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NUMERICAL SIMULATION OF AIR FLOWS - APPLICATION TO THE VENTILATION OF A PAINT-BOOTH.

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ABSTRACT

This paper presents a numerical study of instationary three-dimensional flows. Three methods, a semi-implicit one and two explicit ones were compared and tested on typical flow configurations (lid driven cavity, natural convection and mixed convection in a cavity). These methods were then applied to a problem of ventilation in a paint-booth. The semi-implicit method proved to have a higher accuracy. The explicit method of the M.A.C. type turned out to be more advantageous in calculation time.

INTRODUCTION

When designing a general ventilation system, it is necessary to take into account all the transfer mecanisms of pollutants in the workshop. In most cases, these mecanisms are connected with the convection and diffusion properties of the air flow. However, these properties are generally very difficult to appreciate because they depend very strongly on the local conditions of flow.

Therefore, it seemed interesting to us to develop numerical codes enabling us to predict the velocity fields and later, pollutant concentrations at any point in the workshop. The work presented in this paper is the first step in the development of these codes.

MATHEMATICAL FORMULATION OF THE PROBLEM AND METHODS OF RESOLUTION

Air flow configurations in ventilation problems, are essentially three-dimensional. Considering the air in the workshop as an ideal incompressible gas, the air flow is governed by the set of "Navier-Stokes equations" :

$$\frac{\partial Vi}{\partial t} + Vj.Vi,j = -\frac{1}{\rho}P,i + \frac{1}{Re}\,Ui,jj \qquad i = 1, 2, 3$$

$$Vi.i = 0$$

Many authors have proposed numerical methods for the resolution of these equations in the case of bidimensional flows. In particular, the K-ε method used by Nielsen [7] for the study of the ventilation in a room heated evenly by under-floor heating, must be mentioned. This method consists in solving, besides Navier-Stokes's equations, two extra transport equations, the first one for the turbulent kinetic energy K and the second one for the dissipation rate ε. On the other hand, there are only very few references about three- dimensional flows, especially in confined spaces. Therefore, we were led to test, in configurations of the type of those existing in ventilation problems, various numerical methods extrapolated from those developed in bidimensional flows.

Three methods, using finite differences discretization, were compared :

- the SOLA explicit method [1] of the M.A.C. type (Marker and Cell), adapted for three-dimensional flows by Gaillard [2] ;
- the explicit method of artificial compressibility [3] ;
- the semi-implicit method of artificial compressibility [4].

These three methods differ on the one hand as to the limitation of the time step Δt and on the other hand, as to the processing of the pressure gradient.

The discretization of the Navier-Stokes equations is obtained by a finite difference method described in reference [6], and connected to staggered mesh system of the M.A.C. type (Fig. 1).

Summary of the "SOLA" explicit method

The finite difference scheme is written in the following way :

$$V^{n+1} = \Gamma^n - \Delta t \text{ grad } p^{n+1} \qquad \text{with } = \Gamma^n = V^n + \text{terms of transport} \tag{1}$$

$$\text{div } V^{n+1} = 0 \tag{2}$$

Equation (2) is carried out iteratively in each cell by adjusting pressures and velocities with the quantities ∂p^υet ∂v^υ.

υ : iterative index.

$$\partial p^\upsilon = - \omega \cdot \frac{(\text{div } V^{n+1})^\upsilon}{2 \Delta t(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} + \frac{1}{\Delta z^2})}$$

$1 < \omega < 2$ $\qquad$ ω : coefficient of overrelaxation

$$\partial v^\upsilon = \Delta t \frac{\partial p^\upsilon}{\Delta x_i}$$

The numerical stability of the scheme necessary for convergence was studied by Viecelli [8] and enabled the restriction in the choice of time increment to be known :

$$\Delta t \leqslant \frac{1}{2\upsilon} \frac{1}{\left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} + \frac{1}{\Delta z^2}\right)}$$

$$\Delta t \leqslant \min \left(\frac{\Delta x}{U}, \frac{\Delta y}{V}, \frac{\Delta z}{W}\right)$$

Method of artificial compressibility

Let us introduce the equation for artificial compressibility :

$$\frac{\partial P}{\partial \tau} = - C.\mathrm{div}\ V^{\upsilon}$$

This equation is linked to the Navier-Stokes equations for a compressible fluids. Their state law is written :

$P = a^2\rho$ $\quad a^2 = \mathrm{cte}$

The method of artificial compressibility consists in replacing the equation of continuity div $V = 0$ by the equation $\frac{\partial P}{\partial \tau} = - C.\mathrm{div}\ V^{\upsilon}$.

at the asymptotic state, div $V \to 0$.

The explicit scheme is written in the following symbolic form :

$$M_U\ (U_h^{n+1},\ U_h^n,\ V_h^n,\ W_h^n,\ P_h^n) = 0$$

$$M_V\ (U_h^n,\ V_h^{n+1},\ V_h^n,\ W_h^n,\ P_h^n) = 0$$

$$M_W\ (U_h^n,\ V_h^n,\ W_h^{n+1},\ W_h^n,\ P_h^n) = 0$$

$$P^{n+1} - P^n = -C\ \Delta\tau D\ (U_h^{n+1},\ V_h^{n+1},\ W_h^{n+1})$$

The test of convergence of the scheme is the following :

$$\underset{i.j.k.}{\mathrm{Max}} \left| \xi^{n+1} - \xi^n \right| < 10^{-6}$$

ξ : U, V, W, P

Numerical stability of the explicit scheme

There is no general theory allowing the stability study of resolution methods of linear equations. We studied the tangential system stability using von Neumann's method. The stability conditions chosen were the most restrictive conditions in the field.

It can be considered that a disturbance at the initial moment will remain limited during the iterations.

The details of calculations are to be found in réf.[6]. We obtain the following restrictions on the time step Δt :

$$\frac{\Delta t}{Reh^2} \leqslant \frac{1}{6}$$

$$(U + V + W) \frac{\Delta t}{h} \leqslant 1$$

$$C > 0$$

$$\frac{C\Delta t^2}{h^2} + \frac{2\Delta t}{Reh^2} \leqslant \frac{1}{3}$$

Let us note that these conditions are more restrictive than those obtained by Viecelli [8].

Semi-implicit method of artificial compressibility

The theoretical advantage of this scheme is that it does not have restrictions on the time step unlike the explicit scheme, which could result in a considerable decrease in the calculation time.

The numerical resolution of this scheme is made using an iterative procedure of the Gauss-Seidel type [6].

The system of equations to be solved is written :

$$U_h^{n+1,\upsilon+1} = U_h^{n+1,\upsilon} - K\, L_U\,(U_h^{n+1,\upsilon},\ V_h^{n+1,\upsilon},\ W_h^{n+1,\upsilon},\ P_h^{n+1,\upsilon})$$

$$V_h^{n+1,\upsilon+1} = V_h^{n+1,\upsilon} - K\, L_V(U_h^{n+1,\upsilon+1},\ V_h^{n+1,\upsilon},\ W_h^{n+1,\upsilon},\ P_h^{n+1,\upsilon})$$

$$W_h^{n+1,\upsilon+1} = W_h^{n+1,\upsilon} - K\, L_W(U_h^{n+1,\upsilon+1},\ V_h^{n+1,\upsilon+1},\ W_h^{n+1,\upsilon},\ P_h^{n+1,\upsilon})$$

$$P_h^{n+1,\upsilon+1} = P_h^{n+1,\upsilon} - C\Delta\tau\ D(U_h^{n+1,\upsilon+1},\ V_h^{n+1,\upsilon+1},\ W_h^{n+1,\upsilon+1})$$

τ is the fictitious time associated with the iteration υ.
K and C are constants determined from the stability analysis and optimized so that the quickest convergence can be provided.

The test of convergence of this procedure is the following :

$$\underset{U,V,W}{\text{Max}} \quad \underset{i,j,h}{\text{Max}} \quad \left[\left| L_U \right|, \left| L_V \right|, \left| L_W \right|, \left| D \right| \right] \leqslant \varepsilon$$

Numerical stability of the semi-implicit scheme

Calculations were made the same way as for the explicit scheme. We obtained the following inequalities :

$$0 < K < \frac{Reh^2}{3} \qquad C > 0$$

$$K (U + V + W)^2 \leqslant \frac{12 \Delta t^2}{Reh^2}$$

$$\frac{3K}{h^2} (C\Delta\tau + \frac{1}{Re} + \frac{h^2}{6\Delta t}) \leqslant 1$$

These inequalities show that, whatever the choice of time increment Δt, there are always one K and one C which comply with the stability conditions.

VALIDATION OF CALCULATION CODES ON A TEST CASE

The three schemes were used to determine the field of velocities in a lid driven cavity (Fig. 2), in order to test the accuracy and the stability of the numerical methods and their ability to simulate the flows at high Reynolds's number (Re) or with recirculation zones. Three-dimensional calculations were performed at the Reynolds numbers 10, 100 and 1000.

The comparison of the three methods showed that the SOLA explicit scheme was performing in calculation time (C.P.U.). The gain on time step obtained for the semi-implicit scheme (ten times larger) is not enough to compensate for the necessity of iterating the velocities at each time step. On the other hand, this scheme has a greater accuracy. In the explicit scheme of the artificial compressibility method as great a number of interations is needed.

When applying symmetrical boundary conditions on sides P_1 and P_2, the flow is then bidimensional, which enabled us to compare our results with those in literature.

The flow configuration in the median face for a number of Reynolds Re = 100 is given in Figure 3. The downstream vortex can be seen. The two downstream vortices in the two lower corners cannot be seen in this figure, but they could be detected from velocity fields.

The results obtained (position of the centre of the main vortex Xo, positions and sizes of the right (X_1, Z_1) and left (X_2, Z_2) downstreams vortex) are compared to those published by other authors (see Table 1).

TABLE 1
Coordinates of the centres of vortices and sizes of secondary vortices in a Lid driven cavity at Re = 100.

Authors	Mesh	Xo Zo	X_1 Z_1	X_2 Z_2	L_1	L_2
This study	22x12x22	0.65 0.75	0.02 0.03	0.95 0.05	(0.10 ; 0.11)	(0.05 ; 0.05)
Tuann and Olson	8x8	0.62 0.74	0.05 0.07	0.97 0.04	(0.13 ; 0.16)	(0.07 ; 0.07)
Goda (3D)	20x20x20	0.62 0.75			(0.10 ; 0.13)	(0.11 ; 0.13)
Schreiber and Keller	121x121	0.61 0.74				

APPLICATION TO THE VENTILATION OF A PAINT-BOOTH

The SOLA scheme and the semi-implicit scheme were tested on a real flow configuration. They were applied to the ventilation of a paint-booth of the "car-body work type". Air was introduced into the booth at steady speed (W = 1) through a ceiling filtering dust (plenum). It was discharged through the floor by a draining systems made of two parallel pits (Fig. 4).

The flow is assumed to be laminar and bidimensional. The bidimensional nature of the flow was checked experimentally by another research team from the INRS [5]. The booth was symetrical, which enabled us to restrict calculations by a half.

Numerical results

The flows studied are those corresponding to Re = 100, Re = 1000 and Re = 5000. Calculations were made using the SOLA explicit code and the semi-implicit code. The mesh system is composed of 22 x 6 x 22 nodes, and the time increment is Δt = 0.01. The initial condition of the flow at Re = 100 corresponds to the undisturbed uniform flow, while those of flows at Re = 1000 and Re = 5000 are the steady states obtained at Re = 100 and Re = 1000 respectively. Convergence was very slow, especially for the semi-implicit scheme ; this was due, on the one hand to the discontinuity of the boundary conditions and on the other hand, to the slowness of the Gauss-Seidel iterative procedure. Indeed, when the velocity values calculated using the SOLA code are taken as boundary conditions at the outlet, the convergence of the semi-implicit method is obtained only after 100 iterations.

The three flows studied are differentiated by the length of their recirculation zones (Fig. 5, 6). When Re = 5000, we brought to light the fact that there was actually a zone of dead flow all along the wall (Fig. 7).

The coordinates (X_1 , Z_1) and (X_2 , Z_2) of detachment and reattachment points, and other calculation results made with the AMDHAL V7 are grouped tagether in Table 2.

TABLE 2

Paint-booth - Calculation results.

Re	Number of itera-tions/Δt	CPU/Δt	Number of cycles	Total C.P.U.	Length of the recirculation zones (m)	
					(X_1 ; Z_1)	(X_2 ; Z_2)
100	823	14.5	371	105 mn	(0.92 ; 1.36)	(0.92 ; 0.42)
1000	822	14.5	470	31 mn 05	(0.77 ; 2.10)	(0.77 ; 0.75)
5000	819	14.5	586		(0.77 ; 2.81)	(0.77 ; 0.15)

DEVELOPMENT

Let us now study the case in which air carries a pollutant. We introduce the following assumption : the physical properties of air are supposed to be constant, except the density for which Boussinesq's approximation is applied. In the case where there is only one pollutant, the equation systems to be solved is the following :

$$\frac{\partial Vi}{\partial t} + Vj.Vi,j = - \frac{1}{\rho} P,i + \rho_0 \beta g(T - T_0) \delta i_3$$

$$Vi.i = 0$$

$$\frac{DT}{Dt} = \frac{\lambda}{\rho Cp} T,ii$$

The theoretical analysis of the stability of this systems was made for both schemes of the artificial compressibility method and enabled us to specify the new fields of stability. The additional inequalities which must be respectively met are the following :

- Explicit scheme :

$$\frac{\Delta t}{Re\ Prh^2} \leqslant \frac{1}{6}$$

- Semi-implicit scheme :

$$K_T > 0 \; ; \; \frac{K^T}{h^2} \left(\frac{h^2}{2\,\Delta t} + \frac{3}{Re.Pr} \right) \leqslant 1 \text{ et } (\; U \; + \; V \; + \; W \;\; {}^{2} \leqslant \frac{3}{Re.Pr}$$

Application to natural convection in a cavity

The results obtained, as much for the field velocity as for the pollution field (position of the main vortex centre, position of the downstream vortices, isopollution curves and coefficient of heat transfer at the wall) are in total accordance with those noted in literature (see reference [6], figure 6 and table 3).

TABLE 3
Natural convection in a cavity - Results of calculations.

Rayleigh's number	Schemes	Number of itera tions/Δt	CPU/Δt (s)	total CPU Amdhal V7	$\overline{Nu}$		
					This study	De Vahl Davis	Poots
Ra = 100	SOLA Semi-implicit	64 62	0.68 4.46	1 mn 2 mn 45	1.015 1.0045		
Ra =1000	SOLA Semi-implicit	56 98	0.64 5.85	3 mn 25 4 mn 01	1.10 1.19	1.117	1.041
Ra = 10^4	SOLA Semi-implicit	64 62	0.82 10.63	4 mn 15 5 mn 19	2.285 2.30	2.24	1.706

Application to a mixed convection in a lid driven cavity

The purpose of this calculation is to show the change in the flow configuration due to buoyancy forces (Figure 7).

These programs must now be applied to the paint-booth.

CONCLUSION AND FURTHER DEVELOPMENTS

The three calculation codes developed proved to be appropriate for the resolution of some problems encountered within the framework of general ventilation of rooms. The SOLA code turned out to be superior since its computational time is smaller. They have already provides some most interesting information. However, the development of these codes must be continued, on the one hand in order to overcome some numerical difficulties and on the other hand, to introduce the mechanism of turbulence.

NOMENCLATURE

Vi (U,V,W)	: velocity composants
p	: pressure
T	: température
ρ	: air density
t	: time
Δt	: time step
x, y, z	: coordinates
h ou Δx, Δy, Δz	: space step 5
div	: divergence
indices	
i	: 1, 2, 3
h	: index of space discretisation
n	: index of time discretisation
υ	: indice of iteration
Cp	: heat capacity at constant pressure (per unit mass)
β	: coefficient of volume expansion with temperature
g	: acceleration associated with gravity
λ	: thermal conductivity
Re : $\frac{VL}{\upsilon}$	: Reynold's number
Pr : Cp_μ/λ	: Prandtl number
Ra = $\frac{g\ \beta L^3 O_\Delta t}{\alpha\upsilon}$	: Rayleigh number
Nu	: local Nusselt number
$\overline{Nu}$	: mean Nusselt number

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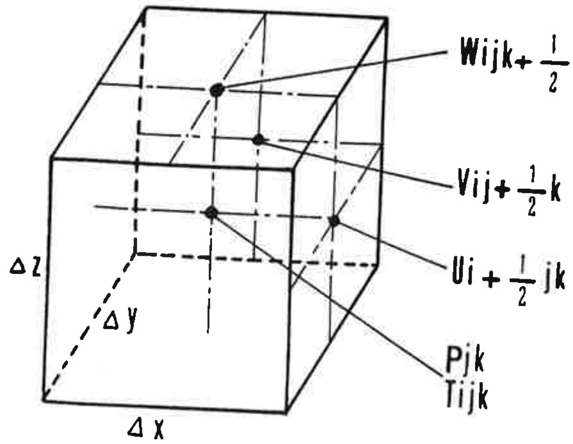

Fig. 1. Mesh i, j, k.

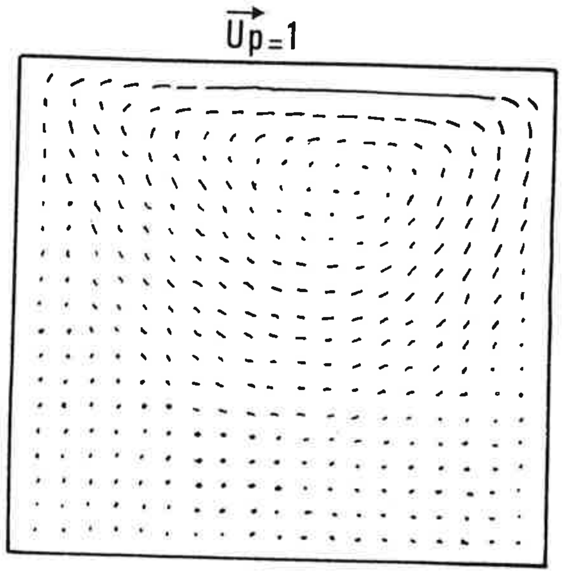

Fig. 2. Lid driven cavity. Velocity field.

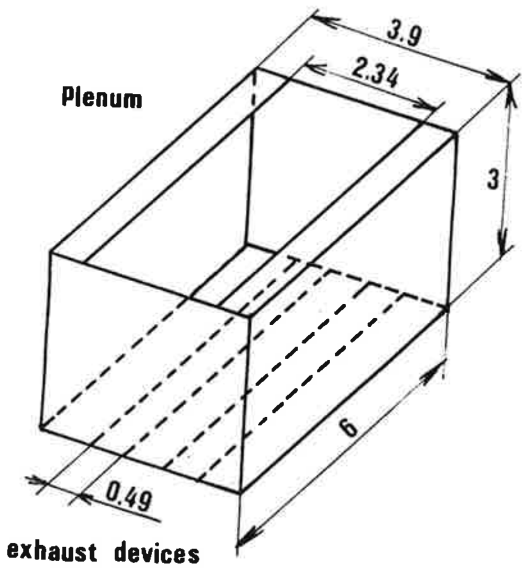

Fig. 3. Paint-booth.

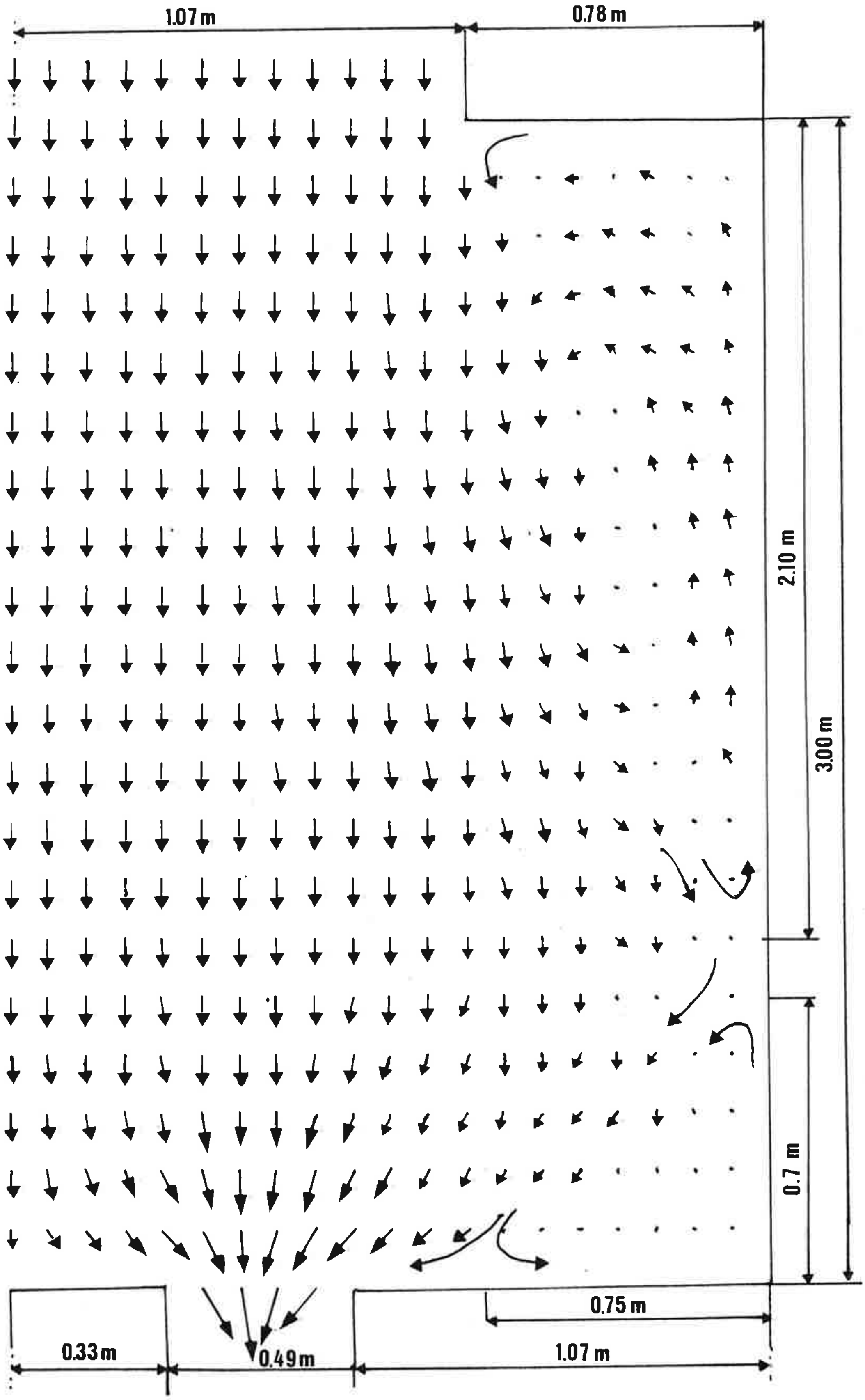

Fig. 4. Flow configuration at Re = 1000.

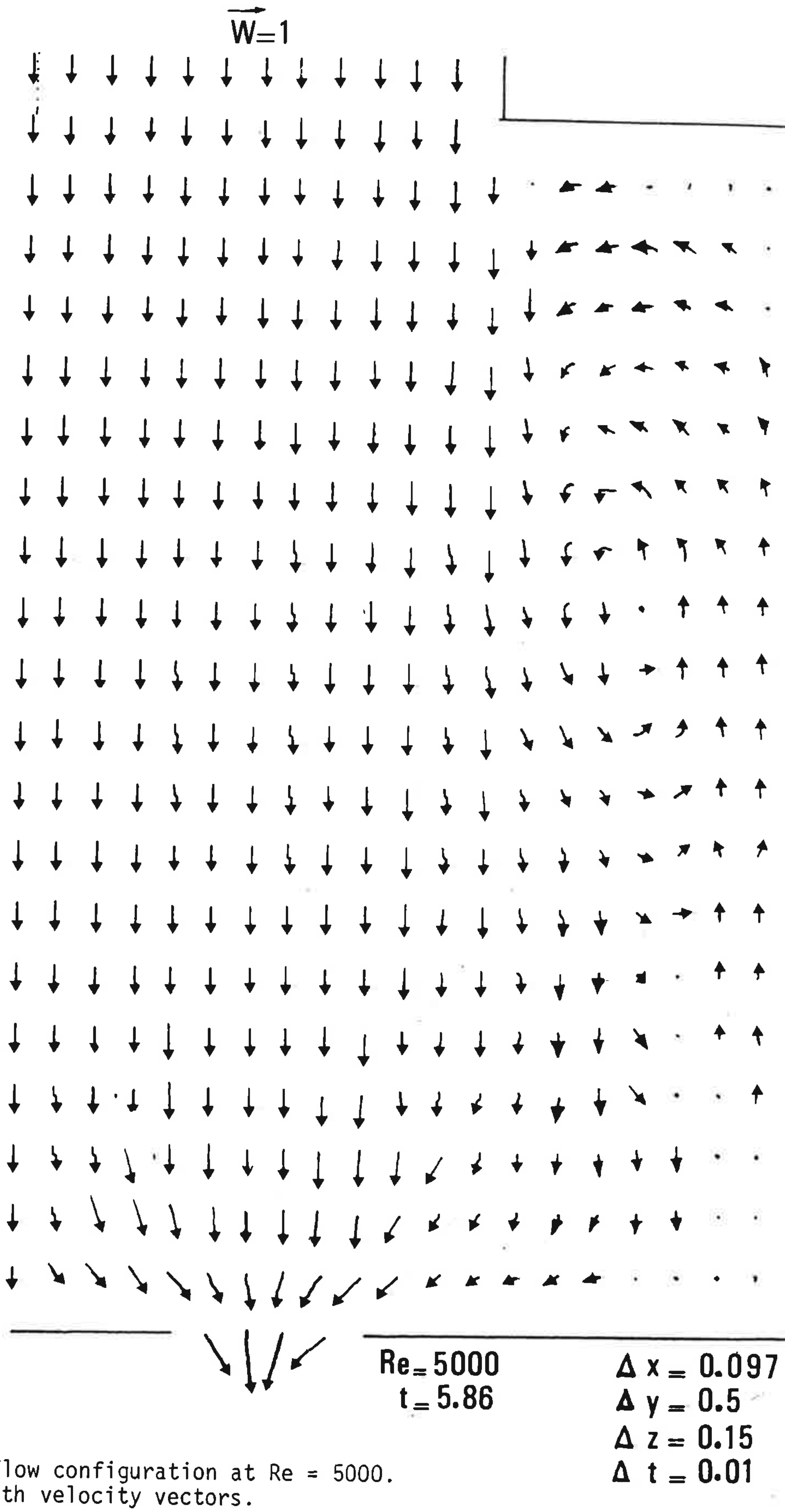

Fig. 5. Flow configuration at Re = 5000.
Paint-booth velocity vectors.

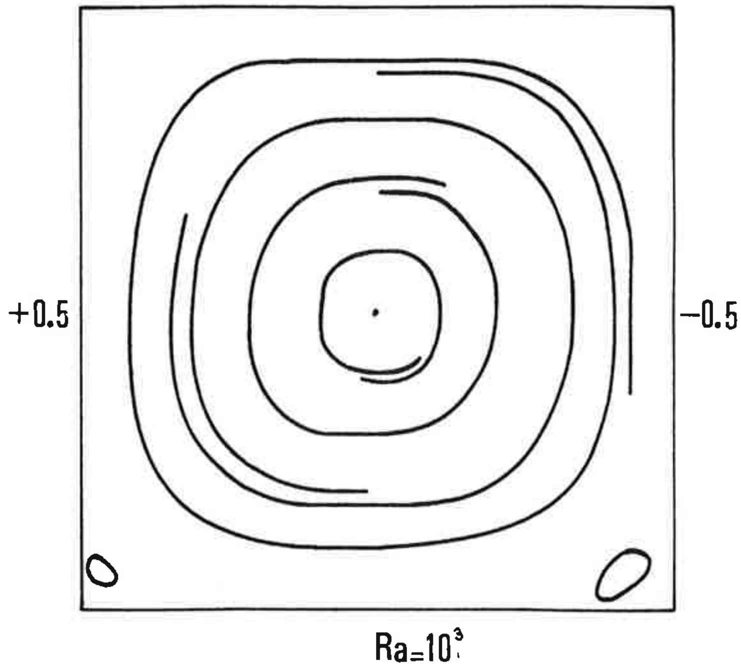

Fig. 6. Natural convection in a cavity.

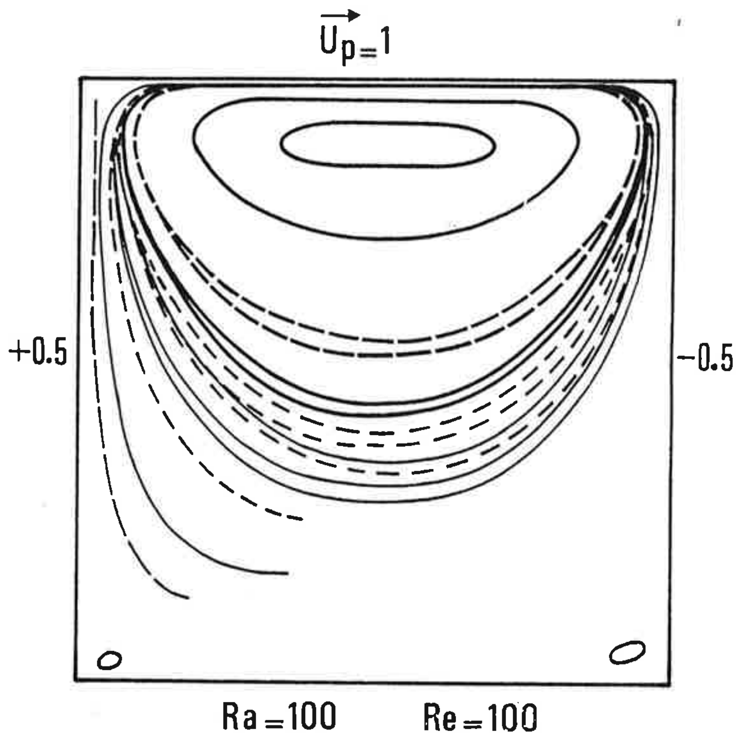

Fig. 7. Lid-driven cavity heated.