

## STOCHASTIC PREDICTION OF VENTILATION SYSTEM PERFORMANCE

F. Haghghat<sup>1</sup>, H. Brohus<sup>2</sup>, C. Frier<sup>2</sup> and P. Heiselberg<sup>2</sup>

<sup>1</sup>Department of Building, Civil and Environmental Engineering  
Concordia University, Montreal, Quebec H3G 1M8, Canada (haghi@cbs-engr.concordia.ca)

<sup>2</sup>Department of Building Technology and Structural Engineering  
Aalborg University, DK-9000 Aalborg, Denmark (hb@civil.auc.dk)

### ABSTRACT

Traditionally, prediction of ventilation systems performance has been based on deterministic approach, which implies that the spread of the input parameters values is zero. The deterministic approach is valid if the effect of fluctuations in the forcing functions (wind speed and direction, temperature, radiation, occupants' behavior, etc.) is negligible when compared to the mean value. When random fluctuations of the forcing functions are significant, the variance of response can no longer be neglected, as for example, for a hybrid-ventilated building, where a part of or the entire required ventilation load is supplied naturally. Most of the parameters influencing the performance of a hybrid-ventilated building are inherently stochastic in nature.

The paper briefly reviews the existing techniques for predicting the airflow rate due to the random nature of forcing functions, e.g. wind speed. The effort is to establish the relationship between the statistics of the output of a system and the statistics of the random input variables and parameters of a model.

### INTRODUCTION

Prediction of the air infiltration and ventilation of buildings is an essential element in the design of energy efficient buildings. Air infiltration in buildings due to wind-induced pressure is a complex process that is highly influenced by the turbulent nature of wind. The turbulent wind can be expressed as a stochastic process composed by a mean value of the wind velocity superimposed by a fluctuating component (Haghghat et al., 1988). In addition to the gustiness of the wind, the turbulence in wind-induced pressure is also a result of the interaction of the wind and the building envelope.

In most of the current multi-zone air flow models, air infiltration is calculated by means of a deterministic approach where the complex wind fluctuations are disregarded and the prediction of the infiltration rate of the building is based solely on average wind velocity (as well as stack and mechanical forces). However, several measurements and detailed analysis of air infiltration reveals a substantial influence of wind fluctuations, especially when the root mean square of the fluctuations is relatively large compared with the

average wind pressures. At the same time there is an increasing demand for proper infiltration models for the purpose of improved prediction of energy consumption, building safety (fire propagation, transport of smoke and other pollutants), indoor air quality and thermal comfort.

Several techniques have been proposed to calculate the resultant fluctuating infiltration. The wind fluctuation has two effects: 1) the pulsation flow and 2) the penetration eddies. The pulsation flow has a variation in pressure at the opening, that will induce a fluctuation flow due to the compressibility of the air in the internal space. Thus, a pulsation flow is produced through the opening which depends on the magnitude of the fluctuating pressure and the size of the enclosure. The penetration eddies have higher frequency fluctuations that enable air exchange at the opening due to smaller scale turbulence with eddy sizes comparable with, or smaller than the opening size. Thus, a turbulent diffusion of air through the opening is produced, and it depends less on the compressibility effects.

#### **Pulsation Model**

Cockroft & Robertson (1976) studied the single-opening pulsation flow of an enclosure with a single opening, subjected to a turbulent impinging air stream, and derived a simple theoretical pulsation model to assist in understanding the physical phenomena causing airflow through the opening. In this model, pulsation flow,  $q$ , of a single opening enclosure was caused by longitudinal fluctuations of wind, and the compressibility of air within the enclosure. This was related to the fluctuating pressure difference,  $\Delta p$ , between the external wind pressure and the internal room pressure in:

$$q = \pm C_D A \sqrt{\frac{2|\Delta p|}{\rho}} \quad (1)$$

where  $C_D$  is the discharge coefficient,  $\rho$  is the air density and  $A$  is the opening area. Assuming an adiabatic process, it was proposed that the fluctuating component of the pulsation flow into a simple opening enclosure can be obtained by:

$$q = \frac{dv}{dt} = \pm C_D A \left| u^2 - \left( \frac{2\gamma P_a}{\rho V} \right) v \right|^{1/2} \quad (2)$$

where  $\gamma$  is the polytropic exponent which is equal to 1.4 to 1.0, respectively for adiabatic and isothermal flows,  $P_a$  is the atmospheric pressure,  $V$  is the volume of the enclosure,  $u$  is the fluctuating component of the wind velocity, and  $v$  is the decrease in volume of the original air inside the enclosure. Cockroft & Robertson (1976) further argued that all the air flowing into the enclosure,  $Q_{in}$ , does not necessarily contribute to the effective ventilation of the enclosure. Some of the inflowing pulsations of air are exhausted before mixing with the interior air occurs. The net mean effective flow rate of ventilation air,  $Q_{eff}$ , into the enclosure is obtained by:

$$Q_{eff} = f_p Q_{in} = \frac{1}{2} f_p \overline{|q|} \quad (3)$$

The proportion of air mixed,  $f_p$ , depends on the frequency spectrum of the turbulent air stream. The  $f_p$  value is determined from results of their experiments as 0.37.

#### **Mechanical-System-Simulation Model**

Sasaki et al. (1987) considered the fluctuation of the airflow through the cracks to be influenced not only by the static resistance of the crack but also by the depth of the crack, the air volume behind the crack, and

the dynamic feature of the wind pressure. A building was modelled by a corresponding mechanical system consisting of mass, resistance, and elastic forces. The airflow is simulated assuming: 1) air in the crack moves as the mass, 2) space behind the crack works as the spring, and 3) dynamic resistance of airflow can be measured by statistical measurement. For an enclosure with a single opening, the corresponding mechanical system can be represented by:

$$M \frac{d^2x}{dt^2} + C \frac{dx}{dt} + Kx = \Delta p A \quad (4)$$

where  $x$  is the displacement of air in the crack,  $M$  is the mass of air in the crack,  $C$  is the airflow resistance of the crack, and  $K$  is a constant accounting for the compressibility of air.

By solving the above second order differential equation, the movements of the air mass displacement  $x$  can be obtained for a given frequency  $f$  of fluctuating wind velocity. The air infiltration through the crack at frequency  $f$  can be written as (assuming inside pressure is constant):

$$Q_{in} = A f (2x^* - l) \quad (5)$$

where  $x^*$  is the magnitude of the  $x$ , and  $l$  is the depth of the opening. Comparison of the theoretical solutions and the measurements resulting from a specially designed experiment shows that the differences were within 20%. However, the input needs for detailed temporal pressure profiles of all the exterior openings and the computation requirement for solving linear differential equations prevent the model be used in realistic applications.

#### ***Correlation Models***

Phaff & De Gidds (1980) proposed an empirical correlation which describes a general definition of the ventilation rate through an open window as a function of the temperature difference, wind velocity, and fluctuating terms.

Etheridge & Stanway (1988), and Etheridge & Alexander (1980) developed a multi-zone model for predicting air infiltration which takes into account the turbulence effect of fluctuating wind.

#### ***Numerical Simulation Model***

Kazic & Novak (1989) developed a dynamic model to simulate the airflow in multi-room buildings subjected to wind pulsation. Their model was based on the steady flow equation, and attempted to calculate the additional airflow due to low frequency variation in the pressure field around the building. Governing equations were derived from several equations, including continuity, energy, Bernoulli's, and the ideal gas equations. Input data must include pressures as functions of time on boundary surfaces of the building. Solutions are obtained by solving a set of non-linear, first-order differential equations in time domain.

#### ***Resonator Model***

Holmes (1980) studied the fluctuating internal pressure mainly for the purpose of wind loading design of building structure and cladding. The case of a single windward opening was modelled as a damped Helmholtz resonator:

$$\frac{\rho l V}{\gamma P_a} \frac{d^2 C_{p_i}}{dt^2} + \frac{\rho^2 V^2 U^2}{4 C_D^2 \gamma^2 A P_a^2} \frac{d C_{p_i}}{dt} \left| \frac{d C_{p_i}}{dt} \right| + A C_{p_i} = A C_{p_e} \quad (6)$$

where  $C_{p_i} = \bar{P}_i / (\frac{1}{2} \rho U_H^2)$  and  $C_{p_e} = \bar{P}_e / (\frac{1}{2} \rho U_H^2)$  are the time varying internal and external pressure coefficients, and  $U_H$  is the mean wind velocity at the roof height.

#### Power Spectrum Analysis Approach

A power spectrum analysis approach to model the pulsating air flows due to turbulent wind-induced pressures was developed and applied to single-opening and two-opening cases (Haghighat et al., 1991). The total air flow is divided into mean and fluctuating components. The equations that govern the fluctuating components were linearized in order to benefit from the spectral analysis methods. Haghighat et al. (1991) showed that for a given Fourier transform of the external wind-induced pressure  $p_w(\omega)$ , the resultant Fourier transform of the airflow through a single opening can be expressed by:

$$q(\omega) = \frac{j A \omega}{\frac{\gamma P_a A}{V} - \rho l \omega^2 + j k \omega} p_w(\omega) = H(\omega) p_w(\omega) \quad (7)$$

where  $j = (-1)^{0.5}$  is the imaginary unit,  $k$  is a fluctuating airflow parameter,  $\omega$  is the frequency, and  $H(\omega)$  is the transfer function. The airflow spectrum can be calculated for a given wind pressure spectrum,  $S_{p_w}(\omega)$ :

$$S_q(\omega) = \|H(\omega)\|^2 S_{p_w}(\omega) \quad (8)$$

The root-mean-square value (RMS) of the fluctuating infiltration can be calculated by integrations of the spectra over the frequency (see Haghighat et al., 1999). The wind pressure spectrum, as input, can be obtained either experimentally or by means of empirical formulae. The internal pressure of this single-opening enclosure, in frequency domain can be expressed as:

$$S_{p_i}(\omega) = \left\| \frac{\gamma P_a}{j \omega V} \right\|^2 S_q(\omega) \quad (9)$$

The RMS value of internal pressure,  $p_i$ , can be calculated in similar fashion, as for the infiltration rate, by integration of the spectrum function.

The results of case studies showed that, for a single-opening case, the turbulence in the airflow rate was concentrated in the higher frequency range (around 0.1 Hz), while for the two-opening case, the fluctuating airflow rates were mainly caused by the instantaneous pressure differences, and the frequency ranges of their turbulence were lower (around the same frequency range of the wind pressure, at about 0.008 Hz) (Haghighat et al., 1991).

The predicted RMS values of fluctuating airflow rates are theoretical, and a conversion is required to obtain the effects on the total air exchange rates. Three factors influence the conversion: the "residue" (or "hidden") air in the openings, the mixing process inside rooms, and the possibility of flow reversal. In the single-opening case, only the first two factors have effects. For the two-openings case, the third factor has a major influence while the other two factors have only negligible effects (Haghighat et al., 1992). Further theoretical and experimental efforts are required to better understand and qualify these phenomena.

## SUMMARY AND CONCLUSION

The importance of considering the fluctuating components of the wind velocity of air infiltration is discussed. Main features of turbulent wind are described together with the physics of fluctuation, especially the two main effects of pulsation flow and penetrating eddies. The different approaches to fluctuating infiltration modelling are reviewed, a summary is found in Table 1. Important main topics for further research in fluctuating infiltration modelling are identified in the following.

TABLE 1  
APPROACHES TO MODELLING

Model	Features
Pulsation (Cockroft & Robertson, 1976)	Longitudinal fluctuations of wind and compressibility of air within enclosure cause pulsation flow of single opening enclosure.
Mechanical-System-Simulation (Sasaki et al., 1987)	Airflow through openings is modelled as a mechanical system. Infiltration can be calculated for pressures of single frequency.
Correlation (Phaff & De Gidds, 1980)  (Etheridge & Alexander, 1986)	Empirical correlation expressing mean airflow rate through an open window as a function of temperature difference, wind velocity, and fluctuating terms. Fluctuating infiltration assumed proportional to RMS value of wind pressure. The coefficients depend on opening characteristics and empirical constants.
Numerical Simulation (Kazic & Novak, 1989)	Governing equations derived from fundamentals are solved numerically.
Resonator (Holmes, 1980)	Fluctuating internal pressures are obtained numerically from second-order non-linear differential equation.
Power Spectrum Analysis (Haghighat et al., 1991)	Power spectrum analysis is used to model pulsating airflow due to turbulent wind-induced pressures. Applied for a single-opening case and a two-opening case.

Some of the models are ready for use in practical engineering design, for instance the correlation models, provided that their range of applications are properly acknowledged. Other models require greater skills among users in terms of understanding and computing. Future incorporation of the models in commercial software for building simulation or as stand alone software demands a higher degree of generalization, i.e. an extension to  $n$  openings and  $m$  enclosures inside an arbitrary oriented building, etc. Simple guidelines for the use of fluctuating models would be highly beneficial, like rules of thumb stating when it is appropriate to use the simple deterministic average wind velocity approach, etc.

An important area is to extend the models to include turbulence in fluctuating pressure differences caused by thermal buoyancy and unsteady operation of mechanical ventilation. In this respect a significant question is whether it is possible to decouple the effect of thermal loads in shape of solar radiation and internal gains from the fluctuations caused by wind, or if there will be substantial mutual interactions that should be acknowledged.

The main purpose of the different techniques reviewed in this paper is to predict the airflow rate. However, as pointed out by Cockroft and Robertson (1976), some of the incoming air is exhausted before mixing with the room air. They found that less than 40% of the airflow rate contributed to the "effective ventilation". This fact raises a crucial need for a kind of ventilation effectiveness that determines the ratio of infiltration air actually contributing to the air change rate and the total air flow rate, where part of the air "short-circuits".

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## REFERENCES

- Cockroft and Robertson (1976). Ventilation of an Enclosure through a Single Opening. *Building and Environment* **11**, 29-35.
- Etheridge D. W. and Alexander D. K. (1980). The British Gas Multi-Cell Model for Calculating Ventilation. *ASHRAE Trans.* **86:2**, 808-821.
- Etheridge D.W. and Stanway R. J. (1988). A Parametric Study of Ventilation as a Basis for Design. *Building and Environment* **23:2**, 81-93.
- Haghighat F., Brohus, H. and Rao, J. (1999). Modelling Air Infiltration due to Wind Fluctuations – A Review. *Building and Environment* (accepted for publication).
- Haghighat F., Fazio P. and Unny T. E. (1988). A Predictive Stochastic Model for Indoor Air Quality. *Building and Environment* **23:3**, 195-201.
- Haghighat F., Rao J. and Riberon J. (1992). Modelling Fluctuating Airflow through Large opening. *Proc. of the 13th AIVC Conference*, pp. 77-85, Nice, France.
- Haghighat F., Rao J. and Fazio P. (1991). The Influence of Turbulent Wind on Air Change Rates - a Modelling Approach. *Building and Environment* **26:2**, 95-109.
- Hill J. F. and Kusuda T. (1975). Dynamic Characteristics of Air Infiltration. *ASHRAE Trans.* **81:1**, 168-185.
- Holmes J. D. (1980). Means and Fluctuating Internal Pressure Induced by Wind. In: Cermak J.E., Editor. *Wind Engineering, Proc. of the 5th Intl. Conf.*, Fort Collins. USA.
- Kazic N. and Novak P. (1989). A Dynamic Model of Air Movement in a Building, *Proc. of CLIMA 2000, The 2nd World Congress on Heating, Ventilating, Refrigerating and Air Conditioning*, ISBN 86-81505-01-7, Vol. 1, pp. 250-255, Sarajevo, Yugoslavia.
- Phaff H. and De Gidds W. (1980). *Building Ventilation, Investigation of the Consequence of Opening one Window on the Internal Climate of a Room*, IMG-TNO Report C448, Delft, Netherlands.
- Sasaki T., Hayashi M. and Aratani N. (1987). On the Ventilating Characteristics of the Space under the Fluctuating Wind Pressure. *Proc. of ROOMVENT '87, Air Distribution in Ventilated Spaces*, Session 4a.