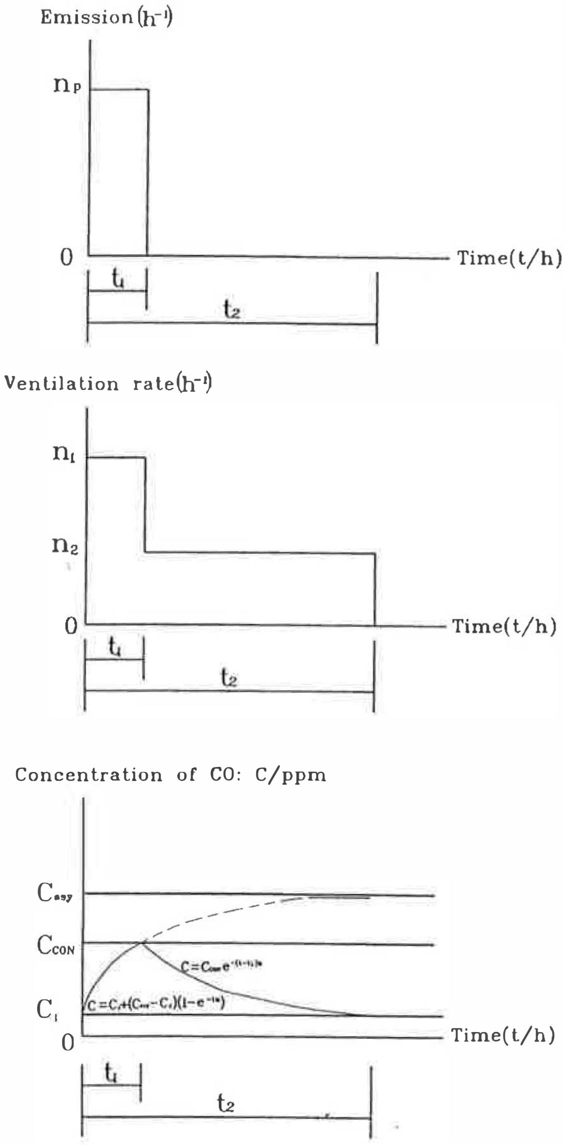

Fig. 1. Emission cycle illustration.

The physical meaning of the nominal time constant is the average residence time in the space of the air supplied to the space. That is, each volume unit of the supplied flow rate will, on average, stay in the space for a time period equal to the nominal time constant. A large time

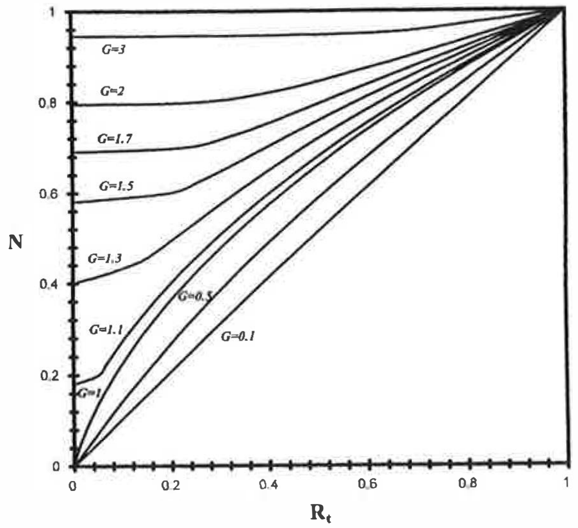

Fig. 2. Nomograph of N against R_t of single level operation.

constant represents the stale air trapped in the space for a longer time, whilst a small time constant represents a quicker replenishment of fresh air.

CO concentration at emission stage is given by solving eqn (12) by taking time period 0–t_1 as the CO emission period in hours, t_1–t_2 as the period when CO emissions cease (Fig. 1), with C_1 the initial concentration:

$$C = C_1 + (C_{\text{asy}} - C_1)(1 - e^{-nt}) \tag{18}$$

The concentration C at eqn (18) eventually equals to C_{CON} (control level of CO, e.g. 50 ppm) when time t approaches t_1.

If the emission ceases ($n_p = 0$), direct integration of the decay equation from time t_1–t_2 with initial concentration C_{CON} will give the instantaneous concentration C at time t:

$$C = C_{\text{CON}} e^{-(t-t_1)n} \tag{19}$$

The target for operating the ventilation system is to have concentration C given by eqn (19) eventually reduced to C_1 at time t_2, that is:

$$C_1 = C_{\text{CON}} e^{-(t_2-t_1)n} \tag{20}$$

Analysis of periodic emission problem reported by Lorenz [13] is based on these assumptions.

The specific ratio of CO generated to match the control level (G) is given by:

$$G = \frac{n_p t_1}{C_{\text{CON}}} \tag{21}$$

The parameter R_t is also defined as:

$$R_t = \frac{t_1}{t_2} \tag{22}$$

Equations (21) and (22) can be combined, with N as given by eqn (16) included, so that:

$$R_t = \frac{-GN}{\ln\left(1 - \dfrac{1 - e^{-GN}}{N}\right)} \tag{23}$$

Equation (23) is solved by computing R_t as a function of G and N. The solution is the optimised single-level ventilation rate to cope with a specific CO generation pattern, as explained in Fig. 2.

7. TWO-LEVEL VENTILATION

The two-level ventilation strategy is an alternative for dealing with emission problems. A ventilation rate of n_1 is employed at CO emission period (0–t_1). A second level of ventilation rate n_2 is applied when the emission of CO ceases (t_1–t_2). This concept was first proposed by Lorenz [13] after taking account that the emission of pollutant is not at maximum levels most of the time, but the ventilation system is operating at full capacity. To cope with two-level ventilation strategy, CO sensors should be installed as part of the ventilation control system. This was not common in the past, but more property management companies are accepting this as having potential for energy saving since 1982 which was reflected in the questionnaire survey.

At present, there has been no detailed investigation or analysis focusing on this strategy for Hong Kong car parks. Some car parks built in the 90's have incorporated dual speed fans, but the strategy is still not widely recognised. From the questionnaire survey, only one out of 22 sites has dual speed fans and CO sensors. Furthermore, the selection of high speed and low speed is entirely based on the management's anticipation of traffic flow, and fire protection requirements, rather than as a compromise of energy saving and CO level. The analysis which follows indicates the potential reduction in power and energy use for the systems surveyed.

A new parameter N_1 may be defined in terms of n_1 (the ventilation rate in terms of air changes per hour during emission stage 0–t_1), C_{CON} (the control level of CO) and n_p (the specific rate of CO generation rate), as follows:

$$N_1 = \frac{n_1 C_{\text{CON}}}{n_p} \tag{24}$$

Combining eqns (15), (18), (20), (22) and (24), gives N_1 the following relationship with the parameters G, t_1 and t_2.

$$N_1 = \frac{1 - e^{-GN_1}}{1 - e^{-(t_2-t_1)n_2} \cdot e^{-GN_1}} \tag{25}$$

The ventilation rate (n_2) in terms of air changes per hour at purging stage (t_1–t_2) is obtained by the following equation (derivation is shown in Appendix B):

$$n_2 = \frac{-1}{(t_2 - t_1)} \ln \frac{N_1 + e^{-GN_1} - 1}{N_1 e^{-GN_1}} \tag{26}$$

Power consumed by fan W_f at the ventilation rate n is given by the equation:

$$W_f = \frac{\Delta p \cdot n \cdot V}{3600 \eta_f \times 1000} \tag{27}$$

where Δp is the pressure developed by the fan; η_f is the overall efficiency of the fan.

$$W_f = \kappa\lambda^2 n^3 \tag{28}$$

where

$$\kappa = \frac{LV^3}{\eta_f(3600)^3(\lambda)^2 \times 1000} \tag{29}$$

L is a constant of ventilation system resistance (kg/m^7). $\lambda^2 n^3$ is expressed in h^{-1} which can be taken as an equivalent ventilation rate [13]. The power consumed (W_f) is converted to kilowatt hour (kWh) because the tariff system in Hong Kong is counted in kWh. λ is fixed as one. The equation is intended to find out the 1 h power consumption.

Total energy consumed by fans (E_T) at two-level ventilation is as follows:

$$E_T = t_1 W_{f1} + (t_2 - t_1) W_{f2} \tag{30}$$

$$\frac{E_T}{\kappa} = (\lambda^2 n_1^3)t_1 + (\lambda^2 n_2^3)(t_2 - t_1) \tag{31}$$

$$\frac{\partial E_T/\kappa}{\partial n_1} = 3\lambda^2 n_1^2 t_1 + 3\lambda^2 n_2^2 (t_2 - t_1) \cdot \frac{\partial n_2}{\partial n_1} \tag{32}$$

where W_{f1} is the power consumed by ventilation fan at the period 0–t_1 (kWh); W_{f2} is the power consumed by ventilation fan at the period t_1–t_2 (kWh). Setting $(\partial E_T/k)/\partial n_1$ to 0 allows the solution for optimum value of n_1.

By substituting appropriate macroscopic parameters:

$$\left(\frac{1-R_t}{R_t}\right)^2 = \left\{\ln^2 \frac{N_1 + e^{-GN_1} - 1}{N_1 e^{-GN_1}}\right\} \cdot \left\{\frac{1 - e^{-GN_1} - GN_1(1 - N_1)}{(N_1 + e^{-GN_1} - 1)GN_1}\right\} \cdot \left\{\frac{1}{(N_1 G)^2}\right\} \tag{33}$$

The solving of eqn (33) is for R_t as a function of G and N_1 which is shown in Fig. 3.

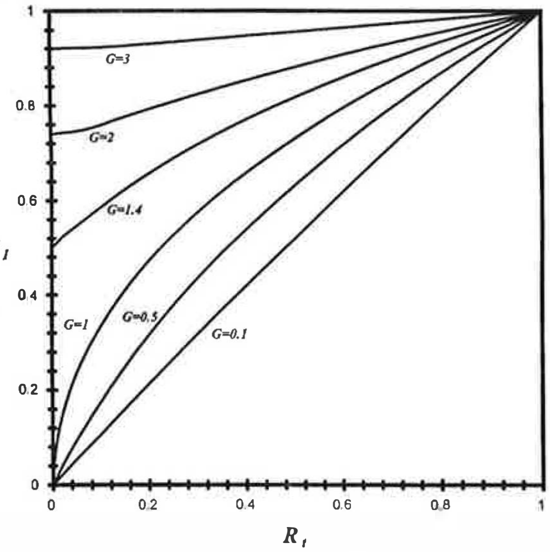

Fig. 3. Nomograph of N_1 against R_t of two-level operation.

8. ENERGY SAVINGS

The above concept was applied to all 54 sites surveyed and the results were compared against existing design values, as shown in Table 5. Few sites have CO sensors, but about half have dual speed fans. It is necessary to install CO sensor control or some other control means if the two-level ventilation strategy is implemented.

The aim is to show that significant savings can be achieved for given CO levels. Whilst this is a mathematical approach, and likely deviates from real circumstances, it suggests energy saving can be up to 90% of the original consumption, if emission periods are very short. This is often the case in residential buildings or office buildings where congested hours are limited to relatively short morning and evening rush-hours.

The R_t value is varied from 0.1–0.9, in order to give a series of reduced air flow rates at different conditions. The emission strength is according to the existing design value; i.e., the emission rate, g_c is 50×10^{-6} times existing fresh air supplied per hour. This enables a comparison of reduced air flow rate, whilst the level of CO is kept below

Table 5. Reduced air flow rate with different R_t value

R_t	Single-level reduction in % with respect to existing design value	Two-level reduction in % with respect to existing design value
0.9	0.6	10
0.7	2.7	24.8
0.5	8.1	40.3
0.3	24.7	55.6
0.1	80.9	84.8

Note: This is the average over 54 sites based on Lorenz model [13].

Table 6. Value of n_p, n_c, n, n_1 and n_2 at $R_t = 0.5$ at 22 surveyed sites

Site no.	n_p (h^{-1})	n_c (h^{-1})	n (h^{-1})	n_1 (h^{-1})	n_2 (h^{-1})
1	2.23×10^{-4}	4.45	3.91	4.12	1.59
2	3×10^{-4}	6.00	5.70	5.81	1.62
3	3.18×10^{-4}	6.37	6.18	6.21	1.63
4	4.7×10^{-4}	9.40	9.32	9.35	1.33
5	3.56×10^{-4}	7.11	6.93	6.99	1.56
6	2.75×10^{-5}	0.55	0.30	0.31	0.28
7	1.08×10^{-3}	21.6	21.6	21.6	0
8	5.61×10^{-4}	11.22	11.20	11.21	0.45
9	3.13×10^{-4}	6.25	5.98	6.08	1.60
10	3.05×10^{-4}	6.10	5.80	5.92	1.62
11	2.92×10^{-4}	5.84	5.55	5.64	1.62
12	2.93×10^{-4}	5.86	5.57	5.66	1.62
13	3.34×10^{-4}	6.69	6.52	6.54	1.61
14	3.03×10^{-4}	6.07	5.76	5.88	1.62
15	1.86×10^{-4}	3.71	3.06	2.07	0.52
16	3×10^{-4}	6.00	5.70	5.81	1.62
17	1.44×10^{-4}	2.89	2.16	2.39	1.30
18	3×10^{-4}	6.00	5.70	5.81	1.62
19	4.1×10^{-4}	8.18	8.07	8.10	1.59
20	6×10^{-4}	12.00	12.00	12.00	0
21	2.59×10^{-4}	5.18	4.79	4.93	1.64
22	3.68×10^{-4}	7.37	7.19	7.26	1.54

Table 7. Saving of power at 22 surveyed sites (kWh) at $R_t = 0.5$ (2-level ventilation strategy)

Site no.	Existing design value (kWh)	Two-level ventilation strategy (kWh)	Power saving in % with respect to existing design value
1	0.16	0.066	58.8
2	87.59	60.39	31.1
3	70.02	44.3	36.7
4	300	147.79	50.7
5	270	130.29	51.7
6	2	0.626	68.7
7	116	58	50
8	459	149.3	67.5
9	13.6	6.17	54.6
10	13.6	6.2	54.4
11	13.6	6.15	54.8
12	8.1	3.74	53.8
13	28	13.34	52.4
14	60.1	28.85	52
15	4.89	2.59	47
16	20	9.23	53.9
17	6	2.3	61.7
18	63	29.18	53.7
19	532	239.4	55
20	48.6	24.3	50
21	95	42.46	55.3
22	70.6	34.8	50.7

Note: Supply air flow rate < exhaust air flow rate at all sites. The saving of power is limited to exhaust fans only.

50 ppm. That is, the proposed two-level ventilation strategy is sufficient to control the concentrations of CO below the control level (50 ppm).

If the geometric configurations of ventilation fans in a car park remain unchanged, the change of volumetric flow rate has a relationship with power consumption, given by:

$$\left(\frac{\dot{Q}_a}{\dot{Q}'_a}\right)^3 = \frac{W_f}{W'_f} \tag{34}$$

Where $\dot{Q}_a$ and W_f are the volumetric flow rate and power consumed, respectively, at condition 1; $\dot{Q}'_a$ and W'_f are the volumetric flow rate and power consumed at condition 2, respectively. The percentage of saving of one-level and two-level ventilation with respect to existing design values is shown in Table 5.

9. DISCUSSION

CO levels in six out of 22 sites exceeded National Institution of Occupational Safety and Health [14] and Occupational Safety and Health Association [11] ceiling limits (Table 1). Momentarily high levels of CO was experienced at these sites (225–667 ppm). At site number eight, air changes per hour was 12 which was twice of average value. Although it was an "exhaust-only system", the energy consumed was as much or even greater than that of a "combined system". The choice of "combined system" or "exhaust-only system" therefore, entirely depends on the physical configurations of the car park. Stagnant zones might be created at some locations if there are cul-de-sacs, particularly long ones far from openings. "Combined system" provides better circulation by careful design of supply grilles.

The correlation between energy and air changes per hour can be used as a reference for estimate of energy consumption at the design stage. From the regression analysis, the energy consumed by an underground car park can be predicted at the inception stage if basic details such as number of stalls, volume of car park, and area are available.

On trial test of the Lorenz [13] model, great savings can be achieved by using single-level or two-level strategy (Table 5). It can be seen that the ventilation rate obtained from the optimisation is much less than the existing design value. The Lorenz model is applicable if the emission is confined within a relatively short period of time, which is often the real situation. The emission was found high at congested hours. The ratio of emission period to a complete cycle is R_t. As the value of R_t tends to one, the value of n_1 comes close to the conventional sizing method n_c ($N = 1$). The physical interpretation is that if the emission is continuous throughout the cycle, ventilation system remains at full load all the time. Energy savings cannot be achieved. On the contrary, saving is significant when R_t tends to be a smaller value (refer to Table 5 sensitivity analysis). If the emission period of CO (t_1) is only a small portion of the whole cycle (t_2), significant savings can be achieved. If G is less than or equal to one, the concentration of CO within the space would never exceed control level (C_{CON}) unless low-level ventilation (n_2) is zero. The physical meaning is that the ventilation system is shut down from t_1–t_2 and CO accumulates. The minimum value of ventilation rate (n) is determined by R_t and G. The physical meaning is explained by the way; increasing rate of emission (g_c), or period of emission (t_1) requires a higher ventilation rate to satisfy dilution. Equation (23) happens to be undefined, if the parenthesis terms are equal to or less than 0. N is a positive number which is greater than zero, but less than unity (if N equals to 0, implies no mechanical ventilation from 0–t_1). If G is smaller than one, the validity of eqn (23) is in no doubt. G is a function of specific rate of emission of CO (n_p) and emission period (t_1), it is possible to have G greater than one. If G is greater than one, the validity of eqn (23) will only be fulfilled under the condition that the terms in parenthesis are greater than zero. This requires a higher N, which means a higher ventilation rate (n).

With a greater ventilation rate (n_1) during the emission period (from 0–t_1), the concentration prevailing within the space is kept below the control level (C_{CON}). Following this period, a comparatively lower ventilation rate (n_2) takes over for the rest time of the cycle from t_1–t_2. The duty of low-level ventilation is to remove the residual CO before the commencement of another cycle. This is the basic principle of two-level ventilation strategy. The energy saving from two-level ventilation is considerable because the ventilation rate from t_1–t_2 is low comparatively to emission period.

10. CONCLUSIONS

Ventilation systems employed in underground car parks are usually either "combined supply and exhaust" systems or "exhaust-only" systems in Hong Kong. For a given car park a "combined system" consumes more energy than an "exhaust-only" system, but gives better fresh air distribution. The average CO level in "combined system" car parks is lower than "exhaust-only" system car parks based on the survey results. Increasing outdoor air supply is a positive step towards better indoor air quality but higher energy consumption. The prudent installation of CO sensors is a practice to reduce energy without sacrificing air quality.

Nearly 30% of the surveyed underground car parks exceeded National Institute of Safety and Health [14] or Occupational Safety and Health Administration [11] criteria. Regression lines were constructed to correlate energy used and dimensions for the surveyed systems. The result can be treated as a rough estimate of energy consumption.

The second purpose of this study was to find out the energy saving using two-level ventilation system with the help of the Lorenz [13] model. The calculations show that reduction of air flow rate up to 80.9% is possible for single-level if the emission period is very short, and as far as 84.8% for two-level when R_t equals to 0.1. As mentioned before, the choice of higher initial cost of dual speed fans and low running cost is highly dependent on the parameters G, N, R_t and ventilation strategy in some borderline cases. A thorough analysis is recommended at inception stage. Although good mixing is difficult to achieve in real situations, under similar imperfect mixing condition this mathematical approach is still useful as a quick reference for design purposes. The advantage of this approach is that a relatively precise ventilation rate can be derived through a rather simple calculation. If the control strategy can match the calculation, energy saved can be remarkable. Apart from running cost, the saving of capital cost for duct work could be considerable. If ventilation effectiveness could be incorporated into the analysis, it is definitely a promising design method in the future.

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APPENDIX A: QUESTIONNAIRE FOR SURVEYING THE DESIGN PARAMETERS

Questionnaire on Ventilation Design in Underground Car Parks
By W. K. Chow and M. Y. Chan, Department of Building Services Engineering, Hong Kong Polytechnic

Please complete the following questions as far as possible. The information surveyed from this questionnaire is used only for academic research and not for commercial purposes. (*Tick as appropriate)

Part A : General Description of the Car Park

(1) Name of the car park (Option) : ____________
(2) Name of Management Company (Option) : ____________
(3) [FS9]Address (Option) : ____________
(4) Nature of the car park :
Open for public/Exclusive for tenants/Both*
(5) Physical dimensions of the car park :
Length____(m) Width____(m) Height____(m)
(6) Shape of the car park :
Rectangular Square/Circular/Elliptic/Irregular* (please specify) ____________
(7) Total number of parking levels : ____________
(8) Total number of parking spaces (please specify parking spaces at each level) : ____________
(9) Categories of vehicles entering the car park :
Motorcycle Private cars/Light vans/Medium goods vehicle-/Heavy goods vehicle*

Part B : Ventilation System

(1) Types of ventilation system (please specify each level separately if necessary) :
Supply fan only/Exhaust fan only/Supply and exhaust fan*
(2) Size of supply grilles :
Length____(m) Width____(m)
(3) Size of exhaust grilles :
Length____(m) Width____(m)
(4) Volume flow rate for each supply grille :____m^3/s or ____cfm
(5) Volume flow rate for each exhaust grille : ____m^3/s or ____cfm
(6) Location of supply grilles : High/Low/Both*
Height above floor level ____(m)
(7) Location of exhaust grilles : High/Low/Both*
Height above floor level____(m)
(8) Grilles are : Side mount/Ceiling mount/Other* ________
(9) Outdoor air supply rate :____m^3/s or____cfm
For more than one level, please specify the air supply rate of each floor level ____________
(10) Exhaust air rate :____m^3/s or____cfm
For more than one level, please specify the air supply rate of each floor level ____________
(11) Number of supply fans installed : ____________
For more than one level, please specify the number of supply fans of each floor level ____________
(12) Number of supply fans installed : ____________
For more than one level, please specify the number of supply fans of each floor level ____________
(13) Fans are Single speed/Variable speed/Two speed/Other*
(14) Any openings for natural ventilation :
Yes/No*
(a) If yes, such openings are :
Vertical/Horizontal/Both*
(b) Total number of such openings are :
Vertical____Horizontal____
(c) Size of opening :
Length____(m) Width____(m)
(d) Please roughly delineate the location of such openings :
____
(e) How the ventilation rate is controlled by: Manual/Operation schedule/CO sensors/Other*
____

This page is the equipment schedule for fans. Please fill in any other equipment on the lower page for a brief description. Attaching schematic diagram for the equipment, drawing layout, catalogue of equipment, and operating schedules together with this questionnaire is welcome.

Fan no.________ Brand :________
Model no.________ For supply/exhaust*
Year installed :________ Location :________
Volume flow rate (m^3/s) :____ Total pressure (Pa) :____
Single speed/Variable speed/Two speed/Other* :________
Power (kW) :____ Efficiency :____ R.P.M. :____
Volt :____ Phase : Single/Three* Frequency (Hz) :____

Fan no.________ Brand :________
Model no.________ For supply/exhaust*
Year installed :________ Location :________
Volume flow rate (m^3/s) : ____ Total pressure (Pa) :____
Single speed/Variable speed/Two speed/Other* :________
Power (kW) :____ Efficiency :____ R.P.M. :____
Volt : ____ Phase : Single/Three* Frequency (Hz) :____

Other equipment

APPENDIX B : DERIVATION OF THE KEY EQUATIONS

Part A : Single level ventilation strategy

The rate of air changes per hour is defined as :

$$n = \frac{3600 \times \dot{Q}_a}{V} \tag{B1}$$

and the rate of carbon monoxide emission rate is :

$$n_p = \frac{3600 \times g_c}{V} \tag{B2}$$

Conservation of mass gives :

$$\frac{dC}{dt} = n_p - nC \tag{B3}$$

By solving the eqn (B3), the instant value of C is :

$$C = C_1 + (C_{asy} - C_1)(1 - e^{-tn}) \tag{B4}$$

where C_1 is the initial concentration at time 0 (see Fig. 1) and C_{asy} is the asymptotic approximation concentration :

$$C_{asy} = \frac{n_p}{n} \tag{B5}$$

For the case with no emission i.e., $n_p = 0$ from the time t_1–t_2, the instant value of C is obtained from the following equation :

$$C = C_{CON} e^{-(t - t_1)n} \tag{B6}$$

At time t_1, eqn (B4) becomes :

$$C_{CON} = C_1 + (C_{asy} - C_1)(1 - e^{-t_1 n}) \tag{B7}$$

At time t_2, eqn (B6) becomes :

$$C_1 = C_{CON} e^{-(t_2 - t_2)n} \tag{B8}$$

Combining eqns (B7) and (B8) gives :

$$C_{\mathrm{CON}} = C_{\mathrm{asy}}\cdot\frac{1-e^{-t_1 n}}{1-e^{-t_2 n}} \tag{B9}$$

and

$$C_1 = C_{\mathrm{asy}}\cdot\frac{e^{t_1 n}-1}{e^{t_2 n}-1} \tag{B10}$$

In view of eqn (21). the specific ratio G is given by:

$$G = \frac{n_{\mathrm{p}} t_1}{C_{\mathrm{CON}}} \tag{B11}$$

Putting in eqn (15) to eqn (16), the parameter N is:

$$N = \frac{nC_{\mathrm{CON}}}{n_{\mathrm{p}}} \tag{B12}$$

and from eqn (22):

$$R_t = \frac{t_1}{t_2} \tag{B13}$$

with the condition

$$0 < R_t \leqslant 1$$

Combining eqns (B5), (B9), (B11)–(B13) would give:

$$N = \frac{1-e^{-GN}}{1-e^{-GN/R_t}} \tag{B14}$$

From this, R_t can be expressed explicitly as:

$$R_t = \frac{-GN}{\ln\left(1-\dfrac{1-e^{-GN}}{N}\right)} \tag{B15}$$

The rate of carbon monoxide generation can be estimated from the size of underground car park. The control level can be derived from various health standards e.g. 50 ppm. N is eventually obtained by solving the above equation by graphical method which is shown in Fig. 2.

Part B: Two-level ventilation strategy

N_1 is defined as:

$$N_1 = \frac{n_1 C_{\mathrm{CON}}}{n_{\mathrm{p}}} \tag{B16}$$

Combining eqns (B7) and (B8) gives:

$$C_{\mathrm{CON}} = C_{\mathrm{CON}}\cdot e^{-(t_2-t_1)n_2} + C_{\mathrm{asy}} - C_{\mathrm{asy}}\cdot e^{-t_1 n_1} - C_1 + C_1\cdot e^{-t_1 n_1} \tag{B17}$$

Re-arranging eqn (B17) and simplification gives:

$$C_{\mathrm{CON}} = C_{\mathrm{asy}}\cdot\frac{1-e^{-t_1 n_1}}{1-e^{-(t_2-t_1)n_2-t_1 n_1}} \tag{B18}$$

Expressing C_{CON} in terms of C_1:

$$C_1 = C_{\mathrm{asy}}\left(1-\frac{1-e^{-(t_2-t_1)n_2}}{1-e^{-(t_2-t_1)n_2-t_1 n_1}}\right) \tag{B19}$$

If $n_1 = n_2$, eqn (B18) and (B19) reduce to eqns (B9) and (B10), respectively. Combining eqns (B5), (B11), (B16) and (B18) gives:

$$N_1 = \frac{1-e^{-GN_1}}{1-e^{-(t_2-t_1)n_2-GN_1}} \tag{B20}$$

Or expressing in terms of n_2:

$$n_2 = \frac{-1}{(t_2-t_1)}\ln\frac{N_1+e^{-GN_1}-1}{N_1 e^{-GN_1}} \tag{B21}$$

Optimised two-level operation is obtained by:

$$\frac{\partial n_2}{\partial N_1} = \partial/\partial N_1\left\{\frac{-1}{(t_2-t_1)}\ln\frac{N_1+e^{-GN_1}-1}{N_1 e^{-GN_1}}\right\} \tag{B22}$$

and

$$\frac{\partial N_1}{\partial n_1} = \frac{C_{\mathrm{CON}}}{n_{\mathrm{p}}} = \frac{t_1}{G} \tag{B23}$$

Therefore,

$$\frac{\partial n_2}{\partial N_1} = \frac{-1}{(t_2-t_1)}\frac{N_1 e^{-GN_1}}{N_1+e^{-GN_1}-1}$$

$$\cdot\frac{(N_1 e^{-GN_1})(1-Ge^{-GN_1})-(N_1+e^{-GN_1}-1)(e^{-GN_1}-GN_1 e^{-GN_1})}{(N_1 e^{-GN_1})^2}$$

$$= \frac{-1}{(t_2-t_1)}\frac{N_1 e^{-GN_1}}{N_1+e^{-GN_1}-1}$$

$$\cdot\frac{(N_1 e^{-GN_1})(1-Ge^{-GN_1})-(N_1+e^{-GN_1}-1)(e^{-GN_1}-GN_1 e^{-GN_1})}{(N_1 e^{-GN_1})^2}$$

$$\cdot\frac{\partial N_1}{\partial n_1} = \frac{-t_1}{(t_2-t_1)}\frac{N_1 e^{-GN_1}\cdot e^{-GN_1}}{N_1+e^{-GN_1}-1}$$

$$\cdot\frac{N_1-GN_1 e^{-GN_1}-N_1+GN_1^2-e^{-GN_1}+GN_1 e^{-GN_1}+1-GN_1}{G(N_1 e^{-GN_1})^2}$$

i.e.

$$\frac{\partial n_2}{\partial n_1} = \frac{-t_1}{(t_2-t_1)}\cdot\frac{1-e^{-GN_1}-GN_1(1-N_1)}{(N_1+e^{-GN_1}-1)GN_1} \tag{B24}$$

Part C: Optimised energy consumption

Power consumed by fans can be calculated by the product of pressure and volumetric flow rate. The pressure can be expressed in terms of volumetric flow rate:

$$\Delta p \propto n^2 V^2 \tag{B25}$$

or

$$\Delta p = L\left(\frac{nV}{3600}\right)^2 \tag{B26}$$

The fan power becomes:

$$W_{\mathrm{f}} = \frac{\Delta p\cdot n\cdot V}{3600\eta_{\mathrm{f}}} \tag{B27}$$

Or simplified into:

$$W_{\mathrm{f}} = \kappa\lambda^2 n^3 \tag{B28}$$

with

$$\kappa = \frac{LV^3}{\eta_{\mathrm{f}}(3600)^3(\lambda)^2} \tag{B29}$$

Total energy consumed by fans at two-level ventilation:

$$Q = t_1 W_{\mathrm{f1}} + (t_2-t_1)W_{\mathrm{f2}} \tag{B30}$$

Putting in eqn (B28):

$$\frac{Q}{\kappa} = (\lambda^2 n_1^3)t_1 + (\lambda^2 n_2^3)(t_2-t_1) \tag{B31}$$

Optimisation requires:

$$\frac{\partial Q/\kappa}{\partial n_1} = 3\lambda^2 n_1^2 t_1 + 3\lambda^2 n_2^2(t_2-t_1)\cdot\frac{\partial n_2}{\partial n_1} = 0 \tag{B32}$$

Or

$$3\lambda^2 n_1^2 t_1 + 3\lambda^2(t_2-t_1)\cdot\frac{-1^2}{(t_2-t_1)^2}\left\{\ln^2\left(\frac{N_1+e^{-GN_1}-1}{N_1 e^{-GN_1}}\right)\right\}\frac{-t_1}{(t_2-t_1)}$$

$$\cdot\frac{1-e^{-GN_1}-GN_1(1-N_1)}{(N_1+e^{-GN_1}-1)GN_1}=0$$

Giving:

$$n_1^2(t_2-t_1)^2=\left\{\ln^2\left(\frac{N_1+e^{-GN_1}-1}{N_1e^{-GN_1}}\right)\right\}\cdot\frac{1-e^{-GN_1}-GN_1(1-N_1)}{(N_1+e^{-GN_1}-1)GN_1} \tag{B33}$$

and

$$\frac{1-R_t}{R_t}=\frac{t_2-t_1}{t_1} \tag{B34}$$

since

$$n_1t_1=N_1G \tag{B35}$$

Therefore

$$\left(N_1G\cdot\frac{1-R_t}{R_t}\right)^2=\left\{\ln^2\left(\frac{N_1+e^{-GN_1}-1}{N_1e^{-GN_1}}\right)\right\}\cdot\frac{1-e^{-GN_1}-GN_1(1-N_1)}{(N_1+e^{-GN_1}-1)GN_1} \tag{B36}$$

$$\left(\frac{1-R_t}{R_t}\right)^2=\left\{\ln^2\left(\frac{N_1+e^{-GN_1}-1}{N_1e^{-GN_1}}\right)\right\}\cdot\frac{1-e^{-GN_1}-GN_1(1-N_1)}{(N_1+e^{-GN_1}-1)GN_1}\cdot\frac{1}{(N_1G)^2} \tag{B37}$$

The solving of eqn (B37) is shown in Fig. 3.