

T-Method Duct Design: Part IV— Duct Leakage Theory

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ABSTRACT

Studies have shown that duct leakage depends on the method of duct fabrication, method of sealing, workmanship, and static pressure differential. An equation that describes leakage as a function of leakage class and static pressure is presented in the "Duct Design" chapter of the 1993 ASHRAE Handbook. This equation is used to calculate the leakage rate through a unit of duct surface for constant duct static pressure. However, the static pressure of a leaking duct does not remain constant. This process is described by a differential equation.

The magnitude of duct leakage for a straight duct depends on internal static pressure and varies uniformly along its length (assumption). Fittings in a system cause sudden changes in static pressure; therefore, duct leakage depends on fitting locations. There are two approaches for duct system leakage calculation:

- *Accurate—For accurate duct leakage calculations, the duct system is divided into single sections between fittings and the leakage rate and pressure loss for each section calculated by the weighing factor method.*
- *Approximate—For most applications, duct leakage can be calculated by approximate formulas based on the average static pressure in a duct section.*

Leakage calculation requires dividing the system into sections between each fitting.

This paper, the fourth in a series on T-method duct design, discusses the theory of calculating air leakage from/into a single duct to incorporate duct leakage into the optimization and simulation calculation procedures.

INTRODUCTION

The following papers have been published for the series of T-method duct design research projects:

- "Part I: Optimization Theory" (Tsal et al. 1988a)
- "Part II: Calculation Procedure and Economic Analysis" (Tsal et al. 1988b)
- "Part III: Simulation" (Tsal et al. 1990)

This paper, part IV, discusses the theory of calculating air leakage from/into a single duct to incorporate duct leakage into the optimization and simulation calculation procedures. Part V covers incorporation of duct leakage into the T-method calculation technique and leakage studies to determine the economics of sealing ductwork.

Studies show that HVAC air duct systems are one of the major energy consumers in industrial and commercial buildings. From electric energy audits, supply, return, and exhaust fans consume 25% to 50% of the energy required to operate high-rise residential and commercial buildings (ASHRAE 1993a). Inefficient design of a duct system means that either energy is being wasted and/or excessive ductwork material is being installed. Duct system optimization offers the opportunity to realize significant owning and energy savings. The T-method was developed as a practical duct optimization procedure. Life-cycle cost was selected as the objective function. The T-method duct design consists of three steps performed in a series: system condensing, fan selection, and system expansion. The papers "T-Method Duct Design, Part I and Part II" present the theory of the method, step-by-step calculation procedures, economic analysis, and examples (Tsal et al. 1988a, 1988b). Part III (Tsal et al. 1990) discusses the simulation of existing systems. Economic analysis showed that significant cost savings are obtainable. For example, if the referenced system is constructed for a residential building in New York City and the material is low-pressure galvanized duct, the economic effect on the life-cycle cost is 53.4%. The

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lowest obtainable economic effect is 12% for commercial buildings in Seattle.

T-method simulation may be used in the solution of many HVAC problems. In addition to the following concerns that can be clarified by simulation, T-method is an excellent design tool for simulating the flow distribution within a system with various modes of operation:

- Flow distribution in a variable-air-volume (VAV) system due to terminal box flow diversity.
- Airflow redistribution due to HVAC system additions and/or modifications.
- System airflow analysis for partially occupied buildings.
- Necessity to replace fans and/or motors when retrofitting an air distribution system.
- Multiple-fan system operating condition when one or more fans shut down.
- Pressure differences between adjacent confined spaces within a nuclear facility when a design basis accident (DBA) occurs (Farajian et al. 1992).
- Smoke control system performance during a fire when certain fire/smoke dampers close and others remain open.

According to Webster, leakage is “to let a fluid out or in accidentally.” Air leakage from or into a duct system means that a part of the air will be lost and the duct system will not be able to attain design conditions. The Associated Air Balance Council (AABC 1983) states that

in the real world, Total System Balance Agencies have measured leakage values of from 30% to 45% in some extensive rectangular duct supply systems, and regularly experience duct leakage rates of 20% of the total quantity supplied by the fan. The effect of air leakage in duct systems has become a major factor in the performance of air distribution systems.

There are many concerns associated with air leakage:

- Potentially inadequate cooling (inadequate zone/room air supply).
- Additional cost of fan power due to increased airflow.
- Oversized fans, filters, coils, chillers, and power supply systems.
- Impossibility to maintain proper pressure relationship between zones/rooms and the ambient atmosphere.

The following analysis explains the practical procedure of calculating duct leakage as a part of the T-method. Ordinary numerical methods for solving differential equations were turned down due to their inability to be combined with the T-method.

LEAKAGE IN A SINGLE DUCT

Duct Leakage Formulas

Research shows that leakage in an assembled duct can be estimated by an exponential equation. According to ASHRAE (1993b) the exponent is 0.65 for turbulent flow.

$$\Delta Q = a_1 C_L P_s^n \quad (1)$$

where

a_1 = coefficient, 1.00 for SI units, $(0.14 \times 10^{-5}$ for IP units),

P_s = average static pressure in duct section, Pa (in. WG).

The constant C_L , leakage class, in this equation reflects the quality of duct construction and sealing method. It is based on experimental data and exists in the range from zero for welded ducts to 110 for rectangular unsealed ducts. The average leakage class C_L for rectangular unsealed ducts is 48 cfm/100 ft² (ASHRAE 1993b). The slope n is related to the type of flow. For laminar type leakage, the flow coefficient n is 1. The n coefficient for turbulent flow leakage is in the range between 0.5 and 1. It is recommended that n be 0.65 (ASHRAE 1993b).

$$\Delta Q = a_1 C_L^{0.65} \quad (2)$$

For typical duct construction and the method of sealing, Equation 2 is not significantly different in either the negative or positive pressure modes. Equation 2 can be used to calculate the leakage at any system point for a known internal static pressure. However, internal static pressure is constantly changing from the static pressure at the fan to the terminal inlets/outlets due to friction, dynamic losses (fittings), and leakage. It is unknown, prior to the pressure loss calculation, what the static pressure is at each duct section. Because pressure loss is a function of leakage, Equation 2 is not usable for practical calculations. The solution requires development and integration of a differential air leakage equation.

Air Distribution Along a Duct

The Darcy-Weisbach equations for round and rectangular ductwork are as follows:

Round:

$$\Delta P = \left(\frac{fL}{D} + \sum C \right) \frac{v^2}{g_c} \quad (3a)$$

Rectangular:

$$\Delta P = \left(\frac{fL}{D_f} + \sum C \right) \frac{v^2}{g_c} \quad (3b)$$

where the equivalent-by-friction diameter (hydraulic diameter) is

$$D_f = 2 \frac{H \cdot W}{H + W} \quad (4)$$

By using the continuity equation ($V=Q/A$) and the aspect ratio for rectangular ducts ($r=H/W$), the following equations result.

For round ducts:

$$V = \frac{4}{\pi} Q D^{-2} \quad (5a)$$

For rectangular ducts:

$$V = \frac{Q}{HW} \quad (5b)$$

or

$$V = \frac{1}{r} Q W^{-2} \quad (5c)$$

Duct width (W) can be interpreted in terms of an equivalent-by-velocity diameter by equating Equations 5a and 5c and solving for W ; thus,

$$W = 0.5 \left(\frac{\pi}{r} D_v \right)^{0.5} \quad (6)$$

Equations 5a and 5b yield the equivalent-by-velocity diameter for rectangular ducts (D_v).

$$D_v = (4/\pi)^{0.5} (HW)^{0.5} \quad (7)$$

Introduce the coefficient μ for round and rectangular duct systems as follows:

Round duct:

$$\mu = f + \Sigma CD/L \quad (8a)$$

Rectangular duct:

$$\mu = \left(\frac{f}{D_f} + \frac{\Sigma C}{L} \right) D_v \quad (8b)$$

Then substitute μ into the Darcy-Weisbach equation (Equations 3a and 3b) using Equation 5a to obtain the following single-duct pressure loss equations for round ductwork of $L=1$ m (ft),

$$\Delta P = 0.8105 g_c^{-1} \mu \rho Q^2 D^{-5}, \quad (9a)$$

and rectangular ductwork,

$$\Delta P = 0.8105 g_c^{-1} \mu \rho Q^2 D_v^{-5}. \quad (9b)$$

Assuming that $\Sigma C=0$ in Equations 8a and 8b, Equations 9a and 9b can be replaced by Equation 10 for a straight duct of $L=1$ m(ft).

$$\Delta P = \sigma Q^2 L \quad (10)$$

where

$$\sigma = 0.8105 g_c^{-1} \mu \rho D^{-5} \quad (11)$$

For simplification reasons, leakage ΔQ is assumed to be distributed equally over the duct length, as noted in Figure 1. Flow upstream of the supply duct is

$$Q_u = Q_d + \Delta Q. \quad (12)$$

An approach that calculates variable flow in a duct was proposed by Konstantinov (1981). Assume, on the section with the elementary length dx , that the initial flow is increasing or decreasing from Q_1 to Q_2 (ΔQ) and that the air velocity is changing from V_1 to V_2 (ΔV). The change of momentum for leaked air projected on the duct axis is

$$\begin{aligned} \Delta M &= \alpha_0 \rho (Q + dQ)(V + dV) - \alpha_0 \rho QV \\ &= \alpha_0 \rho (Q dV + V dQ + dQ dV) \end{aligned} \quad (13)$$

where α_0 is the coefficient of momentum. After the low value of ($dQ dV$) is eliminated, the change of momentum becomes

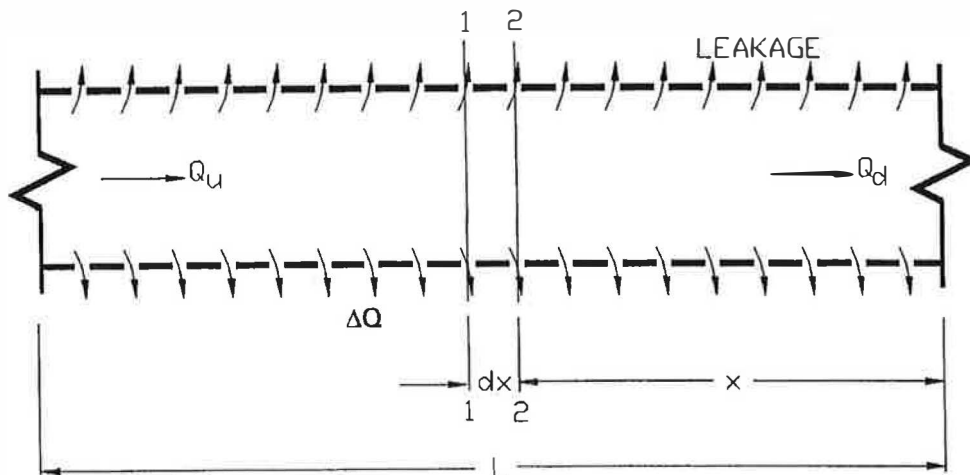


Figure 1 Elemental duct section.

$$\Delta M = \alpha_0 \rho (Q dV + V dQ). \quad (14)$$

As shown in Figure 1, the impulse of pressure forces at cross-section 1-1 and cross-section 2-2 is

$$IM_p = P_s S - (P_s + dP)S = -dP S \quad (15)$$

where

dP = change of pressures between cross sections 1-1 and 2-2, Pa (in. wg);

S = cross-sectional area, m² (ft²).

The impulse of force for air frequency is

$$IM_f = -\tau_0 X dx \quad (16)$$

where

X = duct perimeter, m (in);

τ_0 = tangential stress on duct walls equal to $J \rho g R_h$, Pa/m² (in. wg/in²).

Then

$$IM_f = -J \rho g R_h X dx = -J \rho g S dx \quad (17)$$

where

J = hydraulic slope, m (ft);

R_h = hydraulic radius, m (ft).

Equate the change of momentum (ΔM) to the impulse of forces due to pressure and air frequency ($IM_p + IM_f$), divide by ($\rho g S$), and assume $\alpha_0 = 1$. Then

$$\frac{Q dV}{Sg} + \frac{V dQ}{Sg} = -\frac{dP}{\rho g} - J dx. \quad (18)$$

But $Q = (V)(S)$, $dQ = (dV)(S)$, and $J = dP/dx$; therefore,

$$2 \frac{V dV}{g} + \frac{dP}{\rho g} + dP = 0. \quad (19)$$

Leakage Along a Duct

Two conditions need to be considered for duct leakage: (1) initial conditions (called the Koshi problem) and (2) boundary conditions.

The Koshi problem assigns initial data for duct entry or duct exit, such as pressure, P , and flow, Q . The results are pressure and flow at the other parts of a duct section. The boundary condition problem considers that pressures at both ends of a duct section are known and that airflow is the unknown. The most practical solution is the Koshi problem because for a given diameter, the flow through the duct section is unknown. Equation 19 can be written as follows (multiply by g , divide by dx) and use Equation 3 (only Darcy's part) for $L=1$ and $\Sigma C=0$:

$$2\rho V \frac{dV}{dx} + \frac{dP}{dx} = -\left(\frac{f}{D}\right) \left(\frac{\rho V^2}{2}\right) \quad (20)$$

Equation 20 can be used to develop a new duct leakage equation. Introduce the function

$$T = P_s + \rho V^2 \quad (21)$$

and differentiate. Thus,

$$\frac{dT}{dx} = \frac{d}{dx}(P_s + \rho V^2) = \frac{dP}{dx} + \rho \frac{d(V^2)}{dx}. \quad (22)$$

But

$$\frac{d(V^2)}{dx} = 2 V \frac{dV}{dx}. \quad (23)$$

Substitute Equation 23 into Equation 22; then

$$\frac{dT}{dx} = \frac{dP}{dx} + 2 \rho V \frac{dV}{dx}. \quad (24)$$

Equation 24 then becomes

$$\frac{dT}{dx} = -\left(\frac{f}{D}\right) \left(\frac{\rho V^2}{2}\right). \quad (25)$$

It is convenient for a terminal section to use the length variable not from the end of the duct but from the beginning. This will change the sign of the right part of Equation 25 to plus. Let

$$k_1 = \frac{f \rho}{2 D}. \quad (26)$$

Thus,

$$\frac{dT}{dx} = k_1 V^2. \quad (27)$$

The leakage equation is

$$Q_1 - Q_2 = Q_{ratio} A_s = k_3 P_s^{0.65} A_s \quad (28)$$

where

$$k_3 = 0.14 \cdot 10^{-5} C_L.$$

For round ducts,

$$Q_1 = (\pi/4) D^2 V_1$$

$$Q_2 = (\pi/4) D^2 V_2$$

$$A_s = \pi D L$$

Substitute these formulas into Equation 28; thus,

$$\frac{V_1 - V_2}{L} = \frac{4}{D} k_3 P_s^{0.65}. \quad (29)$$

Substitute $\Delta V = V_2 - V_1$, assume $L = \Delta x$, and

$$k_2 = 4 k_3 / D. \quad (30)$$

Then Equation 28 becomes

$$\frac{\Delta V}{\Delta x} = k_2 P_s^{0.65}. \quad (31)$$

When $\Delta x \Rightarrow dx$ and $\Delta V \Rightarrow dV$, then

$$\frac{dV}{dx} = k_2 P_s^{0.65}. \quad (31a)$$

Combining Equation 31a with Equation 21 yields

$$\frac{dV}{dx} = k_2 (T - \rho V^2)^{0.65}. \quad (32)$$

Finally, Equations 27 and 32, repeated below, are a system of differential equations that describes airflow in a duct with leakage.

$$\frac{dT}{dx} = k_1 V^2 \quad (33)$$

$$\frac{dV}{dx} = k_2 (T - \rho V^2)^{0.65}$$

There is no analytical solution for Equation 33. An approximate numerical solution is necessary.

Let us analyze the conditions at the end of a supply duct where the air velocity is V_0 and the static pressure is $P_s = P_0 = 0$. The variable T_0 at this point in the system is

$$T_0 = P_0 + \rho V_0^2 = \rho V_0^2. \quad (34)$$

The purpose of the following study is to find the variables V and T as a function of x in the range between $x = 0$ and $x = L$. The next step uses the Taylor's series for development of a linear function for static pressure P_s as a function of x .

$$\begin{aligned} P_s &= P_0 + \left. \frac{dP}{dx} \right]_{x=x_0} \left[x - x_0 + \frac{d(T - \rho V^2)}{dx} \right]_{x=x_0} x \\ &= \left. \frac{dT}{dx} x - \frac{d(\rho V^2)}{dx} \right]_{x=x_0} x \end{aligned} \quad (35)$$

Substitute Equation 33 into Equation 35 to obtain

$$P_s = k_1 V^2 x - 2 \rho V \left. \frac{dV}{dx} \right]_{x=x_0} x. \quad (36a)$$

Substitute Equation 32 into 36a; thus,

$$P_s = k_1 V_0^2 x - 2 \rho V [k_2 (T - \rho V^2)^{0.65}]_{x=0} x. \quad (36b)$$

When taking into consideration that $P_s = 0$ at $x = 0$, Equation 21 yields

$$T_0 - \rho V_0^2 = P_0 = 0. \quad (37)$$

Therefore,

$$P_s = k_1 V_0^2 x. \quad (38)$$

Assuming that Equation 38 is valid, find solutions for T and V in Equation 33. Substitute Equation 38 into Equation 32, taking into consideration Equation 21. Equation 32 then becomes

$$\frac{dV}{dx} = k_2 (k_1 V_0^2 x)^{0.65}. \quad (39)$$

Integrating Equation 39 yields

$$\int_{V_0}^{V_1} dV = \int_0^L [k_2 (k_1 V_0^2 x)^{0.65}] dx. \quad (40)$$

After integration,

$$V_1 - V_0 = (1/1.65) k_1^{0.65} k_2 V_0^{1.3} L^{1.65} \quad (41)$$

and

$$V_1 = V_0 + 0.606 k_1^{0.65} k_2 V_0^{1.3} L^{1.65}. \quad (42)$$

Introduce the coefficient α :

$$\alpha = 0.606 k_1^{0.65} k_2 V_0^{1.3} \quad (43)$$

and substitute Equation 42 into Equation 33. Hence,

$$\frac{dT}{dx} = k_1 V_1^2 = k_1 (V_0 + \alpha L^{1.65})^2. \quad (44)$$

The integral form of Equation 44 is

$$\int_{T_0}^{T_1} dT = k_1 V_1^2 dx = k_1 \int_0^L (V_0 + \alpha x^{1.65})^2 dx \quad (45)$$

$$T_1 - T_0 = k_1 \int_0^L (V_0^2 + 2V_0 \alpha x^{1.65} + \alpha^2 x^{3.3}) dx \quad (46)$$

$$= k_1 (V_0^2 L + 0.75 V_0 \alpha L^{2.65} + 0.23 \alpha^2 L^{4.3}).$$

Considering that $T_0 = \rho V_0^2$ (Equation 34), T_1 is

$$T_1 = \rho V_0^2 + k_1 (V_0^2 L + 0.75 V_0 \alpha L^{2.65} + 0.23 \alpha^2 L^{4.3}). \quad (47)$$

Weighing Factor Method. Since leakage is proportional to air velocity, it would be convenient to analyze the change of air velocity instead of the change of leakage rate. Introduce the coefficients k_6 and Γ ; thus,

$$k = 1/1.65 k_1^{0.65} k_2 \quad (48)$$

$$\Gamma = k_6 V_0^{1.3} = 0.606 k_1^{0.65} k_2 V_0^{1.3}. \quad (49)$$

Then, from Equation 42,

$$V_1 = V_0 + \Gamma x^{1.65}. \quad (50a)$$

Therefore,

$$\Delta V = \Gamma x^{1.65}. \quad (50b)$$

Considering that $\Delta Q = (V_1 - V_2) A$ and $x = L$,

$$\Delta Q = \Gamma L^{1.65} A. \quad (50c)$$

Substituting Equation 50a into Equation 27 yields

$$\frac{dT}{dx} = k_1 V_1^2 = k_1 (V_0 + \Gamma x^{1.65})^2. \quad (51)$$

Integrating Equation 51 yields

$$T_1 - T_0 = \int_0^L k_1 (V_0 + \Gamma x^{1.65})^2 dx \quad (52)$$

$$T_1 - T_0 = k_1 \int_0^L (V_0^2 + 2V_0\Gamma x^{1.65} + \Gamma^2 x^{3.3}) dx \quad (53)$$

$$T_1 - T_0 = k_1 L (V_0^2 + 2/2.65 V_0 \Gamma L^{1.65} + 1/4.3 \Gamma^2 L^{3.3}). \quad (54)$$

Considering that $T_0 = \rho V_0^2$ (Equation 34), T_1 becomes

$$T_1 = \rho V_0^2 + k_1 x (V_0^2 + 0.75 V_0 \Gamma x^{1.65} + 0.23 \Gamma^2 x^{3.3}). \quad (55)$$

The comparison between Equations 55 and 47 shows that they are identical. Practically speaking, leakage $Q_1 - Q_0$ is smaller than 25% of Q_0 . The velocity is proportional to leakage; therefore, $V_1 - V_0 < V_0/4$. From Equation 50a, $V_0/4 > \Gamma x^{1.65}$. The square of both sides of this expression results in the following inequality: $x V_0^2/16 > \Gamma^2 x^{3.3}$. Therefore, the term $0.23 \Gamma^2 x^{3.3}$ in Equation 55 is smaller than 2% of V_0^2 . Thus, Equation 55 can be simplified as

$$T_1 = \rho V_0^2 + k_1 x (V_0^2 + 0.75 V_0 \Gamma x^{1.65}) \quad (56a)$$

$$\Delta T = k_1 x (V_0^2 + 0.75 V_0 \Gamma x^{1.65}). \quad (56b)$$

A new formula is developed that shows leakage as a linear combination of $P_s^{0.65}$ at the duct nodes and pressure loss as a linear combination of V^2 . If the combination coefficients are defined correctly, the integral obtained will be the solution. In other words, it is necessary to find weighing average variables between duct nodes that, if used as constants, yield actual pressure loss and flow leakage. Following is the approximating formula, derived from Newton's binomial theorem, that will be used for values of γ and any s .

$$(1 + \gamma)^s = 1 + s\gamma \quad (57)$$

This formula is used to derive equations with respect to the coefficient α and β (linear combination) as a function of $T_1, T_2, V_1,$ and V_2 . The linear combination derived from Equation 33 follows, where dx for finite length is L and dT is ΔT .

$$\Delta T = k_1 L [\alpha V_1^2 + (1 - \alpha) V_2^2] \quad (58)$$

The linear combination derived from Equation 31 for ΔV also follows, where dx is L and dV is ΔV .

$$\Delta V = k_2 L [\beta P_{s1}^{0.65} + (1 - \beta) P_{s2}^{0.65}] \quad (59)$$

It is proven in the final report (Tsal and Varvak 1992) that α changes in the range between 0.5 and 0.62 (1.65/2.65) and that β changes in the range between 0.4 (0.65/1.65) and 0.5.

The Calculation Procedure for the Weighing Factor Method. Using previous equations, the calculation procedure for a duct with $x=L$ length, positive pressure (supply), airflow, and velocity are known at the beginning of the duct (P_1, V_1) and unknown at the end of the duct (P_2, V_2). First, assume $P_1=0$. Then, define $P_0 = P_1$ and $V_0 = V_1$.

A. **Case I.** Terminal section, supply system.

Step 1. Coefficient k_3 :

$$k_3 = 0.14 \times 10^{-5} C_L$$

Step 2. Coefficient k_2 (Equation 30):

$$k_2 = 4 k_3/D$$

Step 3. Coefficient k_1 (Equation 26):

$$k_1 = \frac{f \rho}{2D}$$

Step 4. Coefficient Γ (Equation 49):

$$\Gamma = 0.606 k_1^{0.65} k_2 V_0^{1.3}$$

Step 5. Air velocity at L distance and velocity difference ΔV (Equations 50a and 50b):

$$V = V_0 + \Gamma L^{1.65}$$

$$\Delta V = \Gamma L^{1.65}$$

Step 6. Air leakage is calculated using Equation 50c:

$$\Delta Q = \Gamma L^{1.65} A$$

Step 7. Coefficient ΔT (Equation 56b):

$$\Delta T = k_1 (V_0^2 L + 0.75 V_0 \Gamma L^{2.65})$$

Step 8. Pressure loss (Tsal and Varvak 1992, Equation 2.91):

$$\text{Static: } \Delta P_s = \Delta T - \rho (V_0 + \Delta V/2) \Delta V$$

$$\text{Total: Using } \Delta P = P_s + \frac{V^2}{2} \text{ yields}$$

$$\Delta P = \Delta T + 0.5 \rho V_0 \Delta V$$

where ΔV and ΔT are from steps 5 and 7.

B. **Case II.** Intermediate section.

In the case of $P_1 > 0$ the order of calculation is:

Step 1. Coefficient k_3 :

$$k_3 = 0.14 \times 10^{-5} C_L$$

Step 2. Coefficient k_2 (Equation 30):

$$k_2 = 4 k_3 / D$$

Step 3. Coefficient k_1 (Equation 26):

$$k_1 = \frac{\dot{P}}{2D}$$

Step 4. Coefficient x_f (Tsal and Varvak 1992, Equation 2.78):

$$x_f = \frac{P_1}{k_1 V_1^2}$$

Step 5. Air velocity at L distance (Tsal and Varvak 1992, Equation 2.85) and ΔV (Equation 50b):

$$V_2 = V_1 + k_6 V_1^{1.3} [(x_f + L)^{1.65} - x_f^{1.65}]$$

$$\Delta V = k_6 V_1^{1.3} [(x_f + L)^{1.65} - x_f^{1.65}]$$

Step 6. Air leakage is a function of air velocity; therefore:

$$\Delta Q = k_6 V_1^{1.3} ((x_f + L)^{1.65} - x_f^{1.65}) A$$

Step 7. Coefficient ΔT (Tsal and Varvak 1992, Equation 2.87):

$$\Delta T = k_1 L V_1^2 \{1 + 0.9 k_6 V_1^{1.3} [(x_f + L)^{1.65} - x_f^{1.65}]\}$$

Step 8. Pressure loss (Tsal and Varvak 1992, Equation 2.91)

$$\text{Static: } \Delta P_s = \Delta T - \rho (V_0 + \Delta V/2) \Delta V$$

$$\text{Total: Using } \Delta P = P_s + \frac{\rho V^2}{2} \text{ yields}$$

$$\Delta P = \Delta T + 0.5 \rho V_0 \Delta V$$

where ΔV and ΔT are from steps 5 and 7.

Approximate Method

It is assumed that the variable P_s in Equation 2 is the average static pressure in a duct section.

$$P_{s_{av}} = \frac{P_{s1} + P_{s2}}{2} \quad (60)$$

Since

$$P_{s1} = P_1 \mp \Delta P_1 = P_1 \mp \frac{\rho V_1^2}{2} \quad (61a)$$

$$P_{s2} = P_2 \mp \Delta P_2 = P_2 \mp \frac{\rho V_2^2}{2} \quad (61b)$$

Assuming that dynamic pressure in Equations 61a and 61b is always positive, the "up sign (-)" is for supply and the "down sign (+)" is for return/exhaust duct. Substitute Equations 61a and 61b into 60 to obtain Equation 62a.

$$P_{s_{av}} = 0.5 \left(P_1 + P_2 \mp \frac{\rho}{2} (V_1^2 + V_2^2) \right) \quad (62a)$$

For a terminal section $P_2 = 0$ and $V_2 = 0$; therefore, Equation 62a becomes

$$P_{s_{av}} = 0.5 (P_1 \mp 0.5 \rho V_1^2) \quad (62b)$$

Substitute $V = Q/A$ into Equation 62a; thus,

$$P_{s_{av}} = 0.5 \left(P_1 + P_2 \mp 0.5 \frac{\rho}{A^2} (Q_1^2 + Q_2^2) \right) \quad (63)$$

Substitute Equation 63 into Equation 2 instead of ΔP_s and multiply by the duct surface area A_s to obtain leakage for the section.

$$\Delta Q = 0.89 \cdot 10^{-6} C_L A_s \left(P_1 + P_2 \mp 0.5 \frac{\rho}{A^2} (Q_1^2 + Q_2^2) \right)^{0.65} \quad (64a)$$

For a terminal section where $P_2 = 0$ and $Q_2 = 0$,

$$\Delta Q = 0.89 \cdot 10^{-6} C_L A_s (P_1 \mp 0.5 \rho V_1^2)^{0.65} \quad (64b)$$

The average flow rate can be described as follows:

$$Q_{av} = Q_1 - \frac{\Delta Q}{2} \quad (65)$$

Substituting Equation 65 into Equation 10 and solving with respect to Q_1 ,

$$Q_1 - \frac{\Delta Q}{2} = k' \Delta P^{0.5} \quad (66)$$

where

$$k' = 1.1107 \left(\frac{g_c}{\mu L \rho} \right)^{0.5} D^{2.5} \quad (67)$$

Finally, airflow at the beginning of the section is

$$Q_1 = k' \Delta P^{0.5} + \frac{\Delta Q}{2} \quad (68)$$

Introducing the variable k ,

$$k = k' + 0.5 \Delta Q \Delta P^{-0.5} \quad (69)$$

and substitute k into Equation 68. Note that Equation 69 is similar to the original T-method equation below (Tsal et al. 1990, Equation 19).

$$Q_1 = k \Delta P^{0.5} \quad (70)$$

or

$$\Delta P = \left(\frac{Q_1}{k} \right)^2 \quad (71)$$

Leakage Evaluation

Actual and Approximate Solutions. In order to select a methodology (equation) for duct leakage calculation, a one-section duct system is analyzed for the three air leakage calculation procedures: (1) actual leakage (25 short duct intervals), (2) approximate formula based on average static pressure (Equation 64a), and (3) weighing factor method. Actual solution is impractical for routine duct leakage calculations; however, it is used here to analyze the accuracy of the leakage calculation methods.

Calculation of actual leakage is presented in Table 1 as a result of dividing the total duct length into 25 one-meter increments. The following data are calculated at each step:

- Flow at the beginning (Q_1), end (Q_2), and average (Q_{av}).
- Average air velocity based (V_{av}) on average flow (Q_{av}).
- Reynolds number (Re) and friction factor (f).
- Pressure loss (dP_{av}) based on average air velocity.
- Total pressure at the beginning (P_1) and end (P_2) of each increment and average over the entire length (P_{av}).
- Dynamic pressure (P_{dyn}) based on average air velocity.
- Static pressure at the end of each increment (P_{s2}) and the average over the entire length ($P_{s,av}$).
- Leakage (dQ) based on average static pressure.
- Leakage percent related to the total flow in a particular section ($\%dQ$).

The following data are also presented in the lower part of Table 1:

- Summation C-coefficient (C_{tot} , col. 4) in the section.
- Average air velocity in the section (V_{av} , col. 8).
- Total pressure loss as a sum of pressure losses in the steps (dP_{sum} , col. 13).
- Total leakage calculated as a sum of leakages at each step (Sum , col. 18).
- Leakage percentage
- Flow/surface ratio as used in ASHRAE (1993b, Chapter 32, Table 6, p. 32.15)

A large number of calculations based on previous procedures similar to the calculations presented in Table 1 were performed and statistical data were collected. Eight examples of using an approximate calculation for a single duct with different diameters, flows, and/or C-coefficients are presented in Table 2. The least accuracy is obtained for 40.2% leakage and $C = 127.0$, which is unrealistic. Even when $C = 9.0$ and $D = 0.17$ m and the pressure loss is 1500 Pa (6 in. wg) for a 25 m (82 ft) duct section, the duct leakage is 17.5% and the inaccuracy is 9.75%. It shows that when the C-coefficient is high, the approximate approach creates errors. These errors can be avoided by dividing the system into sections between each fitting (see explanation below).

Weighing Factor Solution. As an example, let us calculate a duct with no C-coefficients using the weighing factor method. Input data are:

$$C_L = 6, D = 0.5 \text{ m}, Q = 0.98 \text{ m}^3/\text{s}, L = 25 \text{ m}, P_1 = 28 \text{ Pa}, \\ \rho = 1.17 \text{ kg/m}^3, R = 0.00015 \text{ m}, f = 0.0181, V_0 = 4.994 \text{ m/s}, \\ A = 0.196 \text{ m}^2.$$

Step 1. Coefficient k_3 (Equation 28):

$$k_3 = 0.14 \cdot 10^{-5} C_L \\ = 0.14 \cdot 10^{-5} \cdot 6 = 0.84 \cdot 10^{-5}$$

Step 2. Coefficient k_2 (Equation 30):

$$k_2 = 4 k_3 / D \\ = 4 \cdot 0.84 \cdot 10^{-5} / 0.5 = 0.67 \cdot 10^{-4}$$

Step 3. Coefficient k_1 (Equation 26):

$$k_1 = f \rho / (2 D) \\ = 0.0181 \cdot 1.17 / (2 \cdot 0.5) = 0.02118$$

Step 4. Coefficient Γ (Equation 49):

$$\Gamma = 0.606 k_1^{0.65} k_2 V_0^{1.3} \\ = 0.606 \cdot 0.02118^{0.65} \cdot 0.67 \cdot 10^{-4} \cdot 4.994^{1.3} \\ = 0.000027$$

Step 5. Air velocity at L distance (Equation 50a) and ΔV :

$$\Delta V = \Gamma L^{1.65} = 0.000027 \cdot 25^{1.65} = 0.00545 \text{ m/s} \\ V = V_0 - \Delta V = 4.994 - 0.00545 = 4.989 \text{ m/s}$$

Step 6. Air leakage (Equation 50c):

$$\Delta Q = \Gamma L^{1.65} A \\ = 0.000027 \cdot 25^{1.65} \cdot 0.196 = 0.001068 \text{ m}^3/\text{s}$$

Step 7. Coefficient ΔT (Equation 56b):

$$\Delta T = k_1 (V_0^2 L + 0.75 V_0 \Gamma L^{1.65}) \\ = 0.021177 (4.994^2 \cdot 25 + 0.75 \cdot 4.994 \cdot \\ 0.000027 \cdot 25^{1.65}) \\ = 13.2 \text{ Pa}$$

Step 8. Pressure loss (Tsal et al. 1992, Equation 2.91)

$$\Delta P_s = \Delta T - \rho (V_0 + \Delta V / 2) \Delta V \\ = 13.2 - 1.17 (4.994 + 0.00545 / 2) 0.00545 \\ = 13.2 \text{ Pa}$$

TABLE 1
Actual Leakage Calculation for a Single Duct Section

D	= 0.50 m	Init. Flow	= 0.980 m ³ /s = 2082 cfm	Length step	= 1.00 m = 3.3 ft
	20 in.	Df	= 0.50 m = 19.7 in.	Surf./step	= 1.57 m ² = 17 ft ²
		Dv	= 0.50 m = 19.7 in.	Leak. class	= 6 = 6
Cross-sec	= 0.20 m ²	Length	= 25 m = 82 ft	μ-coeff.	= 0.45
	= 304 in. ²	Init. P_{tot}	= 28 Pa = 0.11 in. H ₂ O		
		Roughness	= 0.00015 m = 0.0005 ft		
Perimeter	= 1.57 m	Viscosity	= 1.51E-05m ² /s = 0.154 ft ² /s		
	62 in.	Density	= 1.17 kg/m ³ = 0.073 lb/ft ³		

S	x1 m	x2 m	x(av) m	C	Q1 m ³ /s	Q2 m ³ /s	Qav m ³ /s	Vav m/s	RE	f	dPav Pa	P1 Pa	P2 Pa	Pav Pa	Pdyn Pa	Ps ₂ Pa	Ps,av Pa	dQ m ³ /s	%dQ
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	18	
1	25	24	25	0.00	0.980	0.980	0.980	4.994	165567	0.0181	0.53	28.0	27.5	27.7	14.63	12.8	13.1	0.000070	0.007
2	24	23	24	0.00	0.980	0.980	0.980	4.994	165555	0.0181	0.53	27.5	26.9	27.2	14.62	12.3	12.6	0.000068	0.007
3	23	22	23	0.00	0.980	0.980	0.980	4.993	165544	0.0181	0.53	26.9	26.4	26.7	14.62	11.8	12.1	0.000067	0.007
4	22	21	22	0.00	0.980	0.980	0.980	4.993	165533	0.0181	0.53	26.4	25.9	26.2	14.62	11.3	11.5	0.000065	0.007
5	21	20	21	0.00	0.980	0.980	0.980	4.993	165522	0.0181	0.53	25.9	25.4	25.6	14.62	10.7	11.0	0.000063	0.006
6	20	19	20	0.00	0.980	0.980	0.980	4.992	165512	0.0181	0.53	25.4	24.8	25.1	14.62	10.2	10.5	0.000061	0.010
7	19	18	19	0.00	0.980	0.980	0.980	4.992	165502	0.0181	0.53	24.8	24.3	24.6	14.62	9.7	10.0	0.000059	0.006
8	18	17	18	0.00	0.980	0.979	0.980	4.992	165492	0.0181	0.53	24.3	23.8	24.0	14.61	9.2	9.4	0.000057	0.006
9	17	16	17	0.00	0.979	0.979	0.979	4.991	165482	0.0181	0.53	23.8	23.2	23.5	14.61	8.6	8.9	0.000055	0.006
10	16	15	16	0.00	0.979	0.979	0.979	4.991	165473	0.0181	0.53	23.2	22.7	23.0	14.61	8.1	8.4	0.000053	0.005
11	15	14	15	0.00	0.979	0.979	0.979	4.991	165465	0.0181	0.53	22.7	22.2	22.5	14.61	7.6	7.8	0.000050	0.005
12	14	13	14	0.00	0.979	0.979	0.979	4.991	165456	0.0181	0.53	22.2	21.7	21.9	14.61	7.1	7.3	0.000048	0.005
13	13	12	13	0.00	0.979	0.979	0.979	4.990	165448	0.0181	0.53	21.7	21.1	21.4	14.61	6.5	6.8	0.000046	0.005
14	12	11	12	0.00	0.979	0.979	0.979	4.990	165441	0.0181	0.53	21.1	20.6	20.9	14.60	6.0	6.3	0.000043	0.004
15	11	10	11	0.00	0.979	0.979	0.979	4.990	165434	0.0181	0.53	20.6	20.1	2.03	14.60	5.5	5.7	0.000041	0.004
16	10	9	10	0.00	0.979	0.979	0.979	4.990	165427	0.0181	0.53	20.1	19.6	19.8	14.60	4.9	5.2	0.000039	0.004
17	9	8	9	0.00	0.979	0.979	0.979	4.989	165421	0.0181	0.53	19.6	19.0	19.3	14.60	4.4	4.7	0.000036	0.004
18	8	7	8	0.00	0.979	0.979	0.979	4.989	165415	0.0181	0.53	19.0	18.5	18.8	14.60	3.9	4.2	0.000033	0.003
19	7	6	7	0.00	0.979	0.979	0.979	4.989	165409	0.0181	0.53	18.5	18.0	18.2	14.60	3.4	3.6	0.000031	0.003
20	6	5	6	0.00	0.979	0.979	0.979	4.989	165405	0.0181	0.53	18.0	17.4	17.7	14.60	2.8	3.1	0.000028	0.003
21	5	4	5	0.00	0.979	0.979	0.979	4.989	165400	0.0181	0.53	17.4	16.9	17.2	14.60	2.3	2.6	0.000024	0.002
22	4	3	4	0.00	0.979	0.979	0.979	4.989	165396	0.0181	0.53	16.9	16.4	16.6	14.60	1.8	2.1	0.000021	0.002
23	3	2	3	0.00	0.979	0.979	0.979	4.989	165393	0.0181	0.53	16.4	15.9	16.1	14.60	1.3	1.5	0.000017	0.002
24	2	1	2	0.00	0.979	0.979	0.979	4.989	165391	0.0181	0.53	15.9	15.3	15.6	14.60	0.7	1.0	0.000013	0.001
25	1	0	1	0.00	0.979	0.979	0.979	4.989	165389	0.0181	0.53	15.3	14.8	15.1	14.60	0.2	0.5	0.000008	0.001

Ctotal = 0	Vav = 4.991	dPsum = 13 + Pdyn = 27.8 Pa	Sum = 0.001095 m ³ /s = 2 cfm
		Vav = 4.99 m/s	Leakage = 0.112%
		Flow ratio = 25.0 L/s/m ²	
The leakage by approximate equation:		Leakage = 0.0011 m ³ /s	Differ. Q = -3.33%
	a = 13.74	Press. loss = 13.18+Pdyn = 27.8 Pa	Differ. dP = 0.68%

TABLE 2
Approximate Solution for Eight Single-Duct Sections

D (m)	Q (m ³ /s)	C_L	dP (pa)	C	μ	V_{av} (m/s)	Q_{ratio} (L/s-m ²)	Leakage (%)	Diff. Q (%)	Diff. dP (%)
0.500	0.980	6	28	0.00	0.45	4.99	25.00	0.1	-3.33	0.68
0.170	0.134	48	1500	127.00	22.10	4.40	10.00	40.02	-21.90	-2.22
0.170	0.175	48	1500	63.00	11.20	6.15	13.10	32.0	-16.20	-4.10
0.500	0.990	48	28	0.00	0.45	5.02	25.20	0.9	-4.77	-0.93
0.500	3.010	12	250	0.00	0.41	15.30	76.80	0.3	-4.86	-0.30
0.170	.326	48	1500	9.00	2.50	12.80	24.40	17.5	-9.75	-5.03
0.270	0.860	48	1500	9.00	3.26	14.02	40.60	10.7	-7.58	-3.39
0.320	1.170	12	1500	9.00	3.66	14.35	46.60	2.4	-5.47	-0.74

TABLE 3
Weighting Factor Analysis

C_L	6	48	48	48	12	48	48	12	
D	0.5	0.17	0.17	0.5	0.5	0.17	0.27	0.32	
Q	0.98	0.1339	0.1753	0.99	3.015	0.3264	0.86	1.171	
L	25	25	25	25	25	25	25	25	
P_t	28	1500	1500	28	250	1500	1500	1500	
V	4.994	5.81	7.64	5.04	15.36	14.3	14.976	14.558	
f	0.0181	0.024	0.023	0.018	0.0163	0.0403	0.0359	0.0344	
P_s	13.41	1480.25	1465.85	13.14	111.98	1380.37	1368.80	1376.02	
C_t	0	127	63	0	0	9	9	9	
dQ_{tab}	0.001095	0.054109	0.056096	0.008517	0.008569	0.057108	0.09202	0.027909	
$K3$	0.000008	0.000067	0.000067	0.000067	0.000016	0.000067	0.000067	0.000016	
$K2$	0.000067	0.001581	0.001581	0.000538	0.000134	0.001581	0.000996	0.000210	
$K1$	0.021177	0.082588	0.079147	0.02106	0.019070	0.138679	0.077783	0.062887	
Γ	0.000026	0.001865	0.002590	0.000216	0.000216	0.008427	0.003869	0.000685	
dV	0.00545	0.37799	0.52488	0.04395	0.04386	1.70726	0.78384	0.13880	
V_2	4.989	5.432	7.115	4.996	15.316	12.593	14.192	14.419	
dQ	0.001069	0.008575	0.011908	0.008625	0.008607	0.038732	0.044857	0.011157	
dT	13.215	73.097	121.446	13.461	112.726	772.445	453.253	335.585	
dP_s	13.183	70.361	116.431	13.200	111.936	740.471	438.799	333.198	
dP_t	13.23	74.38	123.81	13.59	113.12	786.23	460.13	336.77	
$dQ, \%$	-2.36	-84.15	-78.77	1.27	0.44	-32.18	-51.25	-60.02	
$dP, \%$	58.77	95.04	91.75	51.46	54.75	47.55	69.23	77.55	
Approximate Solution (for Comparison)									
$dQ, \%$	-3.33	-21.90	-16.20	-4.77	-4.86	-9.75	-7.58	-5.47	
$dP, \%$	0.68	-2.22	-4.10	-0.93	-0.30	-5.03	-3.39	-0.74	

$$\begin{aligned} \Delta P &= \Delta T + 0.5 \rho V_0 \Delta V \\ &= 13.2 + 0.5 \cdot 1.17 \cdot 4.994 \cdot 0.00545 \\ &= 13.2 \text{ Pa} \end{aligned}$$

The weighing factor solution is:

$$\Delta Q (\%) = \frac{(0.001068 - 0.001095)}{0.001095} \cdot 100 = -2.36\%$$

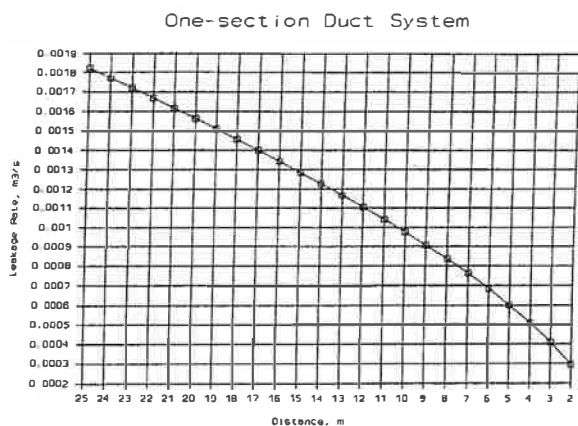
$$\Delta P (\%) = \frac{(28 - 13.2)}{28} \cdot 100 = 52.7\%$$

The results of weighing factor analysis for a number of problems are shown in Table 3. As shown in Table 3, leakage accuracy is very high for all the cases where the C-coefficients are zero. Calculations by the weighing factor method are slightly better for cases when $C_1 = 0$ than by the approximate method. However, the weighing factor method poorly calculates pressure loss.

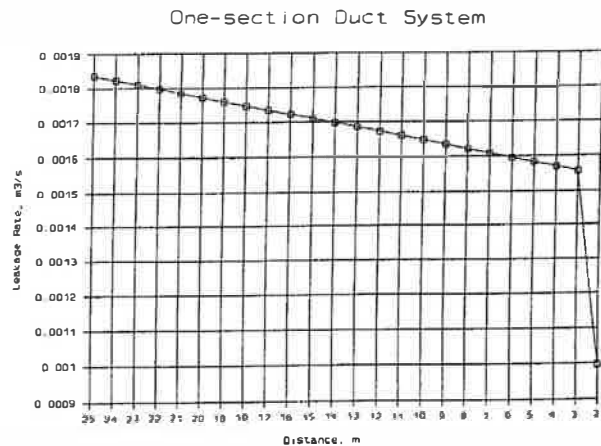
Duct Leakage as a Function of Local Resistance Locations

The amount of duct leakage depends on internal static pressure. For a straight duct, static pressures change uniformly. However, local resistance cause sudden changes of static pressure in a duct. Figure 2a illustrates leakage for a duct section with uniformly distributed local resistances. Local resistance concentrated at the end of a duct section are shown in Figure 2b. Figure 2c shows the resistance at the beginning of a duct section, while Figure 2d illustrates four resistances equally spaced along a duct.

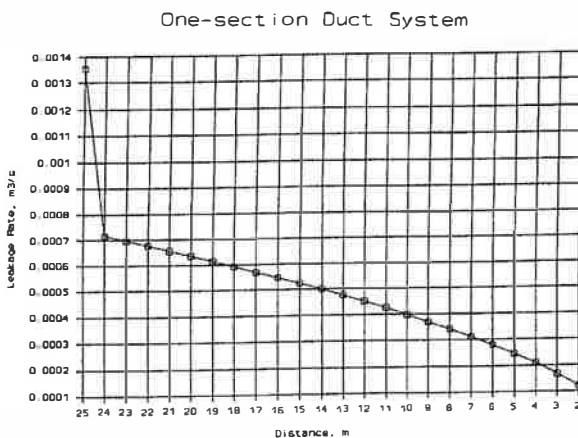
The results show (Table 3) that, when $\Sigma C \neq 0$, using the Darcy-Weisbach equation without locating resistances introduces errors into the duct leakage calculation regardless of the method used. These errors can be avoided by dividing the system into sections between each fitting.



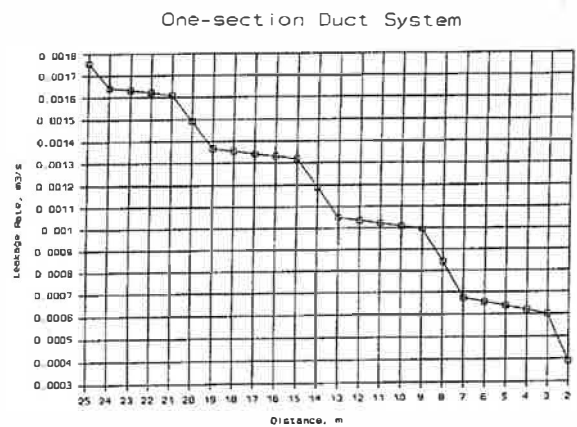
a Local resistance uniformly distributed.



b Local resistance at the end of a duct section.



c Local resistance at the beginning of a duct section



d Four local resistances along the duct length.

Figure 2 Air leakage for various locations of local resistances.

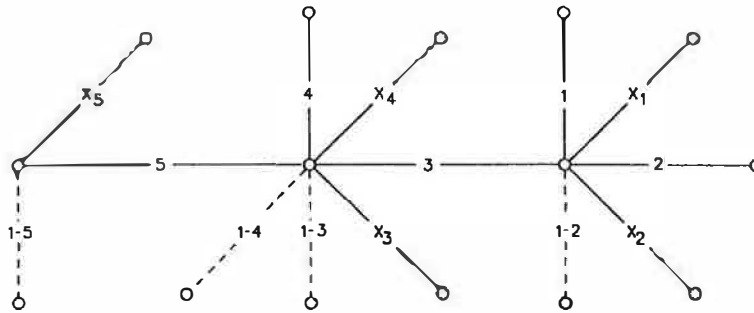


Figure 3 Five-section duct system schematic.

CONCLUSIONS

The magnitude of duct leakage for straight ducts depends on internal static pressure and varies uniformly along its length. Fittings in a system cause sudden changes in static pressure; therefore, duct leakage depends on fitting locations. For practical applications, duct leakage can be calculated using the average static pressure in each duct section if the system is divided into sections between each fitting.

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NOMENCLATURE

- A = duct cross-sectional area, m^2 (ft^2)
- A_s = duct surface, m^2 (ft^2)
- C = local loss coefficient, dimensionless
- C_L = leakage class, $m^3/s \cdot m^2$ ($cfm/100 ft^2$)
- C_{tot} = sum of local resistance coefficients, dimensionless (same as ΣC)
- D = duct diameter, m (in.)
- D_f = equivalent-by-friction diameter of rectangular duct, m (in.)
- f = friction factor, dimensionless
- g = acceleration due to gravity, m/s^2 (ft/s^2)
- g_c = dimensional constant, 1.0 ($kg \cdot m / (N \cdot s^2)$) (32.2 [$lb_m \cdot ft / (lb_f \cdot s^2)$])
- H = duct height, m (in.)
- J = hydraulic slope, m (ft)
- L = duct length, m (in.)
- P = total pressure at section, Pa (in. wg)
- P_{av} = average total pressure, Pa (in. wg)
- P_{dyn} = dynamic pressure, Pa (in. wg)
- P_s = static pressure differential from duct interior to exterior, Pa (in. wg)
- P_{si} = static pressure at point "i," Pa (in. wg)

- P_{sav} = average static pressure, Pa (in. wg)
- Q_1, Q_2 = airflow, m^3/s (cfm)
- Q_{av} = average airflow between upstream and downstream nodes, m^3/s (cfm)
- Q_d = downstream flow rate, m^3/s (cfm)
- Q_{ratio} = duct leakage per unit surface area, $(L/s)/m^2$ ($cfm/[s/ft^2]$)
- Q_u = upstream flow rate, m^3/s (cfm)
- R = absolute roughness factor, m (ft)
- Re = Reynolds number, dimensionless
- r = duct aspect ratio, dimensionless
- R_h = hydraulic radius, m (ft)
- S = duct cross-sectional area, m^2 (ft^2)
- V_1, V_2 = mean air velocity, m/s (fpm)
- V_{av} = average air velocity, m/s (fpm)
- W = duct width, m (in.)
- x = duct length, m (ft)
- $dP, \Delta P$ = total pressure loss of elemental and infinitesimal section, Pa (in. wg)
- dP_{av}
- ΔP_{av} = average pressure loss of elemental and infinitesimal section, Pa (in. wg)
- ΔP_s = static pressure loss, Pa (in. wg)
- $dQ, \Delta Q$ = flow leakage rate of elemental and infinitesimal section, m^3/s (cfm)
- $\%dQ$ = percent flow leakage to sectional flow, %
- dQ_{tab} = tabulated leakage rate by ASHRAE (1993, Table 7, p. 32.16)
- ρ = air density, kg/m^3 (lbm/ft^3)

REFERENCES

- AABC. 1983. Duct leakage and air balancing. Technical Publication No. 2-83. Washington, DC: Associated Air Balance Council.
- ASHRAE. 1993a. ASHRAE professional development seminars, *Air system design and retrofit*. Atlanta: American Society of Heating, Refrigerating and Air-Conditioning Engineers, Inc.

- ASHRAE. 1993b. *ASHRAE handbook—Fundamentals*, Chapter 32. Atlanta: American Society of Heating, Refrigerating and Air-Conditioning Engineers, Inc.
- ASHRAE/SMACNA/TIMA. 1985. Investigation of Duct Leakage, ASHRAE Research Project 308.
- Durfee, R.L. 1972. Measurement and analysis of leakage rates from seams and joints of air handling systems. AISI Project No. 1201-351/SMACNA Project No. 5-71. Contractor: Versar, Inc. Springfield, VA. New York: American Iron and Steel Institute/Chantilly, VA: Sheet Metal and Air Conditioning Contractors National Association.
- Farajian, T., G. Grewal, and R.J. Tsal. 1992. Post-accident air leakage analysis in a nuclear facility via T-method airflow simulation. 22d DOE/NRC Nuclear Air Cleaning and Treatment Conference, Denver, October.
- Horowitz, E., and S. Sahni. 1976. *Fundamentals of data structures*. New York: Computer Science Press.
- Konstantinov, U.M. 1981. *Hydraulics*. Kiev, USSR: Vischa Scola Publishing House.
- Swim, W.B. 1984. Analysis of duct leakage. ETL Data from ASHRAE RP-308 (prepared for TC 5.2).
- Tsal, R.J., and L.P. Varvak. 1992. Duct design using the T-method with duct leakage incorporated. ASHRAE Research Project 641-RP. Contractor: NETSAL and Associates. Atlanta: American Society of Heating, Refrigerating and Air-Conditioning Engineers, Inc.
- Tsal, R.J., H.F. Behls, and R. Mangel. 1988a. T-method duct design: Part I—Optimization theory. *ASHRAE Transactions* 94(2): 90-111.
- Tsal, R.J., H.F. Behls, and R. Mangel. 1988b. T-method duct design: Part II: Calculation procedure and economic analysis. *ASHRAE Transactions* 94(2): 112-151.
- Tsal, R.J., H.F. Behls, and R. Mangel. 1990. T-method duct design: Part III—Simulation. *ASHRAE Transactions* 96(2): 3-31.
- Tsal, R.J., H.F. Behls, and L.P. Varvak. 1998. T-method duct design: Part V—Duct leakage calculation technique and economics. *ASHRAE Transactions* 104(2).