Building Serv. Eng. Res. Technol. **10**(2) 85–88 (1989) Printed in Great Britain

#3518

Technical Note

Summary This paper examines the accuracy of various theoretical methods for the analysis of experimental data on interzone air movement obtained using a single tracer gas technique. Two types of data were examined; (*a*) well conditioned data obtained in the laboratory from measurements of air movement between two chambers and (*b*) less well conditioned measurements made in houses. Errors in calculated flow rates were found to be in the range −8 to 33% for well-conditioned measurements. For less well-conditioned data the estimated values of air flow rates were spread over a wide range and it was not possible to establish the errors in these values as the real flow rates were unknown.

Air flows between two zones: Accuracy of single-tracer gas measurements for estimation

S B Riffat DiplTech MSc DPhil CEng MASHRAE
Department of Civil Engineering, Loughborough University of Technology, Loughborough, Leics. LE11 3TU, UK

Received 8 August 1988, in final form 4 November 1988

1 Introduction

Interzone air movement in buildings has recently received considerable attention since it has important consequences for energy conservation, thermal comfort and air quality control. Air movement has been measured by both single and multiple tracer gas techniques[1]. Inaccuracies in estimating air flows arise as a result of both experimental errors (dependent upon the measuring system and extent of tracer gas mixing) and errors arising from assumptions made in the mathematical analysis of experimental results. To establish errors in the calculated values of air flow rates, the 'real values' must be known. The use of an environmental chamber in which the interzone air flows can be set to known values would be useful for assessing the accuracy of various techniques and analysis methods. Irwin and Edwards[2] have analysed errors on multi-tracer gas measurements. This paper discusses various analysis methods and evaluates the errors in estimating air flow rates for measurement made using a single tracer gas technique.

2 Analysis methods for estimating air flows

The fundamental equations of multi-zone air movement within a building are based upon Sinden's model[3]. The model assumes that a building consists of a number of zones 0, 1, 2, . . . , N, which are connected by air-flow passages. These passages are assumed to allow air to flow in one direction only. In the decay tracer gas method, each zone is initially injected with a known amount of tracer gas and it is assumed that there is no further of generation of tracer gas in the zone after time zero. Applying tracer gas volumetric balance equations to zone j, we obtain:

$$V_j\,dC_j/dt = \sum_{i=0}^{N} F_{ij}C_i - \sum_{i=0}^{N} F_{ji}C_j \quad (\text{for } 1 \leq j \leq N) \tag{1}$$

The total flow into chamber j must be equal the total flow out of the chamber and is given by the conservation equation:

$$\sum_{i=0}^{N} F_{ij} = \sum_{i=0}^{N} F_{ji} \quad (\text{for } 1 \leq j \leq N)(F_{jj} = 0) \tag{2}$$

where V_j is the volume of the jth chamber, C_j is the tracer gas concentration in the same chamber, dC_j/dt is its time derivative, F_{ij} is the air flow rate from the ith to the jth chamber, F_{ji} is the flow rate from the jth to the ith chamber. Zone 0 in the model represents the external air and is assumed to have an infinite volume. The concentration of tracer gas in this zone is assumed to be zero.

The discussion centres upon air flow measurements between two zones as shown in Figure 1. The single-tracer decay method can be applied to each zone in order to determine the infiltration, exfiltration and interzone air flow rates. Assuming that the air and tracer gas are perfectly mixed and that the concentration of tracer gas in the outside air is zero, equation 1 can be then solved using one of various analysis methods, such as that adopted by Penman and Rashid[4], Perera and Walker[5], Wortman and Burch[6], Littler *et al*[7] and I'Anson *et al*[8].

The method used by Penman is based upon numerical integration of equation 1 over a time step between t_1 and t_2. Applying equations 1 and 2 to a two-zone system ($N = 2$) allows the following equations to be obtained:

$$V_1(C_{1(t_2)} - C_{1(t_1)}) = -F_{01}\int_{t_1}^{t_2} C_1\,dt + F_{21}\int_{t_1}^{t_2} (C_2 - C_1)\,dt \tag{3}$$

$$V_2(C_{2(t_2)} - C_{2(t_1)}) = -F_{02}\int_{t_1}^{t_2} C_2\,dt + F_{12}\int_{t_1}^{t_2} (C_1 - C_2)\,dt \tag{4}$$

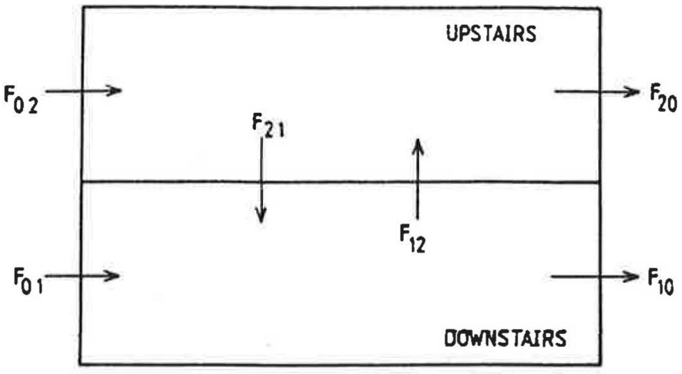

Figure 1 Air movement between two zones

The curve describing the decay of tracer gas is divided into several time intervals. Air flows are then calculated using a least squares approximation[9].

Perera and Wortman employed an analysis method which used two tracer gases (a and b). Consider the schematic of the building shown in Figure 1. If one applies the volumetric balance equations to each zone, then:

$$V_1\, dC_{1a}/dt = -C_{1a}F_{10} - C_{1a}F_{12} + C_{2a}F_{21} \qquad (5)$$

$$V_1\, dC_{1b}/dt = -C_{1b}F_{10} - C_{1b}F_{12} + C_{2b}F_{21} \qquad (6)$$

$$V_2\, dC_{2a}/dt = C_{1a}F_{12} - C_{2a}F_{21} - C_{2a}F_{20} \qquad (7)$$

$$V_2\, dC_{2b}/dt = C_{1b}F_{12} - C_{2b}F_{21} - C_{2b}F_{20} \qquad (8)$$

The four defining equations for the tracer gas concentrations, equations 5–8, can be written in matrix form as follows:

$$\boldsymbol{F} = \boldsymbol{C}^{-1}V\, d\boldsymbol{C}/dt \qquad (9)$$

where

$$\boldsymbol{F} = \begin{pmatrix} F_{10} \\ F_{12} \\ F_{21} \\ F_{20} \end{pmatrix} \qquad (9a)$$

$$\boldsymbol{C} = \begin{pmatrix} -C_{1a} & -C_{1a} & C_{2a} & 0 \\ -C_{1b} & -C_{1b} & C_{2b} & 0 \\ 0 & C_{1a} & -C_{2a} & -C_{2a} \\ 0 & C_{1b} & -C_{2b} & -C_{2b} \end{pmatrix} \qquad (9b)$$

$$\boldsymbol{V}\, d\boldsymbol{C}/dt = \begin{pmatrix} V_1\, dC_{1a}/dt \\ V_1\, dC_{1b}/dt \\ V_2\, dC_{2a}/dt \\ V_2\, dC_{2b}/dt \end{pmatrix} \qquad (9c)$$

This approach introduces a source of error as the term $d\boldsymbol{C}/dt$ cannot be measured directly but is estimated by interpolation between two adjacent data points. Perera relied on one pair of tracer gas concentration/time points to calculate the flow rates while Wortman used a series of concentration/time intervals and the calculated values were then averaged over the entire measurement period to give the mean flow rates.

Littler adopted a numerical technique for analysing experimental data from multi-zone, multi-gas experiments. The technique was based on the Sinden model and considered the multi-zone system to be a series of cells of known and constant volumes. These were assumed to be coupled to another cell of infinitely large volume, i.e. the outside space. The zone volumetric balance equations were expressed in matrix form with the addition of the discrete time model as follows:

Introducing the notation $G_{j,i} = F_{j,i}/V_i$, equation 9 can be rewritten:

$$d\boldsymbol{C}/dt = \boldsymbol{GC} \qquad (10)$$

where

$$\boldsymbol{C} = (C_1 \ldots C_N)^{\mathrm{T}} \qquad (10a)$$

and

$$\boldsymbol{G} = \begin{pmatrix} G_{1,1} & G_{2,1} & \cdots & G_{N,1} \\ G_{1,2} & G_{2,2} & \cdots & G_{N,2} \\ & & \vdots & \\ & & \vdots & \\ G_{1,N} & G_{2,N} & \cdots & G_{N,N} \end{pmatrix} \qquad (10b)$$

The discrete time model is written:

$$C_1(t+1) = D_{1,1}\, C_1(t) + D_{2,1}\, C_2(t) \ldots + D_{N,1}\, C_N$$

$$C_2(t+1) = D_{1,2}\, C_1(t) + D_{2,2}\, C_2(t) \ldots + D_{N,2}\, C_N$$

$$\vdots$$

$$\vdots$$

$$C_N(t+1) = D_{1,N}\, C_1(t) + D_{2,N}\, C_2(t) \ldots + D_{N,N}\, C_N \qquad (11)$$

or, in matrix form:

$$\boldsymbol{C}(t+1) = \boldsymbol{D}\,\boldsymbol{C}(t) \qquad (11a)$$

where

$$\boldsymbol{D} = \exp \boldsymbol{G} \qquad (11b)$$

The exponential of the square matrix $\boldsymbol{G}$ is most conveniently defined by the power series:

$$\exp \boldsymbol{G} = \boldsymbol{I} + \boldsymbol{G} + \boldsymbol{G}^2/2! + \boldsymbol{G}^3/3! + \ldots \qquad (11c)$$

This technique minimises the errors introduced by the uncertainties in tracer gas measurements and estimation of the gradient $d\boldsymbol{C}/dt$ since the variable t is restricted to the values of 1, 2, 3, ... $S - 1$, where S denotes the number of samples taken in each zone.

I'Anson adopted a different approach for that of the previous researchers. The volumetric balance equations for zones 1 and 2 were written as second-order differential equations:

$$\frac{V_1V_2}{F_{21}}\frac{d^2C_1}{dt^2} + \frac{V_1V_2}{F_{21}}(N_1+N_2)\frac{dC_1}{dt} + \left(\frac{N_1V_1N_2V_2}{F_{21}} - F_{12}\right)C_1 = 0 \qquad (12)$$

$$\frac{V_1V_2}{F_{12}}\frac{d^2C_2}{dt^2} + \frac{V_1V_2}{F_{12}}(N_1+N_2)\frac{dC_2}{dt} + \left(\frac{N_1V_1N_2V_2}{F_{21}} - F_{21}\right)C_2 = 0 \qquad (13)$$

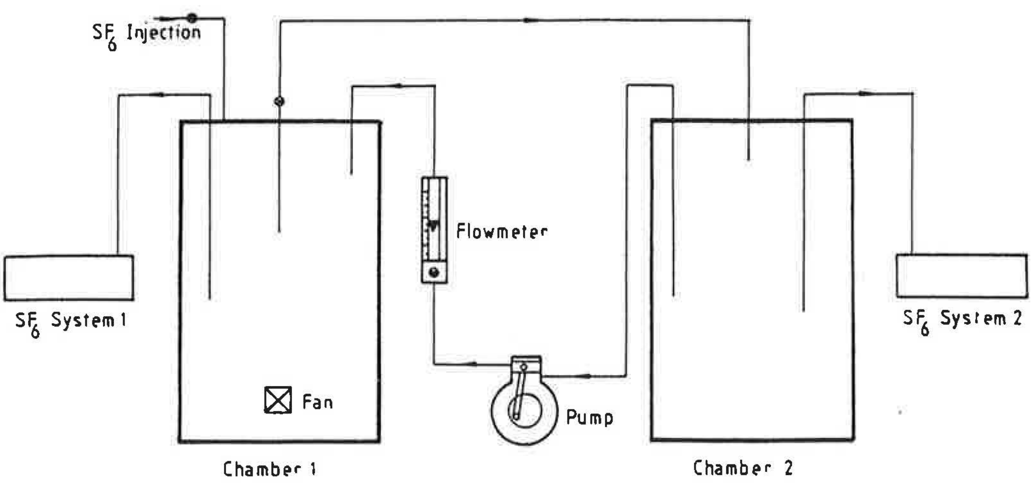

Figure 2 Test rig for validation of the single-tracer gas measuring technique

Equations 12 and 13 were solved by the application of an auxiliary equation and Prony's approximation method[9] was used to estimate the flow rates.

3 Experimental

Air flows were measured using two portable microcomputer-controlled systems. The two systems are identical in construction and are described in detail elsewhere[10]. Each consists of the following components: a sampling and injection unit, a column, a chromatograph oven, an electron capture detector and a microcomputer and interface.

The sampling unit consists of a two-position, six-port valve, connected to a 0.5 cm^3 sampling loop. The valve can be rotated to position 1 or 2 using a small motor. The separation column was made by packing a 1.5 m length × 4.3 mm internal diameter nylon tube with 60–80 mesh aluminium oxide. The column was held at 35 °C in a thermostatically controlled electric oven. The electron capture detector, which uses a Ni-63 radioactive cell, was made by Pye Unicam Ltd.

To evaluate the analysis errors in single tracer gas measurements, air flow tests were carried out under controlled conditions. For this purpose a small-scale test rig was built. This simply consisted of two chambers, a pump, a flow meter and a fan (Figure 2).

The experimental procedure was as follows. At the beginning of each test SF_6 tracer gas was injected into chamber 1 in which a fan mixed tracer gas and air. The concentration of SF_6 was then measured at different heights in the chamber. A mixing period of about one hour was found to be sufficient to achieve a uniform concentration. Following tracer gas mixing, the two chambers were connected and the pump was turned on. SF_6/air samples were taken from the two chambers for analysis.

4 Specimen results and discussion

Tests were carried out at various air flow rates using the two-zone test rig. Figure 3 shows a typical tracer gas concentration decay curve for the well conditioned data. The decay curve in chamber 1 was found to be a simple exponential function of time. This indicates that the tracer gas in the chamber was fully mixed. The flow rates for tests 1 and 2 were 124 and 230 lh^{-1} respectively, as measured using a calibrated flow meter. Table 1 gives flow rates as estimated by various analysis methods.

Table 1 shows that the errors in the calculated air flow rates $(F_{\text{calculated}} - F_{\text{measured}})/F_{\text{measured}}$ are in the range −8 to 33%. Perera's method produced the largest error, as one pair of data points were used to estimate the concentration gradients. The use of concentration gradients at a number of data

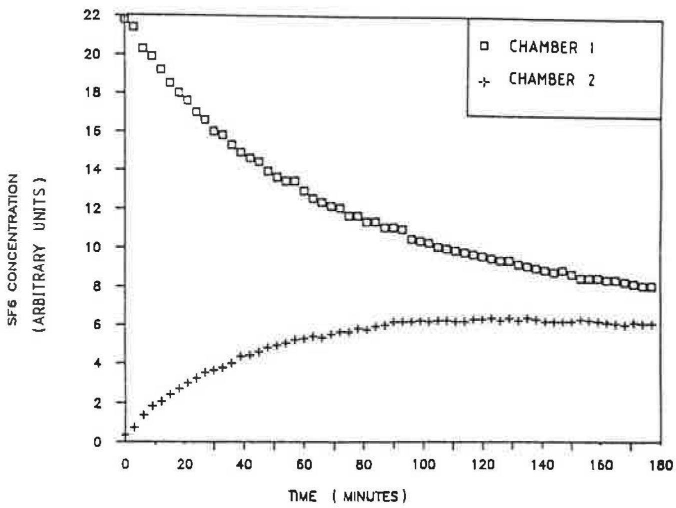

Figure 3 Tracer gas concentration decay curve for the well conditioned data

Table 1 Comparison of various analysis methods for estimating air flows using well conditioned data

Method	Test 1		Test 2	
	Flow rate (lh^{-1})	Error (%)	Flow rate (lh^{-1})	Error (%)
Penman	144	16	264	15
Perera	165	33	300	31
Wortman	146	18	276	20
Littler	114	−8	217	−6
I'Anson	161	30	292	27

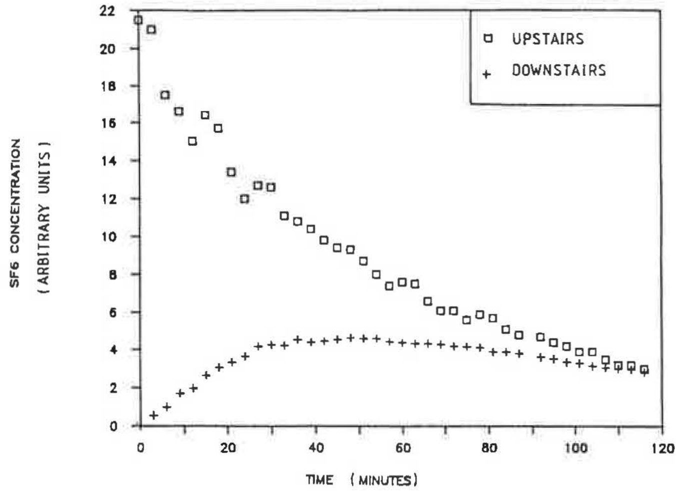

Figure 4 Tracer gas concentration decay curve for the ill conditioned data

points (as by Wortman) was found to improve the accuracy of estimating air flow rates. Even so, the estimation of dC/dt is still subject to potentially large errors, particularly during the transient period, i.e. the first 15–30 minutes of the measurements. Penman's analysis overestimates air flow rate by 15 to 16%. This error arises because it uses the concentration difference between adjacent data points to estimate air flow rates. The accuracy of this method depends on the random error in the data points and also on the number of data points used. Thus to reduce the error in the calculation, the lengths of the integration periods and of the time interval (defined by Simpson's rule[9]) should be small enough to allow the function to be well represented by line segments. Unlike other analysis methods the numerical method adopted by Littler was found to underestimate the flow rate by 6 to 8%. Accuracy has been considerably improved by this method but errors have arisen due to the sensitivity of the analysis to measurement noise associated with incomplete mixing of tracer gas in the zone. I'Anson's method produced a similar error to that of Perera. The major disadvantage of I'Anson's method is that it relies on Prony's approximation technique. This technique may be applied successfully to well conditioned data but cannot be applied to less well conditioned measurements as it produces complex roots for the differential equations.

Measurements of air flow between the lower and upper floors of a two-storey house were carried under a variety of test conditions[11]. Figure 4 shows the variation of tracer gas

Table 2 Comparison of various analysis methods for estimating air flows using less well conditioned data

Method	Flow					
	F_{01}	F_{10}	F_{12}	F_{02}	F_{20}	F_{21}
Penman	9	62	106	37	108	35
Perera	49	17	94	184	216	62
Wortman	68	108	14	106	66	54
Littler	67	59	105	34	42	97
I'Anson	*	*	*	*	*	*

* No solution is possible as the roots of the quadratic equations are complex.

concentration with time for both the downstairs and upstairs. The experimental scatter of the data points is greater than that observed for data obtained from well conditioned measurements. This scatter may result from incomplete mixing of the tracer gas and air in the measurement zone (ill-conditioned data). The use of the gradient of the decay curve for the transient period (i.e., early part of the decay) could lead to an overestimation of the air change rate in zone. Use of the gradient for the dominant period (i.e. later part of the decay) could result in an underestimation. Table 2 shows a comparison of different analysis methods using the ill-conditioned SF_6 decay curve. From this table, it is clear that there is a wide range in the calculated values of flow rates. The large variation in flow rates has arisen due to the way in which each analysis treats the data and the approximations involved. Since the real values of flow rates are unknown it is not possible to establish the errors for ill conditioned data. This problem could be investigated by the use of known air flow rates in well sealed houses, such as superinsulated houses, which provide an independent check on air flow rates.

5 Conclusions

A number of analysis methods have been examined. The method adopted by Littler was found to be the most reliable for estimating air flow rates for well conditioned measurements. For ill conditioned data, the estimated values of air flow rates were spread over a wide range and it was not possible to establish the errors in these values as the real flow rates were known. Errors in the ill conditioned measurements could be reduced by improving both sampling and mixing methods.

References

1 Lagus P and Persily A K A Review of Tracer-Gas Techniques for Measuring Air Flow in Buildings *ASHRAE Trans.* **91**(2) 1075–1087 (1985)

2 Irwin C and Edwards R E The Validation of a Multiple Tracer Gas Technique for the Determination of Airflows Between Three Interconnected Cells *Air Infiltration Rev.* 7 (4) (1986)

3 Sinden F W Multi-Chamber Theory of Air Infiltration *Build. Environ.* **13** 21–28 (1978)

4 Penman J M and Rashid A A M Experimental Determination of Air-Flow in a Naturally Ventilated Room Using Metabolic Carbon Dioxide *Build. Environ.* **17**(6) 253–256 (1982)

5 Perera M and Walker R R Measurement of Interzonal Airflows in a Large Building Using Multiple Tracer Gases *Proc. Int. Seminar Energy Saving in Buildings, The Hague* (14–16 Nov 1983)

6 Wortman D N and Burch J *System Analysis on Two-Tracer-Gas, Two-Zone Airflow Measurements* Solar Energy Research Institute Report (1984)

7 Littler J, Martin C and Prior J *Deducing Interzonal Air Flows from Multi-Zone Gas Measurements* Research in Building Report 84/718/9 (1984)

8 I'Anson S J, Irwin C and Howarth A T Air Flow Measurements Using Three Tracer Gases *Build. Environ.* **17**(4) 245–252 (1982)

9 Hildebrand F B *Introduction to Numerical Analysis* 2nd edn (New York: McGraw-Hill) (1974)

10 Riffat S B, Eid M and Littler J Developments in a Multi-Tracer Gas System and Measurements Using a Portable SF_6 System *Proc. 8th AIVC Conf. FRG* (1987)

11 Riffat S B and Eid M Measurement of Air Flow Between the Floors of Houses Using a Portable SF_6 System *Energy and Building* **12** 67–75 (1988)