Signal attenuation due to cavity leakage

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The propagation of sound waves in fluids requires information about three properties of the system: capacitance (compressibility), resistance (friction), and inductance (inertia). Acoustical design techniques to date have tended to ignore the frictional effects associated with airflow across the envelope of the acoustic cavity (e.g., resistive vents). Since such leakage through the cavity envelope is best expressed with a power law dependence on the pressure, standard Fourier techniques that rely on linearity cannot be used. In this article, the theory relevant to nonlinear leakage is developed and equations presented. Potential applications of the theory to techniques for quantifying the leakage of buildings are presented. Experimental results from pressure decays in a full-scale test structure are presented and the leakage so measured is compared with independent measurements to demonstrate the technique.

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LIST OF SYMBOLS

A	area (m^2)
A_L	area of leak (m^2)
C	capacitance of the cavity (m^3/Pa)
c	speed of sound in the fluid (m/s)
E	acoustic energy of cavity relative to the environment (J)
$F(n)$	form factor (—)
K_L	leak coefficient of leak L (m^3/s-Pa^n)
K	leak coefficient of cavity (m^3/s-Pa^n)
l	characteristic size of cavity (m)
n	leak exponent of cavity (—)
ω_{bp}	breakpoint frequency (rad/s)
ω_c	corner frequency (rad/s)
ω	(drive) frequency (rad/s)
P	fluid pressure (Pa)
P_c	critical pressure for nonlinear leak (Pa)
P_d	pressure at the drive (Pa)
P_L	pressure at leak L (Pa)
$P_{rms}(t)$	spatial root-mean-square pressure (Pa)
P_{rms}	spatial and temporal root-mean-square pressure (Pa)
P_ω	pressure amplitude at frequency ω (Pa)
ϕ	partition angle
Q_d	fluid flow at the drive (m^3/s)
Q_L	fluid flow at leak L (m^3/s)
ρ	density of the fluid (kg/m^3)
S	energy flux (W/m^2)
$\sigma(t)$	spatial standard deviation of pressure (Pa)
T	averaging period for the pressure (s)
t	time (s)
V_ω	drive volume component at frequency ω (m^3)
V	volume of cavity (m^3)
v	particle velocity (m/s)
x	position (vector) (m)
Z_L	impedance of leak (Pa-s/m)

INTRODUCTION

In the study of acoustics, purely resistive phenomena (i.e., frictional losses)—especially resistive losses due to leakage out of the acoustic cavity—play the least interesting role and it is, therefore, not surprising that little effort has been devoted to understanding them. Although acoustic methods have been used previously to *detect* leaks,[1] little research has been done on methods to quantify the relationship. In the field of ventilation, however, air leakage is quite important and much research has been devoted to it over the past decade.[2] The use of acoustic techniques for the quantification of air leakage would represent a major advance in this field.

Linear analysis techniques are commonly used to determine the acoustical properties of enclosures and spaces. In a typical experiment, the test cavity is excited by some means, and the desired properties are determined from the measured pressure response. In a typical analysis or design problem, (linear) Fourier analysis methods are used and any nonlinear leakage of air through the cavity is ignored. This report will focus on the situation in which air leakage is the dominant mechanism for the attenuation of the pressure signal.

In the conventional linear approach, acoustical properties of interior spaces can be determined from the response to an impulsive excitation, which can be quantified by an echogram.[3] The impulse response can be converted to a variety of equivalent representations[4] that may be of more use for particular applications. Additionally, techniques exist for using finite duration excitations to determine acoustical response.[5] Although these techniques do not exclude airflow through the cavity envelope as a loss mechanism, they do implicitly assume it is a small effect—since they use linear techniques.

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The common assumptions in the field of noise abatement are that the acoustical enclosure is airtight[6] and that the insertion loss can be calculated from the acoustic properties of the envelope.

The theory of vented loudspeakers[7] describes the use of apertures through cavity envelopes to improve the low-frequency performance of a speaker/cavity system. In optimizing the design of these systems, it has been found that a large vent area that minimizes the resistance to flow is preferred.[8] Since in this limit the impedance of the vent is dominated by the mass of the air moving through the vent, frictional losses caused by vent resistance will have no effect.

I. DERIVATION

We are considering situations in which the only significant (energy) loss mechanism is leakage between the cavity and the environment. We will, accordingly, assume that the walls of the cavity are perfectly reflective and that viscosity and other frictional losses are negligible. The zero of pressure is taken to be the external (constant) environment, and we will assume that the interior pressure disturbance is small compared to the absolute pressure so that the standard (subsonic) limits of small density changed, etc., used to derive the wave equation are applicable.

Since we are dealing with an inherently nonlinear problem, we cannot at first use straightforward impedance techniques to derive expressions for the evolution of the pressure. Instead, we start with the cavity energy balance relative to an unexcited state; the total energy inside the cavity will be the integral of the kinetic and potential[9] energy densities:

$$E(t) = \frac{1}{2}\rho \int v^2(x,t)dV + \frac{1}{2\rho c^2}\int P^2(x,t)dV. \qquad (1)$$

The partition of energy between potential and kinetic will depend on the nature of the disturbance in the cavity. We define a partition angle ϕ to relate the kinetic to potential energy:

$$\tan^2\phi \equiv \rho^2 c^2 \frac{\int v^2(x,t)dV}{\int P^2(x,t)dV}. \qquad (2)$$

A small partition angle occurs in a system whose energy is stored mainly in compression or rarefaction of the gas in the cavity (i.e., potential energy), and a large partition angle indicates that most of the energy is in kinetic energy (e.g., vortices); a purely acoustic system would have a partition angle of 45° (i.e., equipartition of energy). Because a discussion of rotational flow is beyond the scope of this article, we will assume irrotational flow, for which the partition angle can be no more than 45°. Furthermore, because we have assumed that there are no energy loss mechanisms other than leakage, as the pressure evolves with time, the partition of energy and, therefore, the partition angle will not change.

The total energy in the cavity can be expressed in terms of the (spatial) root-mean-square (rms) pressure over the cavity volume. Combining Eqs. (1) and (2),

$$E(t) = \tfrac{1}{2} C P^2_{\rm rms}(t)/\cos^2\phi, \qquad (3)$$

where we have defined the capacity of the cavity as follows:

$$C \equiv V/\rho c^2. \qquad (4)$$

Since there are no internal losses, the change in energy in the cavity will be equal to the surface integral of the energy flux:

$$\frac{dE}{dt} = \oint S(t)dA \qquad (5a)$$

$$= \frac{C}{\cos^2\phi} P_{\rm rms}(t)\frac{dP_{\rm rms}(t)}{dt}. \qquad (5b)$$

A. Undriven systems

If there is no energy input into the cavity, the only energy flux through the envelope will come from the leakage of air through the cavity envelope:

$$\oint S(t)dA = \sum_L P_L(t) Q_L(t), \qquad (6)$$

where the L subscript denotes specific leaks.

The flow through a leak as a general function[10] of pressure can be given by a power law expression of the following form:

$$Q_L(t) = K_L P_L(t)|P_L(t)|^{n-1}. \qquad (7)$$

Such a form can be derived from simple hydrodynamic arguments,[11] and must have an exponent (n) between 0.5 and 1.0. This expression has proven adequate to describe leakage in buildings and is found to have an exponent near $\frac{2}{3}$ (Ref. 12). Combining Eqs. (5)–(7) yields the following expression for the change in rms pressure:

$$\frac{C}{\cos^2\phi} P_{\rm rms}(t)\frac{dP_{\rm rms}(t)}{dt} = \sum_L K_L |P_L(t)|^{n+1}, \qquad (8)$$

where we have assumed that all leaks have the same exponent.

The right-hand side of Eq. (8) involves the instantaneous pressure at each leakage site, which is, in general, quite difficult to know. If, however, we assume that the rms pressure changes slowly compared to the time it takes for a pressure signal to traverse the cavity, we can select a time T, over which we may average the equation and not affect the left-hand side:

$$\frac{C}{\cos^2\phi} P_{\rm rms}(t)\frac{dP_{\rm rms}(t)}{dt} = \sum_L K_L \langle |P_L(t)|^{n+1}\rangle_T. \qquad (9)$$

Given that such a choice of time interval T is possible (an upper limit on T will be discussed below), the pressure (averaged over the time interval T) will be the same for every leak and we can rewrite the differential equation using the overall cavity leakage and pressure measured at any point in the cavity as follows:

$$C\frac{dP_{\rm rms}(t)}{dt} = K\left[\cos^2\phi \frac{\langle |P(t)|^{n+1}\rangle T}{P^{n+1}_{rms}(t)}\right] P^n_{\rm rms}(t). \qquad (10)$$

The term in brackets is an indication of the functional form of the pressure at the leak sites. If the pressure does not change its relative form over time, we may assume this form factor will be independent of time and may be evaluated at any convenient time. As is shown later, this asssumption can be checked after the fact by examining the standard deviation of the pressure.

We can rewrite the differential equation, using the form factor, as follows:

$$\frac{dP_{\rm rms}(t)}{dt} = \frac{K}{C} F(n) P^{n}_{\rm rms}(t), \quad (11)$$

where the form factor F can be evaluated at any time as

$$F(n) \equiv \frac{\cos^2 \phi}{P^{n+1}_{\rm rms}(t)} \frac{1}{T} \int_{t-T/2}^{t+T/2} |P(t')|^{n+1} dt'. \quad (12)$$

We can now solve the differential equation, Eq. (11), to find the time evolution of the rms pressure:

$$P_{\rm rms}(t) = P_{\rm rms}(0)[1-(1-n)\omega_c t]^{1/(1-n)}, \quad (13)$$

where

$$\omega_c \equiv F(n) K P^{n-1}_{\rm rms}(0)/C. \quad (14)$$

This frequency ω_c will be called the *corner frequency* and is a characteristic of the leakage-cavity system and initial pressure conditions.

A few comparisons can now be made between the linear and nonlinear leakage. In the linear case, the corner frequency is a constant, whereas, in the nonlinear case, it is a function of the applied pressure. In the nonlinear case, the disturbance ($P_{\rm rms}$) goes to zero in a finite length of time, whereas, in the linear case, the equation reduces to an exponential (as can be verified by taking the limit as n approaches unity). Figure 1 shows a *steady decay* for different exponents, but the same corner frequency. (As is shown in the next section, the form factor for a steady decay is unity.)

A further examination of Eq. (14) provides us with an upper bound for the averaging time T. This upper bound stems from the limitations that the averaging time be short compared with the decay time, which can be expressed as $T \ll 1/(1-n)\omega_c$.

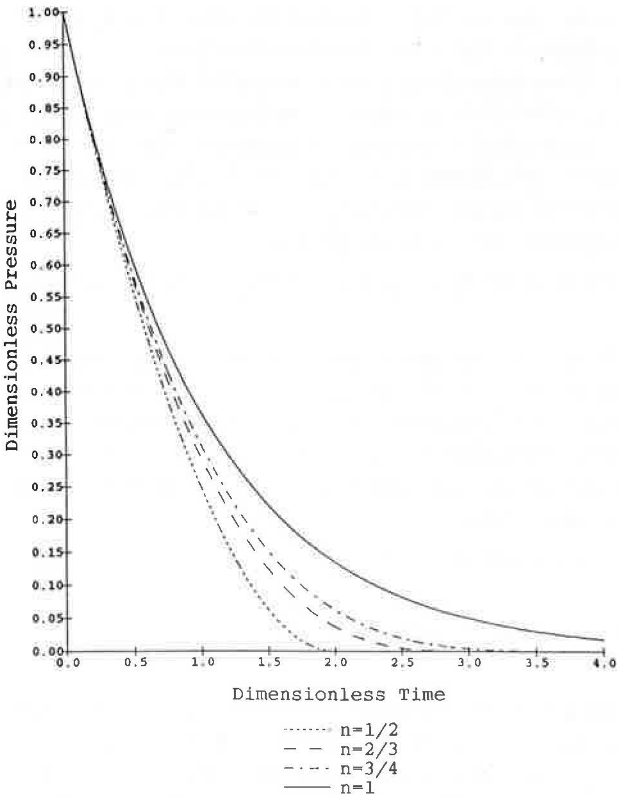

FIG. 1. The effect of exponent on decay shows decay of initially steady pressure for different values of the exponent n at constant corner frequency and initial pressure. The pressure has been normalized by the initial pressure and the time has been normalized by the corner frequency ω_c.

B. Calculation of the form factor

The form factor F is a measure of how much of the acoustic energy takes part in the leakage. In order to evaluate it, we must know something about the partition of energy and the variation around the rms pressure that is experienced by the leaks.

In a constant pressure environment, all of the energy is potential energy. In a cavity full of standing waves (i.e., normal modes), the energy is equally split between kinetic and potential energy. Since we are ignoring the contribution of rotational flow, we can find the partition angle from the (spatial) rms pressure and the standard deviation of the pressure around that mean:

$$\phi = \tan^{-1}[\sigma(t)/P_{\rm rms}(t)]. \quad (15)$$

Thus, from knowledge of the mean pressure and its standard deviation, it is possible to deduce the partition angle.

There are a few specific cases for which the integration implied in the definition of $F(n)$ can be carried out exactly. We have listed four of them below:

Base case: *Steady decay*. If the pressure field is homogeneous, then

$$P(t) = P_{\rm rms}(t), \quad (16a)$$

$$\phi = 0°, \quad (16b)$$

$$F(n) = 1. \quad (16c)$$

Case 1: *Standing wave*. If there is a single standing wave, the relations are as follows:

$$P(t) = \sqrt{2} P_{\rm rms}(t) \cos(\omega t), \quad (17a)$$

$$\phi = 45°, \quad (17b)$$

$$F(n) = (1/\pi) 2^{(1/2)(3n+1)} B(1+n/2, 1+n/2), \quad (17c)$$

where B is the standard beta function.

Case 2: *Standing pulse*. Here, we assume that there is a standing wave on top of an equal, steady pressure:

$$P(t) = \sqrt{2/3} P_{\rm rms}(t)[1+\cos(\omega t)], \quad (18a)$$

$$\phi = 30°, \quad (18b)$$

$$F(n) = (3/4\pi)(128/3)^{(1/2)(n+1)} \times B(3/2+n, 3/2+n). \quad (18c)$$

Case 3: *Random noise*. If there are many different standing waves, we can approximate the pressure by a random (Gaussian) distribution of zero mean whose standard deviation will be the rms pressure:

$$\phi = 45°, \quad (19a)$$

$$F(n) = (1/\sqrt{\pi}) 2^{(1/2)(n-1)} \Gamma(1+n/2), \quad (19b)$$

where Γ is the standard gamma function.

As a practical matter, we may not have accurate knowledge about the exact functional form of the pressure at the leaks. If we know there is zero mean pressure, then we can use Eq. (19b) for the form factor as an approximation, but, if there is an initial nonzero dc component to the pressure, it does not apply. However, we can use a Taylor expansion of a Gaussian of nonzero mean to get an approximate expression for the form factor in the general case:

$$F(n) \approx \cos^2 \phi\{1 - [(1-n^2)/4][\sigma^2(t)/P^2_{\rm rms}(t)]\}. \tag{20}$$

Since the standard deviation is principally caused by acoustic waves, it will have the same value over space and time and can be eliminated in favor of the partition angle using Eq. (15). Performing this substitution, the following is a reasonably accurate expression for the form factor in any undriven situation:

$$F(n) = \cos^2 \phi - [(1-n^2)/4]\sin^2 \phi. \tag{21}$$

C. Slowly driven systems

So far, we have only considered the time evolution of homogeneous (i.e., undriven) systems. For many problems, we would like to know the steady-state solution to the inhomogeneous (i.e., drive) problem. In the driven problem, there will be an extra term to the energy balance equation reflecting the energy input from the drive:

$$\oint S(t)dA = \sum_L P_L(t)Q_L(t) + Q_d(t)P_d(t). \tag{22}$$

If we assume that we are only interested in drive frequencies that are much less than any normal modes in the cavity and that we are only interested in the steady-state solution, there will be no spatial variation in the pressure—the drive, leak, and spatial rms pressure will all be the same:

$$P_L(t) = P_d(t) = P_{\rm rms}(t) \to P(t), \tag{23}$$

and the energy conservation equation becomes the following:

$$\frac{C}{\cos^2\phi} P(t)\frac{dP(t)}{dt} = K|P(t)|^{n+1} + Q_d(t)P(t). \tag{24}$$

The partition angle will tend to zero in the slowly driven case because there is no kinetic energy once steady state has been achieved.

We can follow the development of the undriven case by averaging this expression over a time period T. If the drive is periodic, we can set T to this period and the left-hand side of the equation must vanish, yielding the following expression:

$$K\langle|P(t)|^{n+1}\rangle_T = -\langle Q_d(t)P(t)\rangle_T. \tag{25}$$

This expression allows a direct calculation of the leakage coefficient from the known drive and the measured pressure response. An equivalent expression had been independently derived from continuity arguments.[13]

Although the leakage can be measured with this technique, it will not, in general, be possible to solve Eq. (24) for the pressure because of its nonlinearity. However, we can develop an approximate differential equation for the pressure by *linearizing* the leakage term as follows:

$$|P(t)|^{n+1} \to (\langle|P(t)|^{n+1}\rangle_T/\langle P(t)^2\rangle_T)P(t)^2 \tag{26a}$$

$$= \langle|P(t)|^{n+1}\rangle_T[P(t)^2/P^2_{\rm rms}]. \tag{26b}$$

We can now express Eq. (24) linearly:

$$\frac{dP}{dt} - \omega_c P = \frac{Q_d}{C}, \tag{27}$$

where the corner frequency has a similar definition to that of the undriven case:

$$\omega_c \equiv F(n)KP^{n-1}_{\rm rms}/C, \tag{28}$$

as does the form factor

$$F(n) \equiv \frac{1}{T}\oint_T \left|\frac{P(t)}{P_{\rm rms}}\right|^{n+1} dt, \tag{29}$$

which is time invariant if T is the periodicity.

The linearized differential equation can be solved using Fourier analysis:

$$i\omega P_\omega + \omega_c P_\omega = i\omega V_\omega \tag{30}$$

(where Q_d has been replaced by the time rate of change of a drive volume), which yields the following expression for the Fourier pressure amplitude:

$$P_\omega = (V_\omega/C)/(1 + i\omega_c/\omega). \tag{31}$$

An exact calculation of the form factor will depend on the functional form of the drive, but it will be, in general, quite similar to those form factors used in the undriven case. For example, the form factor for a single frequency is

$$F(n) = (1/\pi)2^{(3/2)(n+1)}B(1+n/2, 1+n/2). \tag{32}$$

If the drive contains many frequencies, then it may be more appropriate to use the random noise form factor

$$F(n) = (1/\sqrt{\pi})2^{(1/2)(n+1)}\Gamma(1+n/2), \tag{33a}$$

or the approximate expression if there is a nonzero mean pressure:

$$F(n) \approx 1 - [(1-n^2)/4](\sigma/P_{\rm rms})^2. \tag{33b}$$

D. Limitations

A fundamental limitation of the above derivation arises from the assumption that leakage does not cause localized pressure reductions relative to the rms pressure of the space. This is equivalent to assuming that the pressure change due to leakage is small during the time it takes for pressure variations to traverse the cavity. This limitation can be expressed mathematically in terms of the corner frequency (which characterizes the rate of leakage) in Eq. (27) as

$$\omega_c < c/l. \tag{34}$$

The characteristic length l will depend on geometry of the cavity and the distribution of the leaks and can vary greatly even for cavities of similar volumes and leakages. For example, if the leakage is concentrated in a single leak, the characteristic length is the path length from the leak to the farthest point of the cavity; if the cavity is a sphere and the leakage is uniformly distributed, the characteristic length is the radius.

In a linear system, the corner frequency is a constant, independent of pressure, and either will or will not meet the criterion. For the nonlinear case, the corner frequency depends on pressure, implying that there will always be a pressure above which this condition is true. We can thus define a critical pressure as follows:

$$P_c \equiv (K\rho c l / V)^{1/(1-n)}, \quad (35)$$

which can be used to express the limitation in Eq. (34) as

$$P_{\rm rms} \gg P_c. \quad (36)$$

For pressures well above the critical pressure, the corner frequency is low enough that the leak acts purely resistively, communicating well with the cavity. At pressures near the critical pressure, the flow through the leak changes faster than it can equilibrate with the cavity; it will therefore lead the cavity pressure in phase. At pressures much below the critical pressure, the leak acts as an open-ended organ pipe (i.e., has negligible resistance), the only impedance from the leak being that due to the mass of air moving, so that the linear techniques used in speaker and microphone design would be appropriate at this limit.

For driven systems, the assumption of being slowly driven means that the drive periods must be long compared to the time for pressure variations to traverse the cavity. In general, this is a redundant restriction as cavity leakage is only a large effect when the drive frequency is near or below the corner frequency, and we have already assumed that the corner frequency will be low compared to the cavity traversal time.

Another assumption that limits the applicability of our model is that there are no other energy loss mechanisms. If other loss mechanisms are not negligible, appropriate terms must be added to the differential equation, but, more relevantly, the partition of energy may change, the form factor may become a function of time, and the corner frequency may no longer be a constant of motion. Figure 2 indicates that such mechanisms may be important near the end points of our experiment.

II. APPLICATIONS

As indicated by the last limitation, the closed-form solutions are not applicable when other loss mechanisms are at work. Thus the most general application for this result may be to include it as one term in a general acoustic circuit design or analysis problem. If we use the linearized expressions from the slowly driven solution, we can cast our results in circuit parlance:

$$P_L = Z_L Q_L \quad (37)$$

and define an equivalent impedance of the leak:

$$Z_L \equiv P_{\rm rms}^{1-n} A_L / F(n) K_L, \quad (38)$$

where the rms pressure must be taken as that across the leak.

The penalty for linearizing the nonlinear equation is that the solution may have to be iterated to get the correct value of the rms pressure in the impedance definition. Furthermore, depending on the problem, it may be desirable to treat either the form factor or the rms pressure as slowly varying in time. While the solution of circuits containing nonlinear leaks may be of interest, it is beyond the scope of this article and we return to the leakage dominated cavity.

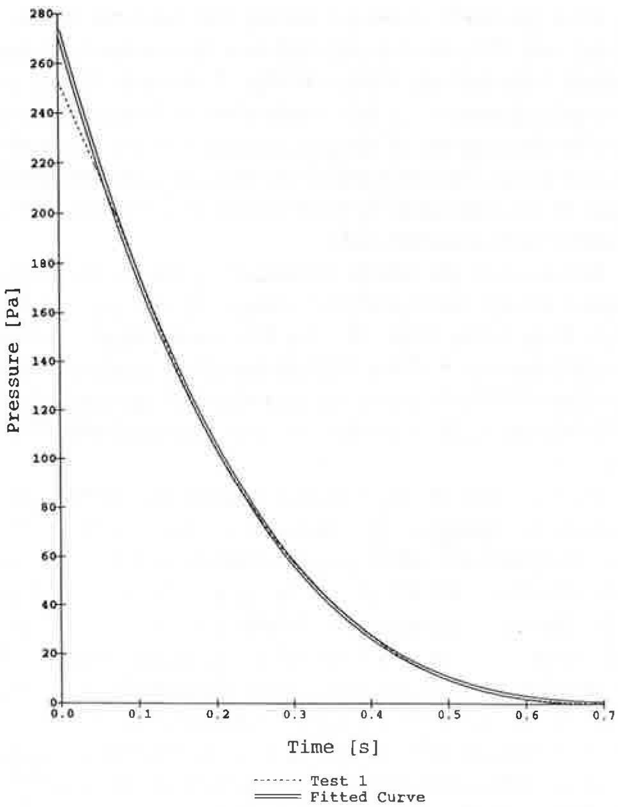

FIG. 2. The measured and fitted values of $P_{\rm rms}$ versus time in the Mobile Infiltration Test Unit resulting from a door slam (test 1). The dotted line represents the measured data; the double line is the fit to the data.

A. Pressurization decay

The decay equation, Eq. (13), suggests that one way to measure the leakage of an enclosure is by creating an initial pressure difference across the envelope of the enclosure and measuring the decay of the pressure after the excitation is removed. Although a similar technique has been previously suggested,[14] an exact analysis technique had been lacking.

To investigate this technique, we conducted several tests with our Mobile Infiltration Test Unit (MITU),[15] which is a room-size (25 m^3) experimental facility. A preliminary set of experiments was done by popping a pressurized tube to create an initial pressure. However, the sharp rise in pressure excited too many normal modes and so, for this investigation, a different type of excitation was used. The initial conditions were created by rapidly closing the access door to the facility.

Figure 2 is an example plot of $P_{\rm rms}(t)$ computed from the measured pressure and $P_{\rm rms}(t)$ obtained from a fit to Eq. (13). The fit line is based upon a nonlinear search routine that extracts $P_{\rm rms}(0)$, ω_c and n from the measured data based upon Eq. (13). The fit follows the measured data very well, the only exception being close to time zero. Part of this deviation is an artifact resulting from limitations in the analysis method near the time origin. The real variation in $F(n)$ results from changes in the partition due to the existence of energy loss mechanisms other than leakage (e.g., acoustic damping by the structure and its contents). To assure that the form factor is constant during the analysis period, $P_{rms}(t)$ and $F(n)$ were computed over the course of three separate tests and are plotted in Fig. 3. As seen in Fig. 3, $F(n)$ quickly reaches a value very close to 1 and remains there for the majority of the test, corresponding to a steady pressure decay. Near the end of the test, the pressure signal begins to be dominated by random noise, for which $F(n)$ should be approximately 0.45.

Based upon the results presented in Fig. 3, the three pressure decays were analyzed using only the periods for which $F(n) \geqslant 0.98$ (Ref. 16). As $F(n)$ was found to vary smoothly between 0.98 and 1.0 during this period, an average value of 0.99 was used to compute the leakage coefficient K. The results from these three tests are summarized in Table I.

If we use the observed standard deviations of these parameters as indicators of precision, it is clear that the technique has provided rather precise estimates of the leakage characteristics of the MITU trailer, $\pm 3\%$ for n and $\pm 5\%$ for K. However, these results should not be viewed as an estimate of the accuracy of the technique, as our estimate of K scales directly with our estimate of the capacity C of the MITU trailer. Thus, although the estimated 10% uncertainty in C does not affect the precision with which K is determined, an inaccurate estimate of C would bias all of the results. However, these results can be compared with estimates of the flow characteristics of the leaks in the MITU envelope

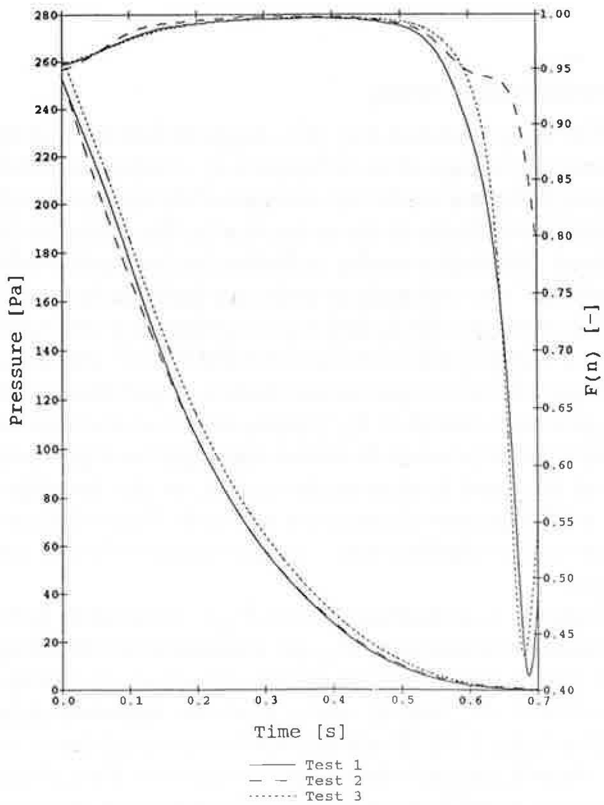

FIG. 3. The evolution of P_{rms} and $F(n)$ with time shows measured pressure decays (P_{rms}) and form factors [$F(n)$] for three door-slam excitations of the Mobile Infiltration Test Unit. The lower three curves correspond to the left axis and the upper curves to the right axis.

TABLE I. Results of fits to pressure decays. The first two columns contain the end points of the time period used for the fit, and the next three columns contain the parameter values determined by the fit; the last two columns contain the fit parameter n and the derived parameter K, which represent the desired cavity leakage characteristics.

Test	Start fit (s)	End fit (s)	$P_{rms}(0)$ (Pa)	ω_c (s^{-1})	n (−)	K ($m^3/s\ Pa^n$)
1	0.11	0.53	273	4.13	0.69	0.0060
2	0.11	0.56	280	3.88	0.67	0.0064
3	0.09	0.54	259	3.91	0.66	0.0066
Average					0.67	0.0063
s.d.					0.02	0.0003

based upon independent measurements using fan pressurization.[17] These estimates are 0.64 ± 0.06 for the flow exponent n and 0.0077 ± 0.002 $m^3/s\ Pa^n$ for the leakage coefficient K. Although these results are not inconsistent, it is clear that experiments under more controlled conditions are warranted.

B. ac pressurization

The pulse pressurization technique described above uses the undriven solution after an initial disturbance. As a practical matter, it may be easier to drive the system in a well-controlled manner and use the steady-state solution. In an earlier report,[18] the authors described a driven process for measuring the leakage of enclosures that is called *ac pressurization*. The fundamental equation of ac pressurization is Eq. (24), and allows real-time calculation of the leakage from the measured volume drive and pressure response. Reference 18 should be consulted for a more detailed description of the method and application as well as examples.

In an independent study by Card *et al.*,[19] the pressure spectrum of a constant drive system was measured, and results in agreement with Eq. (31) were obtained. Card *et al.* defined a *breakpoint frequency* ω_{bp}, which is similar to the corner frequency:

$$\omega_{bp} \equiv K / C^n V_\omega^{1-n}. \tag{39}$$

When the system is driven at the corner frequency and there are no other pressure components, the breakpoint frequency and the corner frequency are close [i.e., assuming $F(n) = 1$]. However, in any other situation, it is the corner frequency that characterizes pressure response.

III. SUMMARY

The theory developed in this article is useful principally for predicting the pressure evolution in a leaky cavity as a function of initial conditions and any driving influences. Based upon this theory, two techniques that could be used to quantify the leakage were examined. An experimental examination of the pressure-decay technique produced repeatable estimates for the leakage of a test cavity. These results were in basic agreement with the leakage estimated by independent means. The theory developed also provides a more compact unified basis for a driven technique known as ac pressurization. Future effort will focus on detailed analysis techniques and development for specific applications such as sonic boom loading, blast waves, and noise transmission in automobiles.

ACKNOWLEDGMENT

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