PREDICTING A TIME-VARYING FLOW RATE USING THE CONSTANT CONCENTRATION AND DECAY TECHNIQUE

Mats Sandberg

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ABSTRACT

This study deals with the accuracy of different tracer gas techniques for predicting the mean flow rate of a time-varying airflow rate, as occurs in naturally ventilated houses. A theoretical analysis of the accuracy is first presented. Experiments were conducted in a test house ventilated by natural ventilation. The methods explored were the constant concentration method and the decay method. The airflow rate in the duct connected to the house was continously recorded by the constant tracer gas flow technique. The oscillations in flow rates that occurred were of a high frequency nature, which should not affect the accuracy. Incomplete mixing of both tracer gas and air seems to be the greatest source of error, even in cases with a time-varying ventilation airflow rate.

INTRODUCTION

The infiltration rate in a house can be measured by different experimental procedures based on the tracer gas technique. Methods in use include the decay method and the constant concentration and constant flow techniques. For situations with constant ventilation airflow rates, the accuracy of the decay method and the constant concentration technique has been determinated in multicell applications (Sandberg and Blomqvist 1985). At a constant ventilation airflow rate, nonuniform mixing is the dominant factor that sets the limit for the accuracy of the decay method. Other factors, such as the type of tracer gas and detector, seem to be of less importance (Shaw 1984). However, the characteristic of infiltration is that the flow rate is time-dependent. The aim of this paper is to discuss methods for estimating the mean flow rate and its accuracy. The paper is divided into two parts: The first part presents a theoretical estimate of the influence of a variable flow rate on the accuracy of the decay technique and the second part presents experimental results.

Mats Sandberg, Head of Heating and Ventilation Laboratory, National Swedish Institute of Building Research, Gävle, Sweden.

DECAY OF CONCENTRATION WHEN THE FLOW RATE IS TIME DEPENDENT

To begin with mixing problems, consider a single cell with complete and instantaneous mixing. We assume that the flow rate varies as a simple harmonic around a stationary mean value:

$$q = \bar{q}\left(1 + \frac{a}{\bar{q}} \sin(2\pi ft)\right) \tag{1}$$

where

a	=	The amplitude	(m^3/h)
f	=	The frequency	$(1/h))$
q	=	The mean flow rate	(m^3/h)
t	=	Time	(h)

If we start from a situation with a uniform initial concentration in the cell at time zero, then the decay of concentration follows the following expression:

$$C(t) = C(0)\, e^{-\frac{t}{\bar{\tau}_n}}\, e^{-\left(\frac{1}{2\pi f \bar{\tau}_n} \frac{a}{\bar{q}} \sin(2\pi ft + \pi)\right)} \tag{2}$$

where

$C(t)$	=	Concentration at time t	(ppm)
$C(0)$	=	The initial concentration	(ppm)
$\bar{\tau}_n$	=	The mean nominal time constant based on the mean flow rate ($\bar{\tau}_n = V/q$)	(h)
V	=	The volume of the cell	(m^3)

Equation 2 shows that the concentration response to a time dependent flow rate exhibits a damped oscillation with a phase shift equal to π. The concentration decay produces the damping, and the phase shift is due to the fact that an increase in flow rate will cause a diminishing concentration, and vice versa. Figure 1 shows the calculated concentration response to oscillating flow rates of different frequencies. The ratio between the amplitude and the mean flow rate is equal to one in each case. This ratio is presumably greater than what one would expect to occur in practice and has only been chosen to enhance the presentation in the figure. The system is linear, and the phase shift is not dependent on the frequency. Therefore, it is theoretically possible to retrieve information about the frequency of the oscillations from a record of concentration versus time. However, in most cases, apart from pure research interest, the frequency of the oscillations is not of vital importance. The mean flow rate is the most important practical quantity. Therefore, we will only deal with methods for determinig the mean flow rate.

METHODS FOR ESTIMATING MEAN FLOW RATE AND THEIR ACCURACY

In this section we assume that the flow rate oscillates with a fixed frequency. In general, to be able to estimate the mean flow rate, the minimum measuring time must not be less than the period of the oscillation. Furthermore, experience shows that the measuring time should not be less than the nominal time constant. Therefore, we assume in this section that the measuring time is not less than the nominal time constant. This is a condition that cannot always be met in practice, since we do not know a priori the magnitude of the mean flow rate.

Concentrations Recorded Only Twice

To begin with, we will consider the by far most common measuring strategy, that is to say, when the concentration is measured only twice. The ratio between the concentrations at time t_1 and t_2 respectively is given by:

$$\frac{C(t_1)}{C(t_2)} = e^{-\frac{(t_1 - t_2)}{\bar{\tau}_n}} \, e^{-\left(\frac{a}{\bar{q}} \frac{1}{(2\pi f \bar{\tau}_n)} (\sin(2\pi f t_1 + \pi) - \sin(2\pi f t_2 + \pi)\right)} \tag{3}$$

where

t_1 = Point of time for the first recorded concentration

t_2 = Point of time for the second recorded concentration

We will assume that the measuring time, t_m, is not less than the mean nominal time constant, that is

$$t_m = (t_2 - t_1) \geq \bar{\tau}_n \tag{4}$$

The first exponential in Equation 3 is the one we should obtain with a constant flow rate equal to the mean flow rate. The second term is the modification caused by the oscillating flow rate. We want to estimate the maximum error. This maximum error occurs when the difference between the arguments in the two sine functions is equal to 2, and the magnitude of the sine functions equal to ± 1. The maximal influence from the oscillating flow on the ratio between the concentrations is obtained for the following frequencies:

$$f_i = \frac{i}{2\bar{\tau}_n} \qquad i = 2, 3, \ldots. \tag{5}$$

or, as expressed in terms of periods T_i

$$T_i = \frac{2\bar{\tau}_n}{i} \qquad i = 2, 3, \ldots. \tag{6}$$

We exclude i = 1 in Equation 5 and 6 because we presuppose that we only have oscillations with a time period less or equal to the mean nominal time constant. The ratio between concentrations with the maximal influence of the oscillations is equal to:

$$\frac{C(t_1)}{C(t_2)} = e^{\frac{t_m}{\bar{\tau}_n}} e^{\pm \frac{a}{\bar{q}} \frac{2}{i\pi}} \qquad (7)$$

The flow rate is estimated from Equation 7 by using the usual two-point estimate:

$$q^e = \frac{V}{t_m} \ln \frac{C(t_1)}{C(t_2)} \qquad (8a)$$

where

q^e = The calculated (estimated) flow rate (m^3/h)

V = The volume of the cell (m^3)

The estimated 8a is correct when the flow rate is constant (time-independent)

By using Equation 7 the relation 8a becomes:

$$q^e = \frac{V}{t_m} \left(\frac{t_m}{\bar{\tau}_n} \pm \frac{a}{\bar{q}} \frac{2}{i\pi}\right) \qquad (8b)$$

and after having adopted the definition of the mean nominal time-constant ($\bar{\tau}_n = V/\bar{q}$), we finally obtain the following relationship between the estimated flow rate, q^e, and the correct mean flow rate, $\bar{q}$:

$$q^e - \bar{q} = \pm \frac{V}{t_m} \left(\frac{a}{\bar{q}} \frac{2}{i\pi}\right) \qquad (9)$$

From expression Equation 9 we can draw the following conclusions:

- If the measuring time goes to infinity, then the difference goes to zero.
- If the frequency of the oscillations goes to infinity ($i \Rightarrow \infty$), then the difference goes to zero.
- If the amplitude goes to zero, then the difference goes to zero.

All of the above three conclusions are intuitively obvious.

The greatest error in Equation 9 we obtain for the shortest measuring time, and, therefore, to obtain an upper limit for the error, we put the measuring time equal to the mean nominal time constant. We obtain

$$\frac{q^e - \bar{q}}{\bar{q}} = \frac{a}{\bar{q}} \frac{2}{i\pi} \qquad (10)$$

As an upper estimate of the ratio between the amplitude and the mean flow ratio, we set it equal to 1, and we finally obtain the following expresssion for the relative error:

$$\frac{q^e - \bar{q}}{\bar{q}} \leq \frac{2}{i\pi} \qquad (11a)$$

Expressed in terms of the frequencies, 11a becomes:

$$\frac{q^e - \bar{q}}{\bar{q}} \leq \frac{1}{f_i \cdot \bar{\tau}_n \cdot \pi} \qquad (11b)$$

We know that the errors due to incomplete mixing in multicell applications amount to 12-18%. If we therefore stipulate that the error due to a oscillating airflow shall be less, let us say 10%, then we obtain the following restrictions on the frequencies.

$$f_i \geq \frac{10}{\pi \bar{\tau}_n} \approx \frac{3}{\bar{\tau}_n} \qquad (12)$$

In terms of periods Equation 12 becomes:

$$T_i \leq \frac{\bar{\tau}_n}{3} \qquad (13)$$

To summarize: under the assumption that the measuring time is equal to the mean nominal time constant, errors caused by an oscillating flow rate can be neglected in comparison to other errors provided that there are no oscillations with a period greater than one-third of the measuring time.

Concentrations Recorded Several Times

By recording the concentrations on several occasions, we possess more information and we should therefore be able to obtain a more accurate estimate of the mean flow rate. The most common method is to fit the data by a least squares fit to a linear relation between ln concentration versus time. As an alternative, when the concentrations are recorded continously, we can calculate the area below the concentration curve and use the following expression:

$$q^e = V \frac{C(t_s) - C(t_e)}{\int_{t_s}^{t_e} C(t)dt} \qquad (m^3/h) \qquad (14)$$

where

t_s = Point of time for start of measurement
t_e = Point of time for end of measurement

For a constant flow rate, Equation 14 gives a correct estimate of the flow rate.

The two methods suggested above should give a more accurate estimate of the mean flow rate, or, alternatively, with the same accuracy, be able to cope with slower oscillations than given by the restriction of Equation 12 or 13. It is, however, more difficult to find a simple analytical expression for the magnitude of their accuracy.

EXPERIMENTS

Description of Test House

The tests reported in this section were carried out in a test house located in the laboratory hall at the Institute (see Figure 2). The house has five "rooms", a total volume of 176 m^3, and the floor area is 70 m^2. Pressurization of the house to 50 pascals gave rise to a flow rate of 0.8 house volumes per hour through the building envelope. One short wall of the house consists of the existing south wall of the laboratory. Against the short wall at the opposite end of the house, there is a cooling chamber. The house is heated by electric radiators. The house and the measuring system belonging to it are designed for studies of the performance of whole house fan powered ventilation systems. It was converted into a naturally ventilated house especially for these tests, by running a duct from the test house and through the ceiling of the laboratory building. The duct was connected to the kitchen and the bathroom (marked as extract points in Figure 2). The height of the duct was approximately 5 meters. During the tests the heating system was functioning. To improve the stack effect, the duct was warmed up by use of a heating coil. The main intake of air was provided by openings in the window frames in the living room. During the tests there was no furniture in the house.

Experimental Procedure

The following quantities were monitored in each room:

- Room air temperature at 0.2 m above the floor and 0.2 m below the ceiling.
- The pressure (in relation to the laboratory hall) in the middle of the room
- Gas concentration in the middle of the room by an IR analyzer with a time constant around 10 seconds.

Tracer gas (N_2O) was released into each room directly into the air stream created by the mixing fan. The temperature in the laboratory hall and the outdoor temperature were continously measured. The pressure difference between the outdoors and the laboratory hall was also recorded.

Tracer gas was continously injected into the duct at a known flow rate, and the gas concentrations both downstream and upstream of the point of injection were recorded continously with an IR-analyzer with two columns. The analyzer was operated in such a way that the upstream concentration (background concentration) was subtracted from the downstream concentration.

The measuring and control sequency in each test was as follows:

1. Start of mixing fans (two in the living room and one each in the other rooms) and release of gas into the duct and measurement of the flow rate of air in the duct
2. Measurement of temperature

3. Measurement of pressure
4. Start of release of gas into the house
5. Measurement of the flow rate of outdoor air to each room by the use of the constant concentration method
6. Measurement of the mean age of air in each room and slope of the decay curve in a graph of ln concentration versus time
7. A repeat measurement of temperature and pressure.

From the pressure mesurements, an equivalent pressure difference, Δp_e, across the building envelope is calculated as:

$$\Delta p_e = \sum_i \left(\frac{A_i}{A}\right) p_i \qquad \text{(Pa)} \qquad (15)$$

where

p_i = Pressure difference across surface no i of the building envelop
A_i = Area of surface number no i of the building envelope
A = The total area of the building envelope.

In an analogus manner, an equivalent temperature difference, ΔT_e, is calculated.

From the continous release of gas in the duct, the time-dependent flow rate is calculated as:

$$q_d(t) = \frac{q_{inj}}{C_d(t)} \qquad (m^3/h) \qquad (16)$$

where

C_d = Concentration in the duct (ppm)
q_d = Airflow rate in the duct (m^3/h)
q_{inj} = Volume flow rate of gas injected (m^3/h)

The response of the constant flow technique to a varying flow rate in the duct is very fast, of the same order as the time constant of the analyzer, that is to say, about 10 seconds. Therefore, the intention behind the design of the tests was that the measurements of the flow rate in the duct with the fast and reliable constant flow technique should give us the total infiltration rate to the house at each time instant. This, in turn, should permit us to make a direct comparison with the predictions from the slower decay technique.

RESULTS

Altogether five tests were carried out. Figure 3 shows two examples of measured flow rates in the duct. The upper part of the figure (test S14) is from a situation with normal wind speed, 2.5±1.0 m/s, while that in the lower part of the figure (test S13) is from a situation with a higher wind speed, 7.9±2.7 m/s. In both cases we have high frequency oscillations around the mean value. However, in the high wind case, the amplitude is approximately five times as great as in the low wind case. If we compare ratios between the amplitude and the mean flow rates, we find that in the high wind case, the ratio is twice as great as in the low wind case. Figure 4 shows, for the same cases as in Figure 3, the flow rate of air to each room as predicted by the constant concentration technique and the mean age of air in each room.

We can see from Figure 4 that, in both cases, almost all outdoor air enters the house through the openings in the living room. This is also reflected by the values of the mean age of the air, which is somewhat lower in the living room than in the other rooms.

Table 1 gives a summary of the results from the prediction of the flow rates by the different methods adopted. We notice that the flow rate measured in the duct is only about half of the flow rate measured in the house. This seems explainable only by the assumption that about half of the air that entered the house left the house through the duct. This implies, unfortunately, that we cannot make a direct comparison between the flow rates measured in the duct and in the house.

From the decay test, an arithmetic mean of the reciprocal of the exponent of the decay curve was calculated from the concentration readings in each individual room. We know that for a constant ventilation airflow rate, the constant concentration method is the most accurate one, with a relative error of 6% (Sandberg and Blomqvist 1985). There is no reason that this should not also be the case for a variable flow rate. We see from Table 1 that, except in test S12, we predict a lower flow rate with the decay method than with the constant concentration method. The difference cannot be fully explained by the difference in accuracy between the two methods at a constant flow rate. However, we must keep in mind that the two methods are not carried out simultaneously (the constant concentration method is carried out before the decay method) and there could be a slight difference in the flow rate on these two different occasions. Therefore, the correspondence between the methods is reasonable.

The last column in Table 1 shows the recorded arithmetic average of the mean age of air recorded in each room. The maximum value and the minimum value recorded in the house are also given. In no case did we obtain uniform mixing. The maximum standard deviation among the five measuring occasions amounts to 8% (test S12). This is more uniform mixing with the same number of mixing fans (less standard deviation) than reported in Sandberg and Blomqvist (1985). This effect can be ascribed to the increased mixing caused by the heating system that was functioning during these tests.

DISCUSSION

The total flow rate predicted by the constant concentration method varied between 47 and 106 m^3/h. The corresponding mean nominal time constants lie in the range 1.7-3.7 (h). The theoretical analysis in this paper shows that we should expect an noticable influence of an oscillating ventilation airflow on the accuracy when we have oscillations with periods greater or equal than one-third of the mean nominal time constant. This implies, in this case, periods in the range 35-75 minutes.

No such periods occured in the flow rate histories recorded in the duct. The oscillations had much shorter periods. It is perhaps unlikely that low-frequency oscillations (of the order of one hour) occur in practice. One support for this surmise is that van der Hover (1957) found a gap in the wind energy spectra in the range 5 minutes to 5 hours.

CONCLUSIONS

The following conclusions can be drawn from the study:

From the theoretical part:

Under the assumption that the flow rate has a stationary mean value, $\bar{q}$, and that the measuring time is not less than the mean nominal time constant $\bar{\tau}_n$ ($\bar{\tau}_n = V/\bar{q}$), then when assessing the mean flow rate.

Errors caused by an oscillating flow rate can be neglected compared to other errors provided that there are no oscillations with a time period greater than one third of the measuring time.

From the experiments:

The oscillations in the flow rate were of a high frequency nature, which should not affect the accuracy, provided that the measuring time is sufficiently long.

There is good correspondence between the constant concentration method and the decay method.

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van der Hoven, I. 1957. "Power spectrum of horizontal wind speed in the frequency range from 0.0007 to 900 cycles per hour." Journal of Meteorology, Vol 14, pp 160-164.

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TABLE 1 RESULTS

Test	Wind speed	T_e	P_e	Total flow rate in house: Constant concentration	Total flow rate in house: Decay	Flow rate in duct	Mean age of air in house
	m/s	°C *	Pa *	m³/h	m³/h	m³/h	h
S10	3.0±1.0	B: 12.3 A: 13.4	B: 3.4 A: 3.2	48	42	25 3	4.26 + 0.18 Min: 4.01 (Lr) Max: 4.43 (K)
S11	5.1±1.8	B: 13.3 A: 13.9	B: 9.0 A: 6.3	63	54	30 4	3.18 + 0.23 Min: 2.82 (Lr) Max: 2.74 (K)
S12	5.1±1.7	B: 13.9 A: 12.9	B: 14.5 A: 9.9	66	79	35 6	2.53 + 0.19 Min: 2.27 (Lr) Max: 2.74 (K)
S13	7.9±2.7	B: 12.9 A: 11.9	B: 15.5 A: 14.5	106	94	56 10	1.86 + 0.108 Min: 1.70 (Lr) Max: 1.96 (K)
S14	2.5±1.0	B: 11.7 A: 12.9	B: 4.9 A: 2.0	47	39	25 2	4.53 + 0.20 Min: 4.24 (Lr) Max: 4.74 (Br)

*) B denotes immediately before test
A denotes immediately after test

Lr = Living room
K = Kitchen
Br = Bath room

Captions

Figure 1. Calculated concentration response to oscillating ventilation air flow rates of different periods.

Figure 2. The test house

Figure 3. Examples of recorded flow rates in duct with the constant flow technique and corresponding wind speed. Mean wind speed in the upper part is 2.5 m/s (test no S14), while in the lower part it is 7.9 m/s (test no S13).

Figure 4. Examples of air flow rates predicted by the constant concentration method. (The same cases as in figure 3).

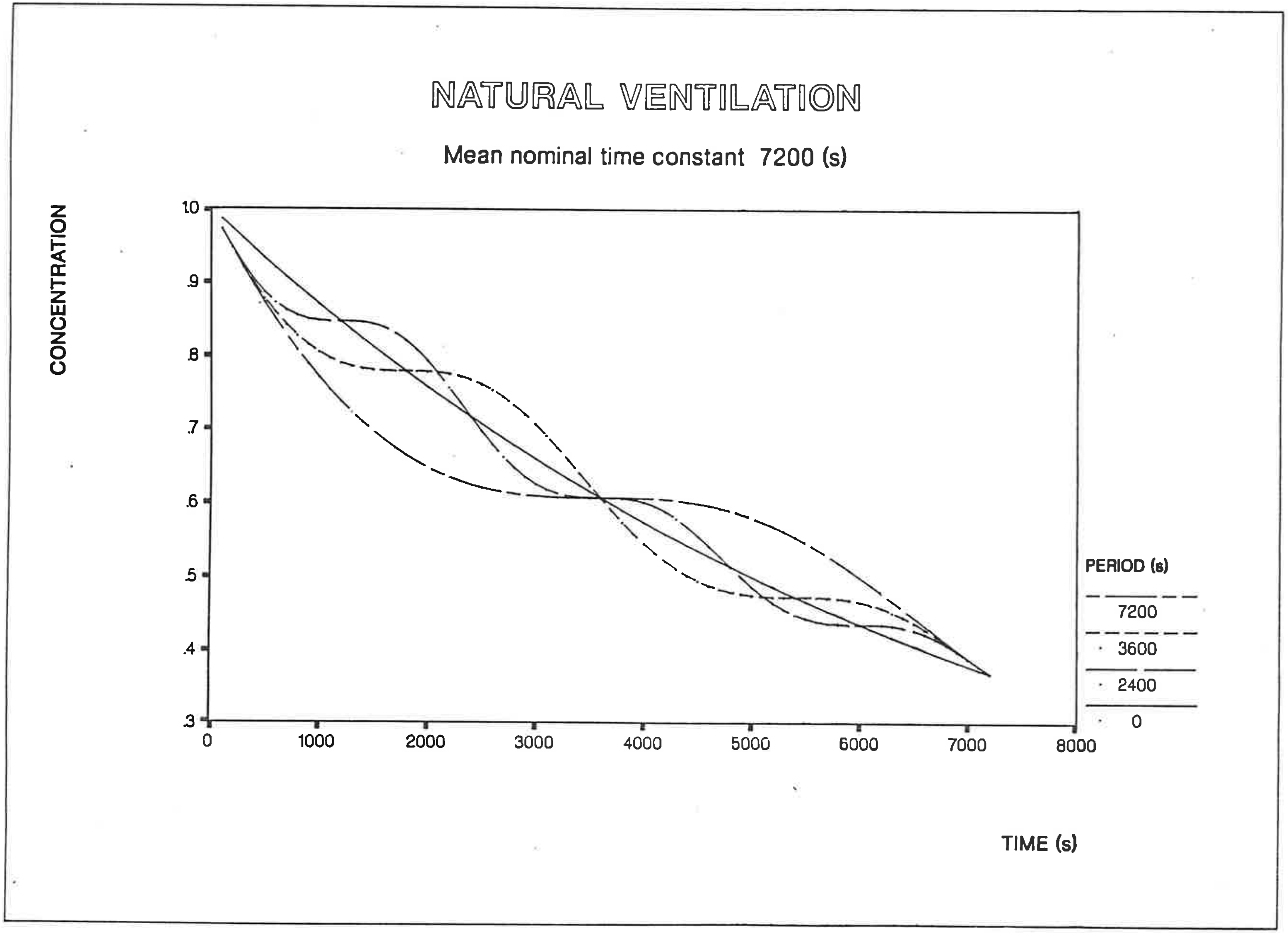
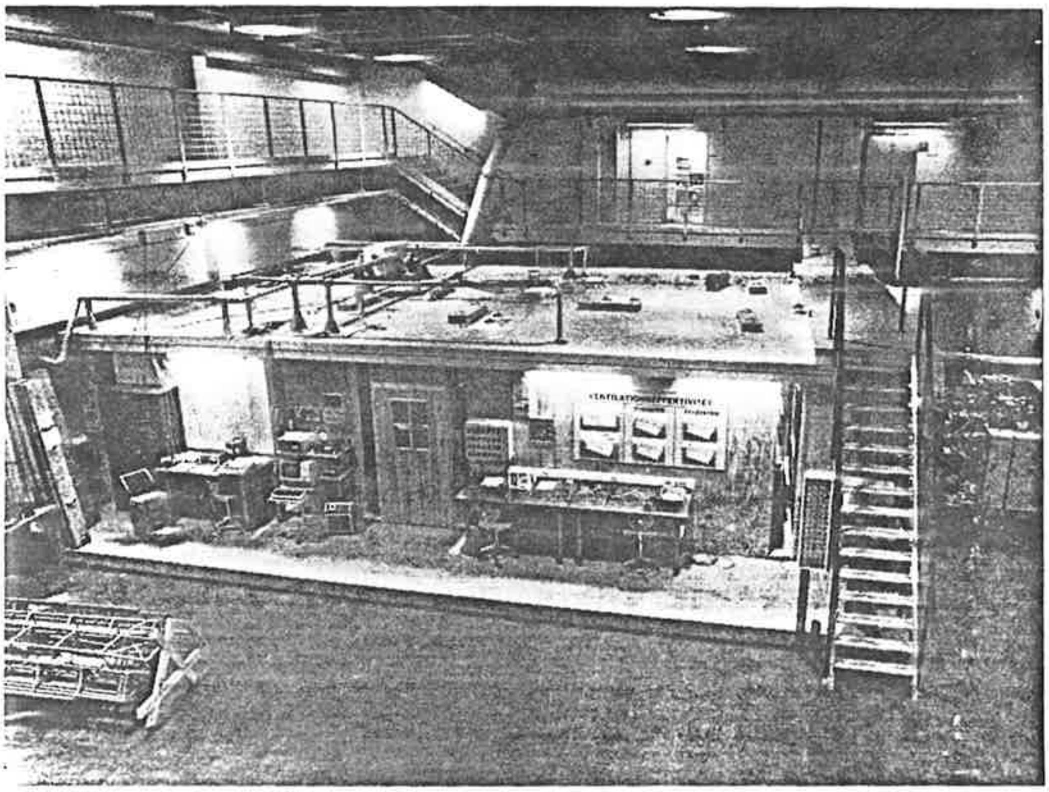
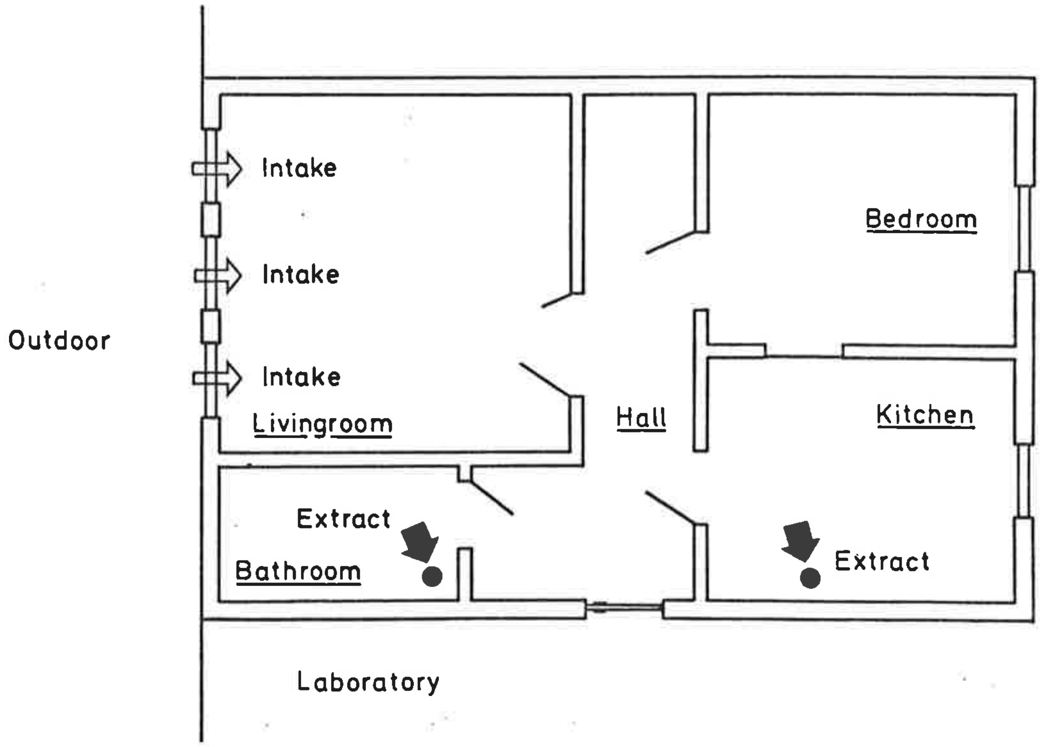

NATURAL VENTILATION
Mean nominal time constant 7200 (s)
CONCENTRATION
1.0
.9
.8
.7
.6
.5
.4
.3
0
1000
2000
3000
4000
5000
6000
7000
8000
TIME (s)
PERIOD (s)
7200
3600
2400
0

Figure 1

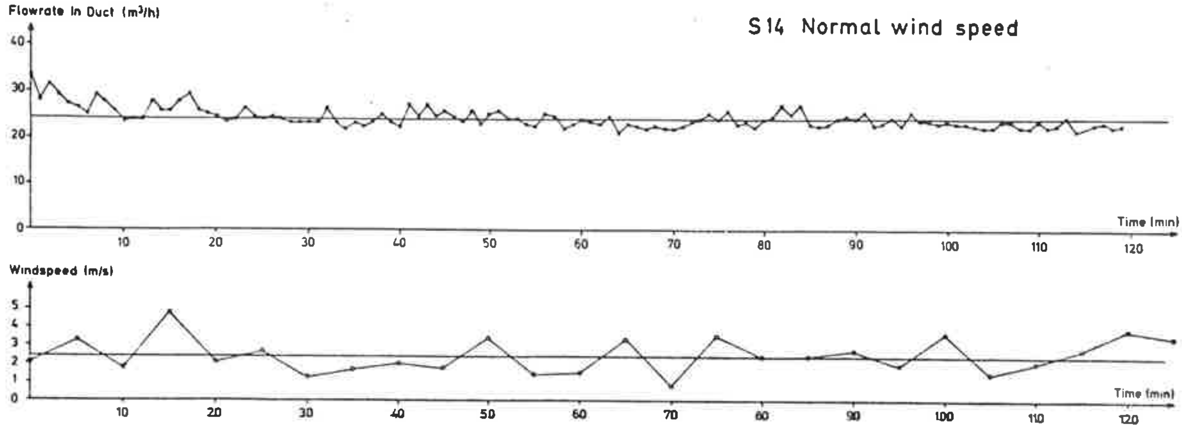
Figure 2



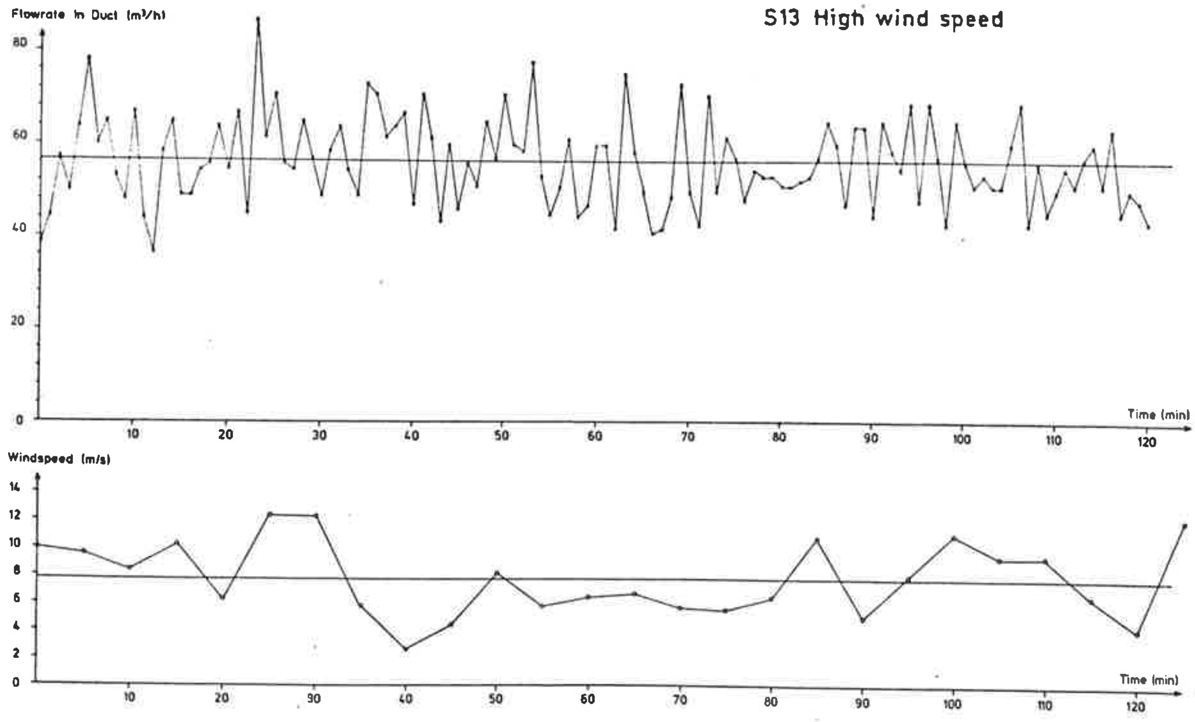


Intake
Intake
Intake
Livingroom
Outdoor
Hall
Bedroom
Kitchen
Extract
Bathroom
Extract
Laboratory

Figure 3

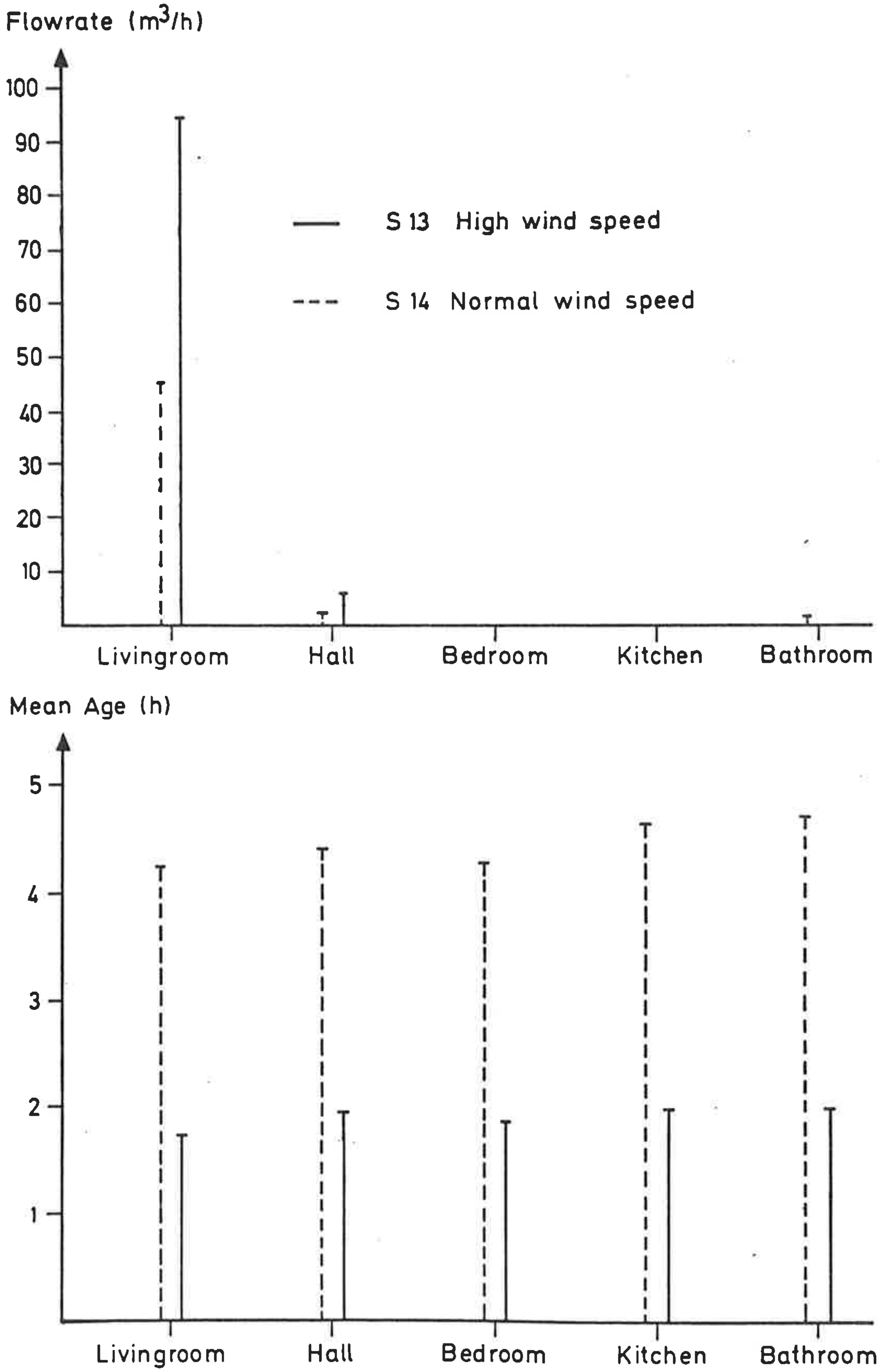


Flowrate in Duct (m³/h)
S14 Normal wind speed
Time (min)
Windspeed (m/s)
Time (min)



Flowrate in Duct (m³/h)
S13 High wind speed
Time (min)
Windspeed (m/s)
Time (min)

Figure 4



Flowrate (m3/h)
100
90
80
70
60
50
40
30
20
10
S 13 High wind speed
S 14 Normal wind speed
Livingroom
Hall
Bedroom
Kitchen
Bathroom
Mean Age (h)
5
4
3
2
1
Livingroom
Hall
Bedroom
Kitchen
Bathroom