Effect of Insulation and Mass Distribution in Exterior Walls on Dynamic Thermal Performance of Whole Buildings

Elisabeth Kossecka, Ph.D.   Jan Kosny, Ph.D.

ABSTRACT

The effect of wall material configuration on dynamic thermal performance is analyzed for six typical wall configurations. Due to different arrangements of concrete and insulation layers, these walls present a wide range of dynamic thermal properties. Newly developed thermal structure factors are used in selection and thermal analysis of these walls. A simple one-room model of the building exposed to diurnal periodic temperature conditions is analyzed to give some basic information about the effect of wall material configuration on thermal stability of the building. Whole building dynamic modeling using DOE-2.1E was employed for energy analysis of a one-story residential building with various walls for six different U.S. climates. Best thermal performance is obtained for walls with massive material layers at the inner side, in good thermal contact with the interior of a building.

INTRODUCTION

It is very important, due to the increasing number of new residential and commercial constructions using massive wall technologies, to optimize the mass and insulation distribution in walls. Comparing several massive walls with the same R-value, some wall configurations are more thermally effective than others (CABO 1995). This better thermal performance can be observed only with a specific distribution of mass and insulation inside the wall.

The annual energy demand for heating and cooling is affected to some extent by the thermal stability of a building itself. Building thermal stability is understood as the ability to hold the internal temperature within a certain interval, with normal external temperature oscillations and constant energy supply from the plant or without any plant action. This building thermal stability depends on the dynamic thermal responses of all building envelope components (exterior walls, internal partitions, ceilings, and floors) to external and internal temperature variations. Dynamic responses are determined by thermal properties of materials, their total amounts, and also specific arrangement in structures.

The important feature of the ambient temperature course is its diurnal character. It can be relatively well approximated by a harmonic function. Analysis of the simple one-room model of a building exposed to periodic temperature oscillations, for which analytic solution is available, gives some insight into the complicated stability problem of real buildings. Such a simple model is examined in this paper.

Newly introduced thermal structure factors are used in selection and analysis of walls of essentially different dynamic thermal properties. Thermal structure factors represent, together with the total thermal resistance and heat capacity, the basic thermal characteristics of walls. Thermal structure factors have their counterparts in structures where three-dimensional heat transfer occurs (Kossecka and Kosny 1997).

Six characteristic exterior wall configurations are considered for this study. These walls are composed of concrete and insulating foam. They have the same R-values. The wall materials, however, are arranged in different ways. Consequently, structure factors have different values for the different walls. Whole building dynamic DOE-2.1E (LBL 1993) modeling was employed for energy analysis of one-story residential buildings. The simulation was performed for six U.S. climatic zones. Three types of whole building performance data were compared for each type of wall: annual heating loads, annual cooling loads, and total annual energy demand.

Elisabeth Kossecka is a professor at the Institute of Fundamental Technological Research, Polish Academy of Sciences, Warsaw, Poland. **Jan Kosny** is a research scientist at Oak Ridge National Laboratory, Oak Ridge, Tenn.

EFFECT OF STRUCTURE ON DYNAMIC THERMAL CHARACTERISTICS OF BUILDING WALLS

Structure Factors for Multilayer Walls

The thermal structure of a wall is understood to be the thermal resistance and capacity distribution in its volume. Formal relationships, which describe in a quantitative way the effect of structure on dynamic thermal behavior of walls, follow from the integral formulae for the heat flow across the surfaces of a wall in a finite time interval (Kossecka 1992). They include quantities called the thermal structure factors. Relationships between the structure factors, response factors, and z-transfer function coefficients have been derived and analyzed by Kossecka (1996, 1999) and Kossecka and Kosny (1997).

Thermal structure factors appear in expressions for the asymptotic heat flow across the surfaces of a wall, for boundary conditions independent of time. Consider the heat transfer in the exterior building wall of thickness L separating the room at temperature T_i from the environment at temperature T_e. Thermophysical properties of this wall include thermal conductivity k, specific heat c_p, and density ρ, as well surface film resistances, R_i and R_e; it is assumed that they do not change with time.

Let θ be the dimensionless temperature for the steady-state heat transfer through a wall, with boundary conditions $T_i = 0$ and $T_e = 1$. For a plane wall, for which one-dimensional heat conduction conditions are satisfied, the function $\theta(x)$ is given by

$$\theta(x) = \frac{R_{i-x}}{R_T} \qquad 1 - \theta(x) = \frac{R_{x-e}}{R_T} \qquad (1)$$

where R_{i-x} and R_{x-e} denote the resistances for heat transmission from point x in a wall to the internal and external environment, respectively, and R_T the total resistance for heat transmission through a wall.

R_{i-x} and R_{x-e} can be expressed by the following integrals:

$$R_{i-x} = R_i + \int_0^x \frac{dx'}{k(x')}, \qquad R_{x-e} = \int_x^L \frac{dx'}{k(x')} + R_e \qquad (2)$$

Consider now the transient heat transfer process for the ambient temperatures held constant for $t > 0$ and initial temperature in a wall equal to zero. For sufficiently large t the asymptotic expressions for the total heat flow in time interval $(0, t)$, across the internal and external surface of a wall, in the direction from the room to the environment, $Q_i(t)$ and $Q_e(t)$, have the following simple form (Kossecka 1992, 1993, 1996, 1999):

$$Q_i(t) = \frac{t}{R_T}[T_i - T_e] + C\varphi_{ii}T_i + C\varphi_{ie}T_e \qquad (3)$$

$$Q_e(t) = \frac{t}{R_T}[T_i - T_e] - C\varphi_{ie}T_i - C\varphi_{ee}T_e \qquad (4)$$

where C is the total thermal capacity of the wall element of the unit's cross-sectional area:

$$C = \int_0^L \rho c_p dx$$

and the quantities φ_{ii}, φ_{ie}, and φ_{ee} are given by

$$\varphi_{ii} = \frac{1}{C}\int_0^L \rho c_p (1-\theta)^2 dx = \frac{1}{C}\int_0^L \rho c_p \frac{R_{x-e}^2}{R_T^2}dx \qquad (5)$$

$$\varphi_{ie} = \frac{1}{C}\int_0^L \rho c_p \theta(1-\theta)dx = \frac{1}{C}\int_0^L \rho c_p \frac{R_{i-x}R_{x-e}}{R_T^2}dx \qquad (6)$$

$$\varphi_{ee} = \frac{1}{C}\int_0^L \rho c_p \theta^2 dx = \frac{1}{C}\int_0^L \rho c_p \frac{R_{i-x}^2}{R_T^2}dx \qquad (7)$$

Dimensionless quantities φ_{ii}, φ_{ie}, and φ_{ee} are called the "thermal structure factors" for a wall. For a plane wall they are determined directly by its structure, represented by the thermal capacity and resistance distribution across its thickness. In transitions between two different states of steady heat flow, they represent fractions of the total variation of heat stored in the wall volume that are transferred across each of its surfaces due to the ambient temperature variations.

The following identity is a consequence of Equations 5 through 7:

$$\varphi_{ii} + 2\varphi_{ie} + \varphi_{ee} = 1 \qquad (8)$$

Structure factors can be determined experimentally in processes with steady-state initial and final states of heat flow (Kosny et al. 1998). As expressed by Equations 3 and 4, they are related to measurable quantities, such as the heat flow, temperature, thermal resistance, and heat capacity. Together with the total thermal resistance R_T and total heat capacity C, they constitute the basic thermal characteristics of a wall and have their counterparts for structures in which three-dimensional heat flow occurs (Kossecka and Kosny 1996, 1997).

Structure factors for a wall composed of n plane homogeneous layers, numbered from 1 to n with layer 1 at the interior surface, are given as follows:

$$\varphi_{ii} = \frac{1}{R_T^2 C}\sum_{m=1}^n C_m\left[\frac{R_m^2}{3} + R_m R_{m-e} + R_{m-e}^2\right] \qquad (9)$$

$$\varphi_{ie} = \frac{1}{R_T^2 C}\sum_{m=1}^n C_m\left[-\frac{R_m^2}{3} + \frac{R_m R_T}{2} + R_{i-m}R_{m-e}\right] \qquad (10)$$
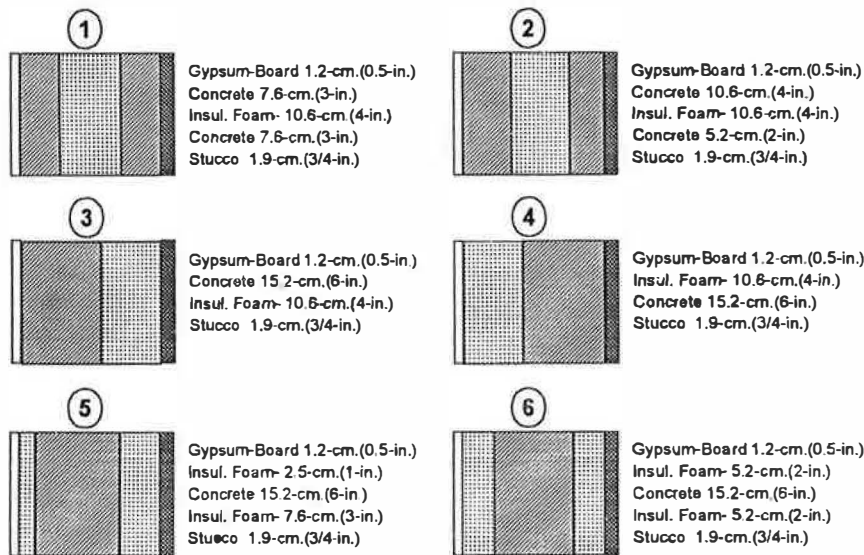
Figure 1 *Walls of different structure composed of concrete and insulatin. See Table 1 for thermal characteristics.*

$$\varphi_{ee} = \frac{1}{R_T^2 C} \sum_{m=1}^{n} C_m \left[\frac{R_m^2}{3} + R_m R_{i-m} + R_{i-m}^2 \right] \quad (11)$$

where R_m and C_m denote, respectively, the thermal resistance and capacity of the mth layer, whereas R_{i-m} and R_{m-e} denote the resistances for heat transfer from surfaces of the mth layer to inner and outer surroundings, respectively.

Structure factor φ_{ii} is comparatively large when most of the total thermal capacity is located near the interior surface $x = 0$ and most of the insulating materials (resistances) reside in the outer part of the wall, located near the surface $x = L$. The opposite holds for φ_{ee}. The following relations are straightforward: $0 < \varphi_{ii} < 1$, $0 < \varphi_{ee} < 1$. Structure factor φ_{ie} is compar-

atively large if most of the thermal mass is located in the center of the wall and the resistance is symmetrically distributed on both sides of it. The following limitations on φ_{ie} result from Equation 10: for a two-layer wall, $0 < \varphi_{ie} < 3/16$; for an n-layer wall, with $n \geq 3$, $0 < \varphi_{ie} < 1/4$. For a homogeneous wall, with negligibly small film resistances R_i and R_e, $\varphi_{ii} = \varphi_{ee} = 1/3$, $\varphi_{ie} = 1/6$. The products $C\varphi_{ii}$, $C\varphi_{ee}$, and $C\varphi_{ie}$ for a multilayer wall are identified as thermal mass factors, introduced by ISO Standard 9869 (ISO 1994).

Structure factors for multilayer walls depend on the arrangement of wall materials. To demonstrate this effect, six examples of walls of the same resistance and capacity but of different structure were examined. Walls (1) through (6) are depicted in Figure 1. Their structure factors are presented in

TABLE 1
Structure Factors for Walls with Cores Composed of Heavyweight Concrete and Insulation, Shown in Figure 1

Structure No.	Layer thicknesses (in.)	φ_{ii}	φ_{ie}	φ_{ee}
Gypsum - Heavyweight Concrete - Insulation - Heavyweight Concrete - Stucco				
1	1/2 - 3 - 4 - 3 - 3/4	0.408	0.048	0.496
2	1/2 - 4 - 4 - 2 - 3/4	0.530	0.053	0.363
3	1/2 - 6 - 4 - 0 - 3/4	0.770	0.068	0.094
Gypsum - Insulation - Heavyweight Concrete - Insulation - Stucco				
4	1/2 - 4 - 6 - 0 - 3/4	0.034	0.040	0.885
5	1/2 - 1 - 6 - 3 - 3/4	0.460	0.187	0.167
6	1/2 - 2 - 6 - 2 - 3/4	0.234	0.222	0.322
Homogeneous Core	1/2 - 10 - 3/4	0.294	0.162	0.382

Table 1. The main part of each wall is a composition of heavy-weight concrete layers of 6 in. (0.152 m) total thickness and insulation layers of 4 in. (0.102 m) total thickness. The interior layer is ½ in. (0.013 m) thick gypsum plaster. The exterior layer is ¾ in. (0.019 m) thick stucco. Total wall thickness is L = 11.25 in. (0.286 m). Results for the wall with homogeneous core, of the same total thermal resistance and capacity, are added for comparison.

Thermophysical properties of the wall materials are as follows:

- *Heavyweight concrete*: k = 10 Btu·in./h·ft^2·°F (1.44 W/m·K), ρ = 140 lb/ft^3 (2240 kg/m^3), c_p = 0.2 Btu/lb·°F (0.838 kJ/kg·K)
- *Insulation*: k = 0.25 Btu·in./h·ft^2·°F (0.036 W/m·K), ρ = 1 lb/ft^3 (16 kg/m^3), c_p = 0.29 Btu/lb·°F (1.215 kJ/kg·K)
- *Gypsum board*: k = 1.11 Btu·in./h·ft^2·°F (0.16 W/m·K), ρ = 50 lb/ft^3 (800 kg/m^3), c_p = 0.26 Btu/lb·°F (1.089 kJ/kg·K)
- *Stucco*: k = 5 Btu·in./h·ft^2·°F (0.72 W/m·K), ρ = 116 lb/ft^3 (1856 kg/m^3), c_p = 0.2 Btu/lb·°F (0.838 kJ/kg·K)

The following wall surface film resistances are assumed: R_i = 0.69 ft^2·°F·h/Btu (0.12 m^2·K/W), R_e = 0.33 ft^2·°F·h/Btu (0.05 m^2·K/W). The total thermal resistance for each wall is R_T = 18.22 h·ft^2·°F/Btu (3.21 m^2·K/W). Overall heat transfer coefficient is U = 0.055 Btu/h·ft^2·°F (0.312 W/m^2·K), total mass M = 79.87 lb/ft^2 (390.27 kg/m^2), and wall thermal capacity is C = 16.146 Btu/ft^2·°F (329.93 kJ/m^2·K).

Relationships Between Structure Factors and Response Factors

Quantities $C\varphi_{ii}$, $C\varphi_{ie}$ and $C\varphi_{ee}$, which in Equations 3 and 4 determine the role of storage effects in transitions between different states of steady heat flow, affect particular modes of dynamic heat flux responses of a wall. They appear in the constraint conditions on dynamic thermal characteristics of walls such as the response factors, z-transfer function coefficients, and also residues and poles of the Laplace transfer functions (Kossecka 1996, 1999; Kossecka and Kosny 1997).

Let $X(m\delta)$, $Y(m\delta)$, and $Z(m\delta)$ denote the response factors for a wall, corresponding to different heat flux response modes. Response factor for number n represents the heat flux due to the unit, triangular temperature pulse of base width 2δ at the time instant $n\delta$ (Kusuda (1969; Clarke 1985). Relationships between the response factors $X(m\delta)$, $Y(m\delta)$, and $Z(m\delta)$ and the structure factors φ_{ii}, φ_{ie}, and φ_{ee} have the following form:

$$\delta \sum_{n=1}^{\infty} n X (n\delta) = -C \varphi_{ii} \qquad (12)$$

$$\delta \sum_{n=1}^{\infty} n Y (n\delta) = C \varphi_{ie} \qquad (13)$$

$$\delta \sum_{n=1}^{\infty} n Z (n\delta) = -C \varphi_{ee} \qquad (14)$$

Analogous conditions are satisfied by the response factors for wall elements of complex structure in which three-dimensional heat flow occurs (Kossecka and Kosny 1997). Equations 12 through 14 refer to the response factors with number $n \geq 1$, which represent the storage effects in form of surface heat fluxes after the duration of the temperature pulse. They indicate that, given total thermal capacity C, the sums of products of response factors of particular kind and number increase with the appropriate structure factors. This means that the role of response factors having large number n increases with values of the appropriate structure factors.

Effect of Wall Material Configuration on the Frequency Responses of Walls

Equations 12 through 14 directly relate structure factors and wall responses to triangular ambient temperature pulses. Wall responses to periodic temperature excitations also depend on the structure factors. This dependence is not represented by formal relations but appears in the form of significant correlations between the frequency-dependent and structure-dependent dynamic wall characteristics.

The general solution of the one-dimensional heat transfer problem in a multilayer slab at periodic temperature conditions is presented in several textbooks (Carslaw and Jaeger 1959; Clarke 1985). The heat flux across the inside surface of a wall is given by

$$q_i(t) = \frac{D(i\omega)}{B(i\omega)} T_i(t) - \frac{1}{B(i\omega)} T_e(t) \qquad (15)$$

where $D(i\omega)$ and $B(i\omega)$ are the complex numbers, elements of the transmission matrix.

The term $1/B$ represents the transmittance response, whereas D/B represents the admittance response. Amplitude of the transmittance response multiplied by R_T is the decrement factor, DF. It represents reduction of the transmission cyclic heat flux amplitude, due to the external temperature excitation, at the inside surface of a wall, as compared to the steady-state value, proportional to $1/R_T$. Time shifts of the transmittance and admittance response are denoted here by τ_{ie} and τ_{ii}, respectively.

$$DF = \frac{R_T}{|B(i\omega)|}, \quad \tau_{ie} = \frac{1}{\omega}\arg\frac{1}{B(i\omega)}, \quad \tau_{ii} = \frac{1}{\omega}\arg\frac{D(i\omega)}{B(i\omega)} \qquad (16)$$

Time shift τ_{ie}, is always negative—its absolute value is thus called the time lag—whereas τ_{ii} is positive.

For very thick walls, $1/B$ amplitude and DF tend to zero, whereas the admittance response amplitude tends to the finite value equal to that for a semi-infinite solid.

The effect of the structure factors, for given total resistance R_T and capacity C, on wall responses to harmonic temperature variations, can easily be demonstrated on simple

TABLE 2
**Decrement Factors, Amplitudes, and Time Shifts of the Transmittance and
Admittance Response for Walls with Cores Composed of Heavyweight Concrete
and Insulation, Shown in Figure 1**

Wall No	$\varphi_{ie}/\varphi_{ii}$	1/B Response			D/B Response	
		DF	Amplitude Btu/h·ft²·°F	τ_{ie} (h)	Amplitude Btu/h·ft²·°F	τ_{ii} (h)
Gypsum - Heavyweight Concrete - Insulation - Heavyweight Concrete - Stucco						
1	0.048/0.408	0.270	0.0148	-8.831	0.758	1.565
2	0.053/0.530	0.251	0.0138	-8.524	0.764	1.231
3	0.068/0.770	0.205	0.0112	-7.478	0.746	0.908
Gypsum - Insulation - Heavyweight Concrete - Insulation - Stucco						
4	0.040/0.034	0.356	0.0196	-6.761	0.153	4.072
5	0.187/0.460	0.070	0.0038	-8.237	0.226	1.905
6	0.222/0.234	0.059	0.0032	-8.288	0.171	2.998
Homogen. Core	0.162/0.294	0.039	0.0021	-20.548	0.398	2.386

examples of multilayer walls (as shown in Figure 1). Decrement factors, amplitudes of the responses, and the appropriate time shifts, calculated for harmonic oscillations of the exterior temperature with the period of 24 hours, are summarized in Table 2. Results for the wall with homogeneous core, of the same total thermal resistance and capacity, are added for comparison.

For the same total resistance and heat capacity, the transmission heat flux response amplitude and decrement factor decrease with the structure factor φ_{ie}. It is worth noticing that for configurations 5 and 6, which represent the type "thermal mass in the center," the value of DF it is about six times lower than for configurations 1 through 4, of type "insulation in the center" or "insulation only on the one wall side."
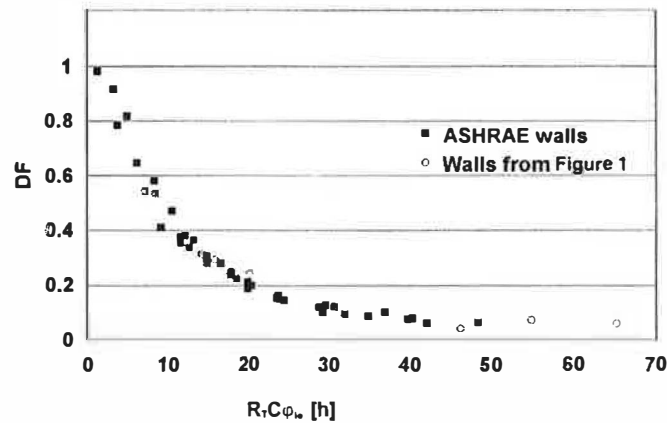
Correlations between the frequency responses and structure factors for building walls were examined by Kossecka (1999) for the set of walls selected by Harris and McQuiston (1988) to represent groups of walls with similar transient heat transfer characteristics. The paper of Harris and McQuiston became the basis for the transfer function method calculation procedure, presented in chapter 26 of *ASHRAE Fundamentals* (ASHRAE 1989). These correlations are represented by the plot of DF vs. structure-dependent time constant $R_T C \varphi_{ie}$ in Figure 2 and D/B amplitude vs. the thermal mass factor $C \varphi_{ii}$ in Figure 3. The results for all 41 walls from the 1989 Handbook (ASHRAE 1989) are analyzed herein.

Figure 2 shows that the decrement factor decreases with $R_T C \varphi_{ie}$, very rapidly in the interval (0,10 h), from 1 to 0.5 approximately, and less rapidly in the interval (10 h, 30 h),
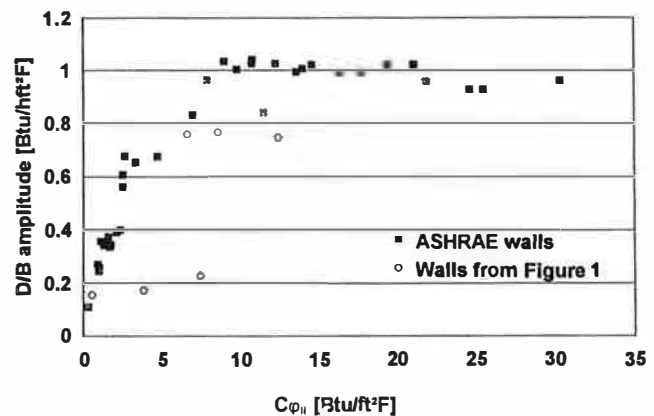

Figure 2 Decrement factor versus structure dependent time constant $R_T C \varphi_{ie}$.

Figure 3 Admittance response amplitude vs. thermal mass factor $C \varphi_{ii}$.

from 0.5 to 0.1 approximately. Above the $R_TC\varphi_{ie}$ value of 30 h it decreases very slowly. The existence of such a correlation is not trivial in light of the lack of a clear correlation between DF and the time constant R_TC. The nonlinear dependence of DF on $R_TC\varphi_{ie}$ may be approximated by a smooth curve with high accuracy. The following function, DF_{est}, found by Jedrzejuk (1997), gives a very good fit ($r^2 = 0.985$ when transformed to the linear dependence):

$$DF_{est}(x) = \frac{1}{\sqrt{1 + ax^b}}; \qquad x = R_TC\varphi_{ie} \qquad (17)$$

where $a = 0.014$ and $b = 2.495$.

As shown in Table 2, amplitudes of the admittance response D/B are several times higher for walls 1, 2, and 3, with massive concrete layers located at the inner side and a comparatively high value of the structure factor φ_{ii}, than for walls 4 and 6, with insulation placed on the inner side and a low value of φ_{ii} (Table 1). Wall 5, with a thin insulation layer at the inner side, is an exception. It has almost as high a value of φ_{ii} as wall 2, but its admittance response amplitude is only 0.227 when for wall 2 it is 0.746.

Amplitudes of the heat flux responses to periodic temperature excitations depend on square roots of the products of k, ρ, and c_p (Carslaw and Jaeger 1959; Shklover 1961), which for light materials with low thermal conductivity differ in order of magnitude from those for heavy materials with high thermal conductivity. The admittance response amplitude is sensitive to the values of $\sqrt{k\rho c_p}$ for the innermost layers with significant thickness.

For a variety of building walls with different thermal properties, the admittance response D/B amplitude shows a significant correlation with the $C\varphi_{ii}$ product (thermal mass factor, according to the ISO standard [ISO 1994]); it is represented by the plot in Figure 3. The D/B amplitude increases very rapidly with $C\varphi_{ii}$ up to the value of about 10 Btu/ft$^2 \cdot$ °F (204 kJ/m$^2 \cdot$K) and is almost steady above that value. However, the dependence of the admittance response amplitude on the $C\varphi_{ii}$ value shows larger scatter than the dependence of the decrement factor on the structure-dependent time constant $R_TC\varphi_{ie}$, especially due to walls 5 and 6, where the thermal mass layers are separated both from the interior of the building and exterior air by insulating material. Material configuration of this kind can be found in insulating concrete form (ICF) walls.

SIMPLE MODEL OF A BUILDING EXPOSED TO PERIODIC TEMPERATURE EXCITATIONS

Consider the very simple model of a building in the form of a rectangular box, with walls identical to each other, exposed to the influence of the external temperature T_e. One-dimensional heat transfer through the walls is assumed. The building is ventilated, and the air exchange velocity is constant in time. All other effects are neglected.

Let the external temperature T_e be a harmonic function of time, with angular frequency ω and amplitude A_{Te}. The steady-state periodic temperature T_i is also a harmonic function of time with angular frequency ω but amplitude A_{Ti} and some time shift τ_{Ti} of the maximum with respect to the maximum of T_e:

$$T_e(t) = A_{Te} e^{i\omega t}, T_i(t) = A_{Ti} e^{i\omega(t+\tau_{Ti})} \qquad (18)$$

The lower the value of the A_{Ti}/A_{Te} ratio, the better the thermal stability of the system.

The equation of the heat balance for this system has the following form:

$$C_V \frac{dT_i}{dt} = -S_w q_i - C_V n(T_i - T_e) \qquad (19)$$

where q_i is the heat flux across the internal surfaces of walls, S_w is the total surface area of the walls, $C_V = \rho c_p V$ is the air volume thermal capacity, and n (h^{-1}) is the air exchange frequency.

Solving Equation 19 with respect to T_i, with q_i given by Equation 15, the following expression is obtained:

$$T_i(t) = T_e(t) \frac{C_V n + \frac{S_w}{B(i\omega)}}{C_V[n + i\omega] + S_w \frac{D(i\omega)}{B(i\omega)}} \qquad (20)$$

The response function $1/B$ is in the numerator whereas D/B is in the denominator, both multiplied by the wall area. Therefore, in general, amplitude of the temperature T_i increases with the amplitude of $1/B$ and decreases with the amplitude of D/B. A simple recipe for the good thermal stability of this system is thus low response to the external temperature variations and high response to the internal temperature variations.

Values of the amplitude ratio A_{Ti}/A_{Te} and the time shift τ_{Ti} calculated for walls 1-6, assuming room dimensions: of 15 ft by 15 ft by 9 ft ($4.5 \times 4.5 \times 2.7$ m) and air exchange frequency $n = 1$ (h^{-1}), are presented in Table 3.

Results of the analysis shown in Table 3 for the simple building model indicate that buildings having walls with massive concrete inside layers (structures 1, 2, 3) are stable. The amplitude ratio A_{Ti}/A_{Te} for walls 1 through 3, with high values of the structure factor φ_{ii} and admittance response amplitude, is about 5.5 times lower than for wall 4 and about 4 times lower than for wall 6 with low values of φ_{ii} and D/B amplitude. Wall 5 is again an exception; a comparatively high value of φ_{ii} does not guarantee a high value of the admittance response amplitude (Table 2) as well as low internal and external amplitude ratio (Table 3).

Results presented in Tables 2 and 3 indicate that the high value of the internal admittance response amplitude of the external walls appears to be more important than the low value of the decrement factor for the transmission heat flux

TABLE 3
Amplitude Ratio and Time Shift of the Internal and External Temperature Oscillations for the One-Room Building with Walls 1-6, Shown in Figure 1

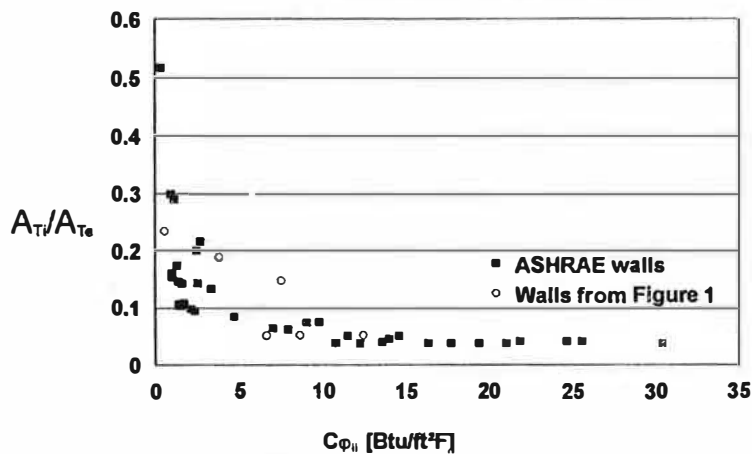| Structure No	Layer thicknesses (in.)	A_{Ti}/A_{Te}	τ_{Ti}
Gypsum - Heavyweight Concrete - Insulation - Heavyweight Concrete - Stucco			
1	1/2 - 3 - 4 - 3 - 3/4	0.040	−2.878
2	1/2 - 4 - 4 - 2 - 3/4	0.041	−2.490
3	1/2 - 6 - 4 - 0 - 3/4	0.047	−1.996
Gypsum - Insulation - Heavyweight Concrete - Insulation - Stucco			
4	1/2 - 4 - 6 - 0 - 3/4	0.222	−5.330
5	1/2 - 1 - 6 - 3 - 3/4	0.142	−2.087
6	1/2 - 2 - 6 - 2 - 3/4	0.184	−2.880
Homogeneous core	1/2 - 10 - 3/4	0.094	−2.101

Figure 4 *Correlation of the internal and external temperature amplitude ratio, calculated for the simple building model, with the thermal mass factor $C\varphi_{ii}$.*
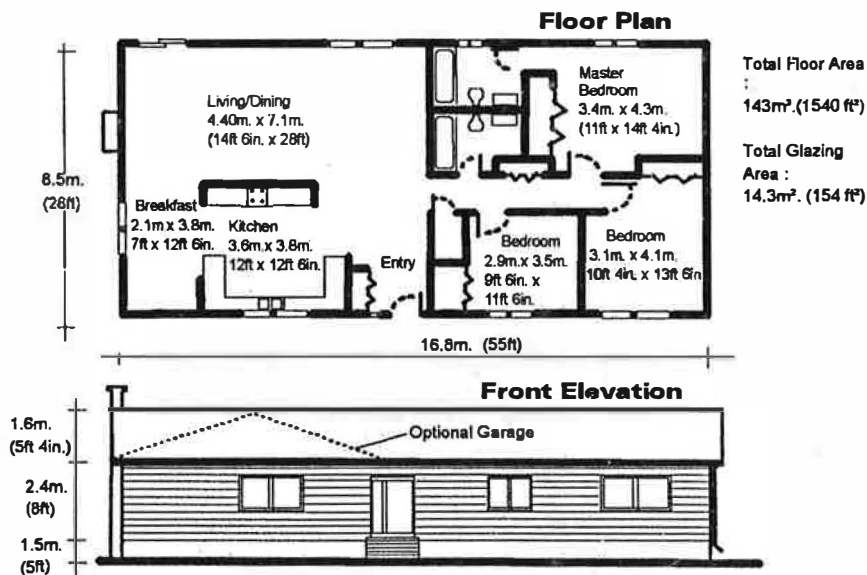

Figure 5 *One-story residential building used in thermal analysis.*

response. Modifications of the model, by adding interior massive walls and changing the air exchange velocity, have no effect on this general conclusion.

Figure 4 depicts the dependence of the internal and external temperature amplitude ratio on the thermal mass factor $C\varphi_{ii}$, calculated using Equation 20, for the representative set of walls from the *1989 ASHRAE Handbook—Fundamentals* and Figure 1. A_{Ti}/A_{Te} decreases with $C\varphi_{ii}$ up to the value of 10 Btu/ft^2·°F (204 kJ/m^2·K). This is the level of $C\varphi_{ii}$ at which admittance response amplitude stopped increasing in Figure 3. Like D/B amplitude, A_{Ti}/A_{Te} is approximately constant for $C\varphi_{ii} > 10$ Btu/ft^2·°F (204 kJ/m^2·K).

EFFECT OF WALL MATERIAL CONFIGURATION ON DYNAMIC THERMAL PERFORMANCE OF THE WHOLE BUILDING

Calculations of the annual heating and cooling energy demands were performed for a one-story residential building. Six types of exterior walls, structures 1-6 presented in Figure 1, were considered. The annual whole building energy analysis program DOE-2.1E was used for the dynamic simulations of the heating and cooling loads. The analyzed house is presented in Figure 5. This house was the subject of previous energy-efficiency studies (Huang at al. 1987; Kosny and Desjarlais 1994; Christian and Kosny 1996). It has approxi-

mately 1540 ft^2 (143 m^2) of living area, 1328 ft^2 (123 m^2) of exterior wall area, eight windows, and two doors (one door is a glass slider; its impact is included with the windows). The elevation wall area includes 1146 ft^2 (106 m^2) of opaque (or overall) wall area, 154 ft^2 (14.3 m^2) of window area, and 28 ft^2 (2.6 m^2) of door area. The following building design characteristics and operating conditions were used during computer modeling:

Interior walls: 3.57 lb/ft^2 (17.4 kg/m^2) of floor area; specific heat, 0.26 Btu/lb·°F (1.09 kJ/kg·K)

Furniture: 3.30 lb/ft^2 (16.1 kg/m^2) of floor area; specific heat, 0.30 Btu/lb·°F (1.26 kJ/kg·K); thickness, 2 in. (5.04 cm) total equivalent floor area

Thermostat set point: 70°F (21.1°C) heating, 78°F (25.6°C) cooling

Window type: double-pane clear glass, transmittance 0.88, reflectance 0.08

Roof insulation: R-30 h·ft^2·°F/Btu (5.3 m^2·K/W)

For the base case calculation of infiltration, the Sherman-Grimsrud infiltration method in the DOE 2.1E whole building simulation model (Sherman and Grimsrud 1980) was used. The average total leakage area, expressed as a fraction of the floor area of 0.0004, was assumed. Typical Meteorological Year (TMY) data for six U.S. climates—Atlanta, Denver, Miami, Minneapolis, Phoenix, and Washington D.C.—were used for the whole building thermal modeling. Values of the annual heating and cooling energy demand are collected in Tables 4 and 5.

TABLE 4
Annual Cooling Energy Demand for the Typical Family House with Different Types of Exterior Walls (MBTU per year)

Wall No.	Atlanta	Denver	Miami	Minneapolis	Phoenix	Washington
1	5.76	0.74	33.06	1.39	27.34	3.09
2	5.68	0.74	32.80	1.32	27.27	3.00
3	5.60	0.73	32.30	1.26	27.24	2.87
4	7.53	1.88	34.70	2.19	29.15	4.54
5	6.79	1.53	33.60	1.80	28.53	3.83
6	7.05	1.70	33.87	1.93	28.76	4.08

TABLE 5
Annual Heating Energy Demand for the Typical Family House with Different Types of Exterior Walls (MBTU per year)

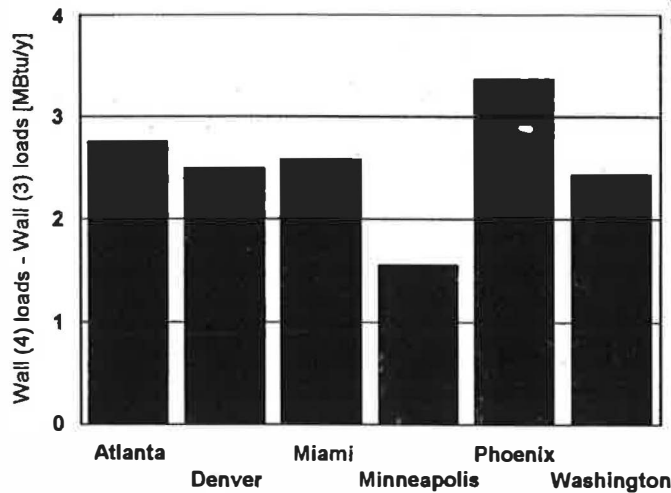| Wall No.	Atlanta	Denver	Miami	Minneapolis	Phoenix	Washington
1	18.91	37.78	0.37	66.75	3.40	33.26
2	18.88	37.76	0.35	66.75	3.37	33.26
3	18.89	37.84	0.32	66.85	3.41	33.34
4	19.72	39.18	0.53	67.48	4.88	34.11
5	19.37	38.70	0.39	67.20	4.50	33.88
6	19.50	38.91	0.42	67.26	4.73	34.01

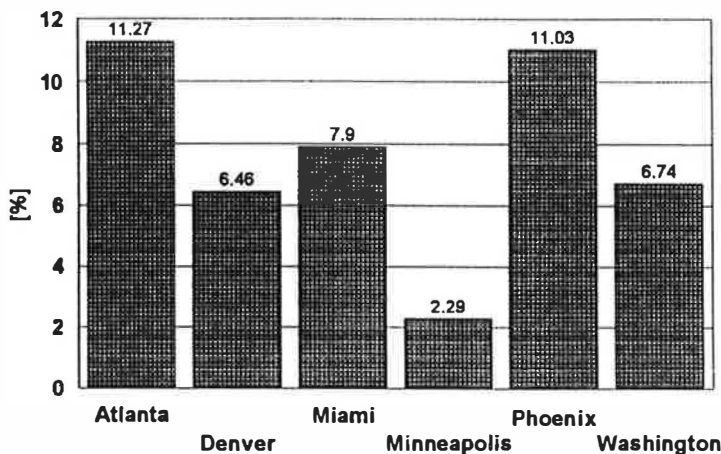**Figure 6** *Differences between total loads calculated for walls (3) and (4).*

Figure 7 *Percent differences between loads calculated for walls (3) and (4).*

Results of the whole building dynamic modeling showed that walls containing massive internal layers, (1, 2, and 3) have the best annual thermal performance for the climates considered. The lowest annual heating and cooling loads are noticed for wall 3, where all the thermal mass is concentrated in its interior part. Wall 4, with insulation material concentrated on the interior side, generates the largest energy demand. Differences between total building loads, for walls 3 and 4, are shown in Figure 6. Percent differences between wall 3 and 4 in annual energy demands are depicted in Figure 7. The highest differences (over 11%) are observed for Atlanta and Phoenix, the lowest for Minneapolis (over 2%). The average difference for all locations is 7.6%. This means that in U.S. residential buildings containing massive walls, up to 11% of heating and cooling energy can be saved by the proper arrangement of the exterior wall materials.

Energy demands for other wall configurations, with the concrete wall core and insulation placed on both sides of the wall (5 and 6), fall between the energy demands for wall 4 and the most efficient walls (1, 2, and 3).

CONCLUSIONS

An analytical solution for the response of a simple building exposed to periodic temperature conditions indicates that the most effective wall assemblies are walls where thermal mass is in good contact with the interior of the building. These walls have high values of the structure factor φ_{ii} and the internal admittance heat flux response amplitude, which enter as parameters in the solution. A high value of the internal admittance response amplitude definitely improves the thermal stability of a building, expressed as the amplitude of internal temperature periodic oscillations in response to the exterior temperature oscillations.

Whole building energy modeling using DOE-2.E was performed to predict annual heating and cooling energy demands for the one-story residential building. Results of the computer simulation lead to the conclusion that walls with massive internal layers, with high values of the structure factor φ_{ii}, show the best thermal performance for all U.S. climatic zones: minimum annual heating and cooling energy demand.

Wall material configuration can significantly affect annual thermal performance of the whole building. There is the potential to save up to 11% of heating and cooling energy in U.S. residential buildings containing massive walls by optimization of the mass and insulation distribution in the walls.

NOMENCLATURE

A_{Ti}, A_{Te} = interior and exterior temperature amplitude

$B(i\omega)$, $D(i\omega)$ = elements of the transmission matrix

$1/B$, D/B = transmittance and admittance response

c_p = specific heat

C, C_m = thermal capacity

DF = decrement factor

$X(m\delta)$, $Y(m\delta)$, $Z(m\delta)$ = response factors with number m

k = thermal conductivity

L = thickness

M = mass

q_i, q_e = heat flux across the internal and external surface of a wall

Q_i, Q_e = heat flow across the internal and external surface of a wall

R_T = total resistance of a wall

R_m = resistance of the wall layer m

R_i, R_e = surface film resistance

R_{i-x}, R_{x-e} = resistance from the interior and exterior air to point x

R_{i-m}, R_{m-e} = resistance from the interior and exterior air to the surface of the layer m

t = time

T_i, T_e = interior and exterior air temperature

δ = time decrement

φ_{ii}, φ_{ie}, φ_{ee} = structure factors of a wall

ρ = density

τ_{ii}, τ_{ie}, τ_{Ti} = time shift

ω = angular frequency

REFERENCES

ASHRAE. 1989. *1989 ASHRAE Handbook —Fundamentals*, I-P Edition. Atlanta: American Society of Heating, Refrigerating and Air-Conditioning Engineers, Inc.

CABO. 1995. *Model energy code*. Council of American Building Officials.

Carslaw, H.S., and J.C.Jaeger. 1959. *Conduction of heat in solids*. Oxford University Press.

Christian, J., and J. Kosny. 1996. Thermal performance and wall rating. *ASHRAE Journal*, March.

Clarke, J.A. 1985. *Energy simulation in building design*. Adam Hilger Ltd.

LBL. 1993. *DOE-2, version 2.1E*. Energy and Environment Division, Lawrence Berkeley National Laboratory, University of California, Berkeley, Nov.

Harris, S.M., and F.C. McQuiston. 1988. A study to categorize walls and roofs on the basis of thermal response. *ASHRAE Transactions* 94(2): 688-715.

Huang, Y.J., J. Ritschard, S. Bull, I. Byrne, D. Turiel, C. Wilson, H. Sui, and D. Foley. 1987. Methodology and assumptions for evaluating heating and cooling energy requirements in new single-family residential buildings. Technical Support Document for the PEAR Microcomputer Program, Lawrence Berkeley National Laboratory Report No. LBL-19128, Berkeley, Calif.

ISO. 1994. *ISO 9869, International Standard. Thermal insulation—building elements—In situ measurement of thermal resistance and thermal transmittance*. Geneva: International Organization for Standardization.

Jedrzejuk, H. 1997. Private communication.

Kosny, J., and A.O.Desjarlais. 1994. Influence of architectural details on the overall thermal performance of residential wall systems. *Journal of Thermal Insulation and Building Envelopes* 18: 53-69.

Kosny, J., J.E. Christian, A.O. Desjarlais, E. Kossecka, L. Berrenberg. 1998. The performance check between whole building thermal performance criteria and exterior wall, Measured clear wall R-value, thermal bridging, thermal mass, and airtightness. *ASHRAE Transactions* 104 (2): 1379-1389.

Kossecka, E. 1992. Heat transfer through building wall elements of complex structure. *Arch. Civ. Engn.* 38 (1-2): 117-126.

Kossecka, E. 1993. Problem of the measured R-value error estimation. *Arch. Civ. Engn.* 39(4): 413-427.

Kossecka, E. 1996. The effect of structure on dynamic thermal characteristics of multilayer walls. *Arch. Civ. Engn.*, 42(3): 351-369.

Kossecka, E. 1998. Relationships between structure factors, response factors, and z-transfer function coefficients for multilayer walls. *ASHRAE Transactions* 104(1A), 68-77.

Kossecka, E. 1999. Correlations between structure dependent and dynamic thermal characteristics of building walls. *Journal of Thermal Env. and Bldg. Sci.*, Vol. 22.

Kossecka, E., J. Kosny, and J.E. Christian. 1994. A simple algorithm for determining the thermal resistance of a wall. *Build. Res. Journ.* 3(1): 57-66.

Kossecka, E., and J. Kosny. 1996. Relations between structural and dynamic thermal characteristics of building walls. *Proceedings of 1996 International Symposium of CIB W67, Energy and Mass Flows in the Life Cycle of Buildings, Vienna, 4-10 August*, pp. 627-632.

Kossecka, E., and J. Kosny. 1997. Equivalent wall as a dynamic model of the complex thermal structure. *J. of Thermal Insul. and Bldg. Envs.*, Vol. 20.

Kusuda, T. 1969. Thermal response factors for multi-layer structures of various heat conduction systems. *ASHRAE Transactions* 75(1): 241-271.

Sherman, M.H., and D.T. Grimsrud. 1980. Measurement of infiltration using fan pressurization and weather data. LBL-10852, Lawrence Berkeley National Laboratory, October.

Shklover, A.M. 1961. *Heat transfer at periodic thermal conditions* (in Russian). Moscow: Gosenizdat.