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Airflow Network Models for Element-Based Building Airflow Modeling

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ABSTRACT

In spite of its importance, the analysis of airflows has significantly lagged behind the modeling of other building features because of limited data, computational difficulties, and incompatible methods for analyzing different flows. Methods have been developed to analyze airflows in HVAC ducts and to estimate infiltration, but the interaction between building HVAC systems and infiltration airflows has seldom been studied. This paper emphasizes the numerical aspects of an airflow network method that would provide a unified approach to building airflow calculations. It also discusses the limitations of the method and poorly understood factors that could profit from further research.

INTRODUCTION

Air movement models have been developed for estimating airflows in buildings. These airflows include infiltration, natural ventilation, interroom airflows through various openings including doorways, and flows through the HVAC system. The numerical estimation of average characteristics of such airflows is useful for moisture and contaminant dispersal analysis, including the design of smoke control systems, and heat transfer analysis including load and energy calculations. In spite of its importance, the analysis of airflows has significantly lagged behind the modeling of other building features because of limited data, computational difficulties, and incompatible methods for analyzing different flows. This is particularly true of the combined building and plant simulation. Methods have been developed to analyze airflows in ducts (ASHRAE 1985) and to estimate infiltration (Liddament and Thompson 1982) and ventilation (ASHRAE 1985), but the intimate relationship between these processes has seldom been studied. When it has, the results have sometimes been surprising (Persily 1985).

Relatively few methods that could be applied to both processes have been developed within the building simulation community and described in detail. Several computer models developed for smoke control analysis are reviewed by Said (1988). Models for building energy analysis have been developed by Clarke (1985) and Walton (1984). All of these methods are based upon the idea that there is a simple nonlinear relationship between the flow through an opening and the relative air pressure difference across it, and that a building can be considered to be composed of a large number of rooms which are connected by openings to each other and to the outside. This is a network of rooms (nodes) and openings (connections) that is conceptually similar to the air-handling system network where the connections are the ductwork and the nodes are the joining points. Conservation of mass for the flows into and out of each node leads to a set of simultaneous nonlinear equations that are solved iteratively for the airflows. This can be called an "airflow network" method. Its relationship to pipe network methods will be discussed. Such an analysis is also sometimes referred to as a multi-chamber or multi-cell method (ASHRAE 1985). This work draws extensively on Axley's airflow element (1987) and contaminant element (1988) methods, which are, in turn, based on numerical methods associated with finite element modeling techniques.

Modeling of airflows requires: (1) determination of the location and mathematical characterization of the airflow paths, (2) determination of the boundary conditions (primarily wind pressure), (3) calculation of the resulting airflows, and (4) a user-friendly framework in which to do the analysis. Progress has been made in such vital areas as wind pressure estimates (Swami and Chandra 1988) and interroom airflows (Barakat 1987). Unfortunately, it is often thought that a network model is so complex that it requires a mainframe computer for its solution (ASHRAE 1985) and is, therefore, impractical. This apparent impracticality discourages the gathering of data which are necessary for the use of network models.

This paper will emphasize the numerical aspects of the airflow network method, which would allow it to provide a practical, unified approach to building airflow calculations. Some details of the program AIRNET, a microcomputer implementation of this airflow network method, will be discussed. It will also discuss the limitations of the method and poorly understood factors that could profit from further research.

AN AIRFLOW NETWORK METHOD

An airflow network consists basically of a set of nodes connected by airflow elements. The nodes may represent rooms, connection points in ductwork, or the ambient environment. The airflow elements correspond to discrete airflow passages such as doorways, construction cracks, ducts, and fans. Figure 1 is a sketch of a portion of a build-

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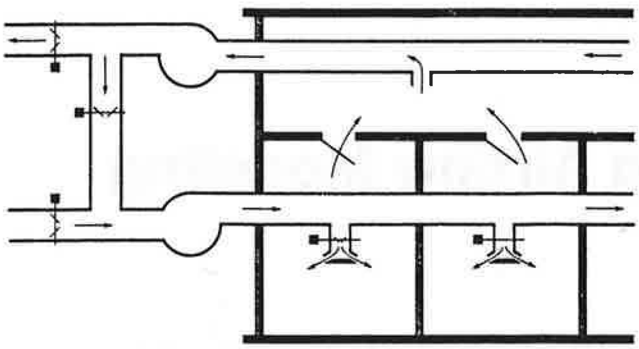

Figure 1 Plan of a portion of a building served by a VAV system

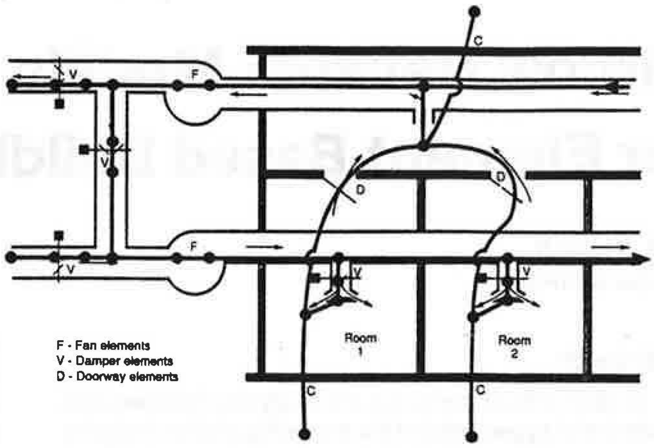

Figure 2 Airflow network for VAV system of Figure 1

ing consisting of two rooms, a hallway, and air distribution equipment representing a VAV system. Figure 2 shows an airflow network superimposed on the physical structure of Figure 1. The large dots are nodes and the connecting lines are the various airflow elements.

Modular Approach

The network approach makes the development of element models, excitation models, and solution methods somewhat independent. The computer program modules will obviously mirror the theoretical, with input and output modules added to create a useful simulation tool. The various models provide a tool kit for the analyst to consider a practically infinite variety of system models.

For this study, an airflow network simulation computer program, AIRNET, was developed from an earlier airflow analysis program (Walton 1984). A more complete description of the AIRNET program is available (Walton 1989). The new program contains:

(1) a process for establishing an initial set of values to start the iterative solution process,

(2) a solution method for nonlinear equations consisting of a traditional Newton's method combined with Steffensen iteration to accelerate convergence,

(3) airflow element subroutines which compute the flow rate and derivative of the flow with respect to pressure difference needed to form the Jacobian matrix used in Newton's method,

(4) a separate process for transferring the above data into the Jacobian matrix (called the element assembly process), and

(5) a solution of the simultaneous linear equations involving the Jacobian matrix.

This discussion will begin with the solution method.

Newton's Method

Each airflow element, i, relates the mass flow rate, w_i, through the element to the pressure drop, ΔP_i, across it. Conservation of mass at each node is equivalent to the mathematical statement that the sum of the mass flows equals zero at each node. The flows are related nonlinearly to the pressures at the nodes, thus requiring the iterative solution of a set of nonlinear equations. In Newton's method (Conte and de Boor 1972), a new estimate of the vector of all node pressures, $\{P\}^*$, is computed from the current estimate of pressures, $\{P\}$, by

$$\{P\}^* = \{P\} - \{C\} \tag{1}$$

where the correction vector, $\{C\}$, is computed by the matrix relationship

$$[J]\ \{C\} = \{B\} \tag{2}$$

$\{B\}$ is a column vector with each element given by

$$B_n = \sum_i w_i \tag{3}$$

where n is the node number and i indicates all flow paths connecting node n to other nodes, and [J] is the square (i.e., N by N for a network of N nodes) Jacobian matrix whose elements are given by

$$J_{n,m} = \sum_i \frac{\partial w_i}{\partial P_m} \tag{4}$$

In Equations 3 and 4, w_i and $\partial w_i/\partial P_m$ are evaluated using the current estimate of pressure $\{P\}$. The AIRNET program contains subroutines for each airflow element which return the mass flow rates and the partial derivative values for a given pressure difference input.

Solution of the Equations

Equation 2 represents a set of linear equations that must be set up and solved for each iteration until a convergent solution of the set of nonlinear equations is achieved. In its full form [J] requires computer memory for N^2 values, and a standard Gauss elimination solution has execution time proportional to N^3. Sparse matrix methods can be used to reduce both the storage and execution time requirements.

A skyline solution process following the method of Dhatt (1984) was chosen. This method can be used to solve equations with symmetric or nonsymmetric matrices. It stores no zero values above the highest nonzero element in the columns above the diagonal and no zero values to the left of the first nonzero value in each row below the diagonal. Analysis of the element models will show that

$$|J_{n,n}| = \sum_{m \neq n} |J_{n,m}| \tag{5}$$

This condition allows a solution without pivoting, although scaling may be useful. Modularizing the equation

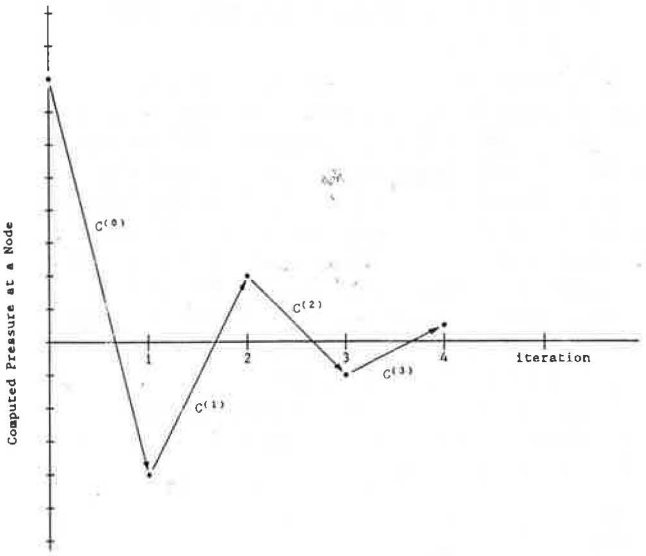

Figure 3 *Oscillating pressure corrections*

solution process and the matrix assembly process will make it easy to substitute other solution processes.

Note that the degree of sparsity of Jacobian matrix is dependent on the ordering of the nodes. Ordering can be improved by various algorithms or rules-of-thumb. Also note that it is easy to define an airflow network which has no unique solution. One requirement for solution is that at least one of the node pressures be known. This is usually the ambient node. All nodes must be linked, through some path, to a known pressure. There may be several known pressure nodes. The airflow network method allows two types of nodes: those with known or unknown pressures. In AIRNET, the constant pressure nodes are included in the system of equations and Equation 2 is processed so as not to change those node pressures. The form of the equations for known pressure nodes, combined with the condition in Equation 5 for unknown pressure nodes, is a sufficient condition for the Jacobian to be nonsingular (Axley 1987). AIRNET is presently set up so that the ambient node pressure is zero, causing the computed node pressures to be values relative to the true ambient pressure. This helps maintain numerical significance in calculating ΔP.

Convergence Criteria

Conservation of mass at each node provides the convergence criterion. That is, if $\Sigma w_i = 0$ for all nodes for the current system pressure estimate, the solution has converged. Many iterations can be saved and sufficient accuracy attained by testing for relative convergence at each node

$$|\Sigma w_i| / \Sigma|w_i| < \epsilon \tag{6}$$

with a test to prevent division by zero. The magnitude of ϵ can be established by considering the use of the calculated airflows, such as in an energy balance.

Convergence Acceleration

Numerical tests of Newton's method solution indicated occasional instances of very slow convergence, always with oscillating corrections on successive iterations. This is depicted graphically for the successive values of pressure at a single node in Figure 3. In the case shown, each successive pressure correction is a constant ratio of the previous correction. The observed corrections come close to this pattern. By assuming a constant ratio, it is simple to extrapolate the corrections to an assumed solution:

$$P_n^* = P_n - C_n/(1-r) \tag{7}$$

where r is the ratio of C_n for the current iteration to its value for the previous iteration. This extrapolated value of node pressure is used in the next Newton iteration. At every other iteration, there are two pressure correction values that may be used for an extrapolation. This method is similar to a Steffensen iteration (Conte and de Boor 1972), which is used with a fixedpoint iteration method for individual nonlinear equations.

The oscillating corrections have been observed by other investigators (Wood 1981; Demuren 1986). Demuren uses a constant relaxation factor of 0.5 to prevent the oscillations. The iteration correction method presented in Equation 7 gives a variable factor. When the solution is close to convergence, Newton's method iterations converge quadratically. By limiting the application of Equation 7 to cases where r is less than some value such as −0.5, it will not interfere with the rapid convergence. Tests by the author confirm that this is faster than the constant relaxation factor. It has not been proven that Equation 7 will always lead to convergence, but it can be shown that it will not prevent convergence. Newton's method converges when the estimated solution values are within some distance, called the radius of convergence, of the correct solution. Applying Equation 7 when $-1 < r < 0$ will cause a smaller correction than Newton's method, which, therefore, cannot force the iterations outside the radius of convergence.

The meanings of other values of r are also interesting. When $r < -1$, the solution diverges in an oscillatory fashion. When $r > 1$, the solution also diverges, but in a nonoscillatory manner. For $0 < r < 1$, the solution is approached from one direction. In all three cases, Equation 7 applies as long as r is truly constant over several iterations. However, for the last case, this involves a true extrapolation of the correction factor, which is very sensitive to the accuracy of r. This is most extreme for the case of $r = 1$, which would cause an infinite correction.

Linear Initialization

Newton's method requires an initial set of values for the node pressures. These may be obtained by including in each airflow element model a linear approximation relating the flow to the pressure drop:

$$w_i = c_i + b_i \cdot \Delta P \tag{8}$$

Conservation of mass at each node leads to a set of linear equations of the form

$$[A]\ \{P\} = \{B\} \tag{9}$$

The coefficient matrix [A] in Equation 9 has the same sparsity pattern as [J] in Equation 2, allowing use of the same sparse matrix solution process for both equations. This initialization handles stack effects very well and tends to establish the proper directions for the flows. The linear approximation is conveniently provided by the laminar regime of the element models described below, but it also may be provided by a secant approximation to the actual nonlinear behavior.

The initialization has been made optional in AIRNET. When solving a set of similar problems, such as when the node temperatures or wind pressures are changed by small amounts, it may be preferable to use the previous solution for the node pressures as the initial values for the new problem.

ELEMENT MODELS

Flow within each airflow element is assumed to be governed by Bernoulli's equation:

$$\Delta P = (P_1 + \rho V_1^2/2) - (P_2 + \rho V_2^2/2) + \rho g(z_1 - z_2) \quad (10)$$

where

ΔP = total pressure drop between points 1 and 2
P_1, P_2 = entry and exit static pressures
V_1, V_2 = entry and exit velocities
ρ = fluid density
g = acceleration of gravity (9.81 m/s²)
z_1, z_2 = entry and exit elevations

The following parameters apply to the nodes: pressure, temperature (to compute density and viscosity), and height. The node height values are used to determine stack effect pressures. When the node represents a room, the airflow elements may connect with the room at other than its reference height. An earlier paper (Walton 1982) shows how to use the hydrostatic equation to relate the pressure difference across a flow element to the heights of the element ends and the node heights, assuming the air in the room is at constant temperature. Pressure terms can be rearranged and a possible wind pressure for building envelope openings added to give

$$\Delta P = P_n - P_m + PS + PW \quad (11)$$

where

P_n, P_m = total pressures at nodes n and m
PS = pressure difference due to density and height differences
PW = pressure difference due to wind

Equation 11 establishes a sign convention for direction of flow: positive is from node n to node m. Since the airflow elements will be described by a relationship of the form w = f(ΔP), the partial derivative needed for [J] in Equation 4 is related by $\partial w/\partial P_m = -\partial w/\partial P_n$, which establishes the relation in Equation 5.

Powerlaw Flow Resistances

Most infiltration models are based on the following empirical (powerlaw) relationship between the flow and the pressure difference across a crack or opening in the building shell:

$$w_i = C \sqrt{\rho} \, (\Delta P_i)^x \quad (12)$$

where

w_i = mass flow rate of air through element i
C = flow coefficient
ρ = air density
ΔP_i = total pressure loss across the element
x = flow exponent

Theoretically, the value of the flow exponent should lie between 0.5 and 1.0. Large openings are characterized by values very close to 0.5, while values near 0.65 have been found for very small openings. Equation 12 should be considered a correlation rather than a physical law. It can be used with the element leakage area formulation which has been used to characterize openings for infiltration calculations (ASHRAE 1985). It can also be used to describe flows through ducts to an accuracy of about 2% over a range of flow rates that vary by a factor of four. Such a variation would be found in a VAV system.

The primary advantage of Equation 12 for describing airflow elements is the simple calculation of the derivatives needed for Newton's method:

$$\frac{\partial w_i}{\partial P_n} = x \, w_i \, / \, \Delta P_i \quad (13)$$

However, there is also a problem with Equation 13: the derivative becomes undefined as the pressure drop (and the flow) go to zero. A simple way to avoid this problem is suggested by what physically happens at low flow rates: the physical character of the flow (and the form of the equation) changes. It goes from turbulent to laminar. Equation 12 is replaced by

$$w_i = K \, \rho/\mu \, \Delta P_i \quad (14)$$

where

K = constant
and μ = viscosity

The partial derivative is a simple constant. The origin of this relationship is shown in the next section. This technique has been independently discovered and used by several researchers (Axley 1987; Isaacs 1980).

The subroutine in AIRNET for powerlaw resistance elements calculates flows using both the laminar and the turbulent models, Equations 12 and 14, and selects the method giving the smaller magnitude flow. There is a discontinuity in the derivative of the w(ΔP) curve where the two equations intersect. This discontinuity is a violation of one of the sufficient conditions for convergence of Newton's method (Conte and de Boor 1972). However, numerical tests conducted by the author for flows at that point using a small airflow network have shown no convergence problem.

Ducts

The theory of flows in ducts (and pipes) is well established and summarized in the ASHRAE Handbook of Fundamentals (ASHRAE 1985). More extensive treatment is given by Blevins (1984) in a long chapter on pipe and duct flow. Analysis is based on Bernoulli's equation and its assumptions. The friction losses in a section of duct or pipe are given by

$$\Delta P = f \cdot L/D \cdot \rho V^2/2 \quad (15)$$

where

f = friction factor
L = duct length
D = hydraulic diameter

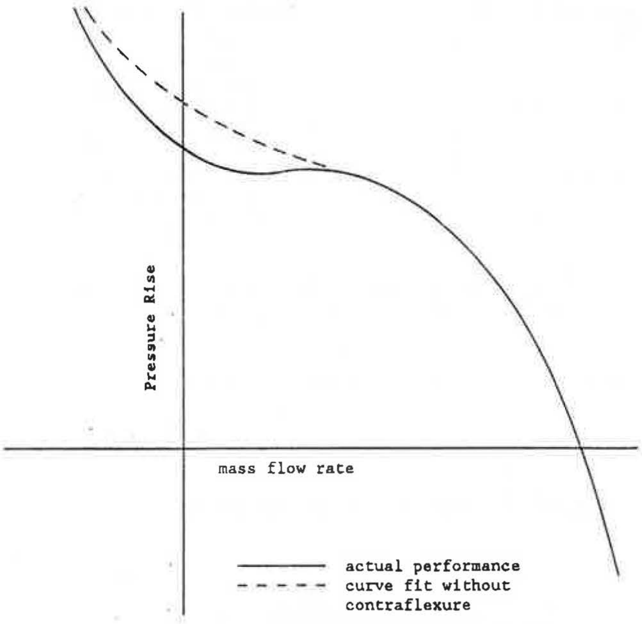

Figure 4 *Typical fan performance curve*

Since $V = w/\rho A$, where A is the cross-sectional area,

$$w = [2\rho A^2 \Delta P \;/\; (fL/D)]^{½} \quad (16)$$

The friction factor can be computed from the Colebrook equation (ASHRAE 1985), which applies in the fully turbulent flow regime above a Reynolds number of about 4000. Dynamic losses can be added to Equation 16 without changing the essential character of the equation.

When the Reynolds number is below about 2000, the flow is laminar with the laminar friction loss described by

$$\Delta P = (k/Re)\cdot(L/D)\cdot(\rho V^2/2)$$

$$= (\mu/\rho)\cdot(kL/2AD^2)\cdot w \quad (17)$$

where

k = laminar friction factor

The derivative is constant. Adding laminar dynamic losses leads to a derivative that is still finite. Although there is physical reason for using Equation 17 at low pressure drops, its real purpose is to ensure convergence of the equations when ΔP approaches zero for one of the many flow paths in a complex network, instead of accurately representing airflows that are too small to be of interest.

Fans

The theory of flows induced by fans is summarized in the ASHRAE Equipment Handbook (ASHRAE 1983). More extensive treatment is given by Osborne (1977). Fan performance is normally characterized by a performance curve. such as that shown in Figure 4. This curve relates the total pressure *rise* to the flow rate for a given fan speed and air density.

The fan performance curve is well represented by one or more cubic polynomials:

$$P = a_0 + a_1 w + a_2 w^2 + a_3 w^3 \quad (18)$$

Multiple polynomials might conveniently be obtained by a cubic spline fit to the performance data. There are two important factors to note on the shape of the fan performance curve. First, it is described by a relationship of the form P(w) instead of w(P), which would be more appropriate for the calculation of flow and partial derivatives. The basic shape of the performance curve cannot be well represented by a simple polynomial with P as the independent variable. Equation 18 requires an iterative solution to determine the flow rate. A modified false-position method (Conte and de Boor 1972) works quickly and reliably. Fortunately, the partial derivative $\partial w/\partial P$ is simply the inverse of $\partial P/\partial w$, which is a simple expression for a polynomial.

Second, it is common for the performance curve to contain points of contraflecture, with up to three different flow rates possible at certain values of fan pressure. This causes difficulty in solving for the flow rate and, more importantly, has points where the derivative goes to infinity. However, it is usually not recommended that the fan operate in the region of the contraflecture points. Therefore, the fan can be modeled with a performance curve that does not include the contraflecture as long as the user checks that the air distribution system does not permit operation in that region.

The performance of a given fan at various speeds and air densities can be related to a single fan performance curve through the "fan laws":

$$w_1 \;/\; w_2 = (N_1\rho_1) \;/\; (N_2\rho_2) \quad (19)$$

and

$$P_1 \;/\; P_2 = (N_1{}^2\rho_1) \;/\; (N_2{}^2\rho_2) \quad (20)$$

where

w = volume flow rate
P = total pressure rise
N = rotational speed
ρ = density

These laws are valid if all flow conditions at the two speeds are similar. In particular, they will not apply at very low flows where fully turbulent conditions have not been developed.

Numerical tests with AIRNET for flows at the laminar-turbulent transition indicate some convergence difficulty: about twice as many iterations as usual are needed for convergence. In one case the iterations showed potential divergence with $r < -1$, but the convergence acceleration algorithm saved the cases tested and produced a solution.

Doorways

Flows through large openings (e.g., doorways) tend to be more complex, with the possibility of flows in opposite directions in different parts of the opening. The temperature and resulting density differences between two rooms may mean that the stack effect causes a positive pressure difference at the top of the doorway and a negative pressure difference at the bottom (or vice versa). A summary of research on heat transfer through doorways is presented by Barakat (1987). Most research has attempted to develop dimensionless correlations (using Nusselt, Prandtl, and Grashof numbers) of the form

$$Nu_D/Pr = C\cdot Gr_D{}^b \quad (21)$$

where b is approximately 0.5 and C lies between 0.22 and

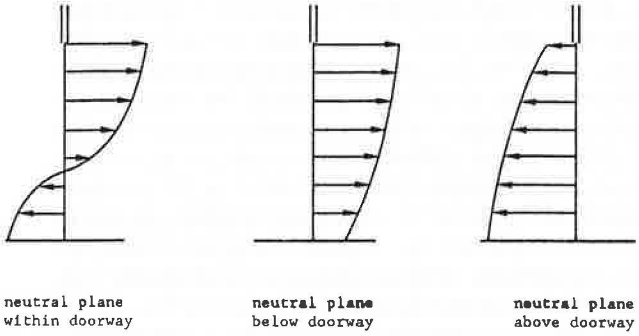

Figure 5 *Three doorway flow patterns*

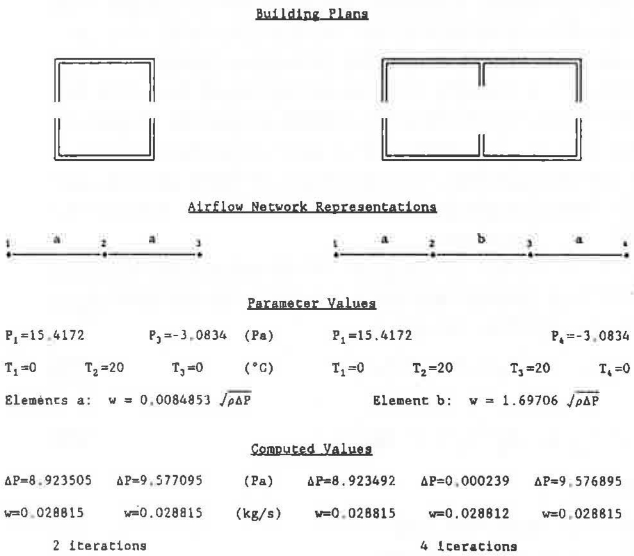

Figure 6 *Powerlaw airflow elements in series*

0.33, depending on the temperature difference used for the correlation. It has been shown that such a heat transfer is equivalent to an airflow that can be modeled by powerlaw elements (Walton 1982) by dividing the total opening into several smaller openings having the same total area but configured to properly account for the magnitude and direction of airflows at different heights in the opening.

An alternative approach is to create a single airflow element that accounts for the flow over the entire opening. A simple theory that estimates the stack-induced airflow through a large opening in a vertical partition is given by Brown and Solvason (1962). The volumetric flow through an infinitesimal area across the opening is given by

$$dQ = C\ W\ dz \sqrt{2\ \Delta P/\rho} \qquad (22)$$

where

C = discharge coefficient
W = width of opening
ΔP = pressure difference across the opening at the given height
dz = layer infinitesimal height

Equation 22 can be integrated from the neutral plane (the height at which ΔP equals zero) to the top and bottom of the door to determine the total airflow. Figure 5 shows

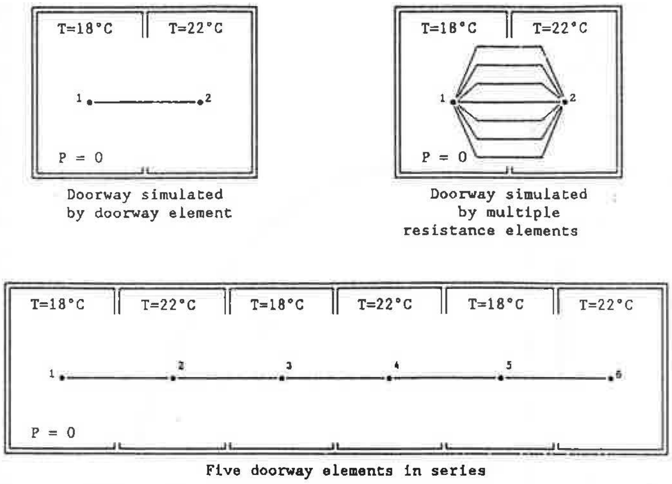

Figure 7 *Three airflow networks for doorways*

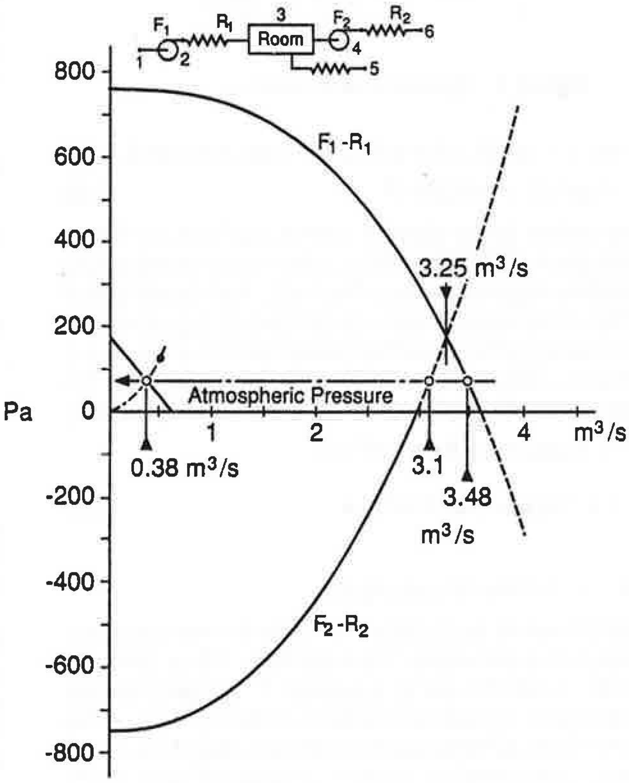

Figure 8 *Fans in parallel test case*

three different flow conditions that must be handled by the integration: (1) neutral plane within the doorway, (2) neutral plane below the doorway, and (3) neutral plane above the doorway. The height of the neutral plane is shifted by flows entering the two rooms through other openings.

This model of a doorway tends to be faster than the multiple opening approach. However, it also complicates the assembly process for the Jacobian matrix because one or two flows may exist. More importantly, development of the doorway element model requires knowledge of the vertical temperature profile used in the node model (here assumed to be constant) in order to compute the pressure difference as a function of height across the opening. This

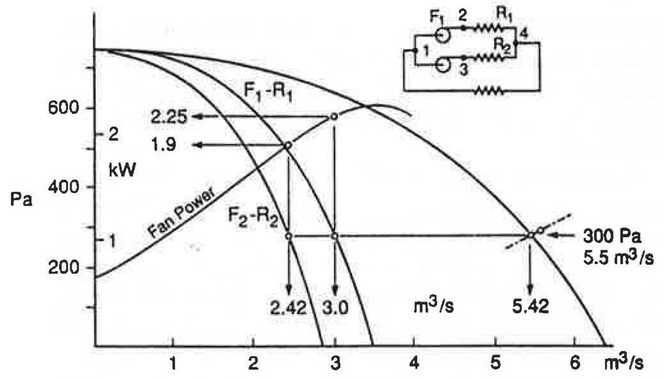

Figure 9 *Fans in series test case*

requirement compromises the independence of the modularity of airflow network program.

SAMPLE CALCULATIONS

Several simple airflow networks have been analyzed to demonstrate the procedure described in the previous sections.

Simple Test Cases

Figure 6 shows two cases involving powerlaw resistances in series. The first case, consisting of three nodes and two flow resistances, can be considered as modeling a room with small 0.01 m² openings on opposite sides with wind pressure driving flow through the room. The second case divides the single room into two rooms with a partition containing a large 2.00 m² opening. This case, with a very low resistance (large opening) mixed with large resistances (small openings), is difficult to solve with some methods (Walton 1982; Clarke 1985). In both cases AIRNET required only two iterations and computed the expected nearly identical flows.

Figure 7 shows three cases involving doorway elements. In the first case, a 0.8 m by 2.0 m doorway connects two rooms with a 4°C temperature difference. The computed two-way airflow is 0.259 kg/s. In the second case, 10 0.16 m² resistance openings at different heights are used to represent a doorway. The computed two-way airflow is 0.261 kg/s. The third case represents six rooms in series connected by doorway elements. The computed flows are identical to the first case. All three cases were solved in two iterations.

Figure 8 shows a test involving two fans in parallel. This is a problem in the textbook by Osborne (1977). The computed pressures agree with the text to within 2 Pa (about 0.5%) and the flows to within 1.5%. These differences are probably due to the inaccuracies in the polynomial fit for the fan performance curve and in the graphical solution used in the text.

Figure 9 shows a test with two fans in series (Osborne 1977). The computed room pressure differs from the text value by 2.5 Pa and the airflows differ by less than 1%.

Figure 10 shows a 36-room airflow network created to test execution time for a larger network. This test case describes a four-story building with six rooms, a hallway, an elevator shaft, a stairwell, and a node representing ambient on each floor. The nodes representing the elevator

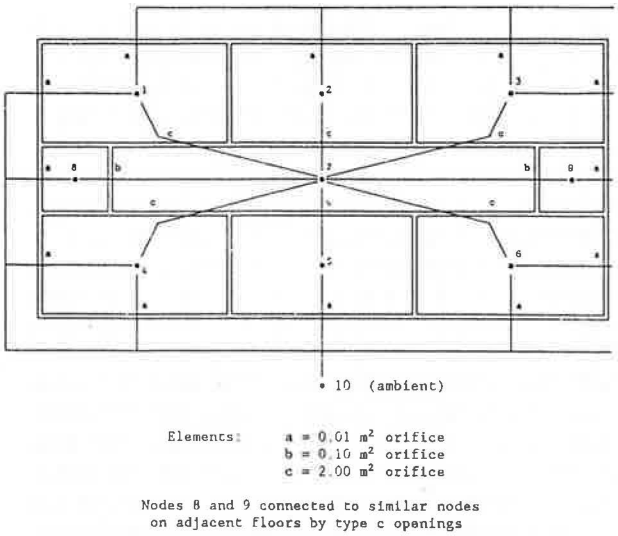

Figure 10 *One floor of the four-floor, 36-room timing test case*

shaft and stairwell on each floor are connected by very large (2.0 m²) openings. Similar openings connect each room to the hallways. Very small (0.01 m²) openings connect the building nodes to the outside. Intermediate size openings (0.1 m²) connect the large vertical shafts to the hallways. This case was solved, with a 0.01% convergence criterion, in five iterations and 2.89 seconds on a PC-compatible computer (4.77 MHz 8088 CPU with 8087 math coprocessor).

Comparison of Methods

The ESP building thermal simulation program (ABACUS 1986) includes a separate program, ESPAIR, for calculating airflows. ESPAIR was compared to the AIRNET program. Both programs were recompiled and run on a workstation computer using the 36-room test case. ESPAIR solved this case using default 5% convergence in about 8800 iterations requiring a total of 150 seconds. AIRNET solved it using the default 0.01% convergence in five iterations requiring 0.16 seconds, or about 1000 times faster.

This extreme difference in calculation times occurs partially because of the difficulty the ESPAIR algorithm has with large openings (Clarke 1985). Limiting all the openings to an area of 0.01 m² allowed ESPAIR to reach a solution in only 137 iterations and 2.10 seconds. AIRNET was also somewhat faster for this case: two iterations and 0.06 seconds, or about 35 times faster than ESPAIR. Greater accuracy in the ESPAIR solution (0.5% convergence) required more iterations (22,000) and more time (400 seconds). The ESPAIR results may explain why airflow network calculations have a reputation for being slow.

DIRECTIONS FOR FUTURE RESEARCH

Alternate Solution Methods

Although the simple tests of the AIRNET program and its comparison to ESPAIR look very promising, some important questions remain. The most important question concerns the reliability of the method for solving the airflow network equations. Solution of the nonlinear equations has been demonstrated in several tests but has not been mathematically proven. The literature for the solution of similar equations may be helpful. The airflow network is very similar to a pipe network with the flow resistance of openings and ducts corresponding to the resistance of pipes and fans corresponding to pumps.

Much of the theory for computing fluid flows in pipe networks is described by Jeppson (1976). The basic flow phenomena are nonlinear and must be described by a set of nonlinear algebraic equations. These equations may be expressed in terms of the unknown flows in the pipes (referred to as loop equations) or the unknown heads at the junctions (node equations). The equations are derived from a form of Kirchoff's circuit laws: (1) the sum of flows into a junction equals the sum of outward flows, and (2) the total headloss around any loop in the system must be zero. Wood and Rayes (1981) give an excellent comparison of several algorithms. Five methods are described and tested; three are based on the loop equations and two on the node equations. The least reliable methods (those least likely to converge to the correct solution) are the method that adjusts each loop flow individually, the method that adjusts each node head individually, and the method that adjusts the node heads simultaneously.

It is interesting to note that ESPAIR solves the airflow problem with a version of the algorithm that adjusts node head individually. Among the airflow algorithms used in smoke control algorithms, Klote and Fothergill (1983) use individual node head adjustments, while Sander (1974) uses the simultaneous node head adjustment algorithm, both of which are among the least reliable methods, according to Wood and Rayes. The method used in AIRNET also does simultaneous node head adjustment, but it is so different that it should be evaluated separately. It addresses the two problems observed by Wood and Rayes: (1) Large openings (low resistances) give inexact flows because small differences in the computed pressures lead to large differences in the flows. This is solved by stringent requirements on mass balance convergence at each node. Such accuracy is not costly, because Newton's method is quadratically convergent—near the solution each iteration greatly improves the accuracy. (2) Failure of the node adjustment method to converge because of oscillating corrections is handled by the Steffensen iteration applied to the Newton's method correction factors.

The two simultaneous loop methods have a good history of convergence for pipe network problems. On the other hand, they are more difficult to set up than the node methods since independent loops must be defined; they tend to require the solution of more simultaneous equations; the equations do not have the very desirable feature of diagonal dominance; they tend to be less sparse than the node equations; and some airflow elements may be difficult to implement. The doorway model may be difficult because it can have either one or two flows, which may make it especially difficult to define the loops.

Of particular interest to the idea of establishing a general modular program is that the loop methods require the airflow elements to compute pressure drop as a function of flow rate, which is opposite the requirement for the node method. For some of the airflow elements, such as powerlaw resistances, the transformation is simple. Others are described more naturally in one form than the other. For example, the duct and fan models are described more naturally for the loop method. This indicates the need to consider the solution technique in the development of element models.

The work of Wood and Rayes indicates that several apparently reasonable solution methods for the simultaneous nonlinear equations are not very reliable. The ideal solution to the question of reliability would be mathematical proofs of the convergent nature of the solution algorithm and the limitations on the element models. Such proofs may be difficult to achieve for nonlinear systems. Alternatively, extensive tests of different methods would give some confidence as to their reliability.

Other Element Models

The modular structure of AIRNET would allow many more airflow elements to be developed. These elements could provide either new capabilities or more accurate simulation.

Baker (1987) indicates that infiltration openings can be more accurately modeled by a quadratic relationship of the form

$$\Delta P = A\,Q + B\,Q^2 \qquad (23)$$

This form can be used as an airflow element by solving the quadratic equation for w (= ρQ). There is no problem with the derivative at $\Delta P = 0$ as long as A is not equal to zero. It appears possible to represent dampers as variable flow resistance elements in an airflow network. The relationship between resistance and actuator position could be represented by a polynomial.

The flow characteristics of some airflow elements may depend significantly on the direction of flow. In pipe networks, check valves act in such a manner. These could be represented by elements with separate performance curves applied to different pressure drop or flow regimes.

Much more work could be done on the development of the doorway models. Complex flow patterns involving boundary layer flows can exist. These patterns are related to the geometries and surface and air temperature distributions in the adjoining rooms. For example, Hill (1986) uses a model that incorporates nonuniform temperatures in the rooms, which leads to multiple neutral pressure levels in the doorway and compares the computed airflows to measured flows. Here the intimate relation between the doorway element model and the node models is important. The constant temperature node model could be rather easily expanded to three more complex models: (1) temperature varies uniformly with height; (2) two uniform temperature layers in the room; and (3) two layers, each having uniformly varying temperature. It may be necessary to develop several doorway models to account for different types of airflow. Detailed doorway calculations would then involve methods to identify which model to use.

The experimental data base for two-way flows between nodes at different heights (through stairs and elevator shafts) appears insufficient to develop element models. It should be possible to extend the airflow network method to include two- and three-dimensional fluid elements for the detailed modeling of airflows within rooms. Of course, this would greatly increase computation time.

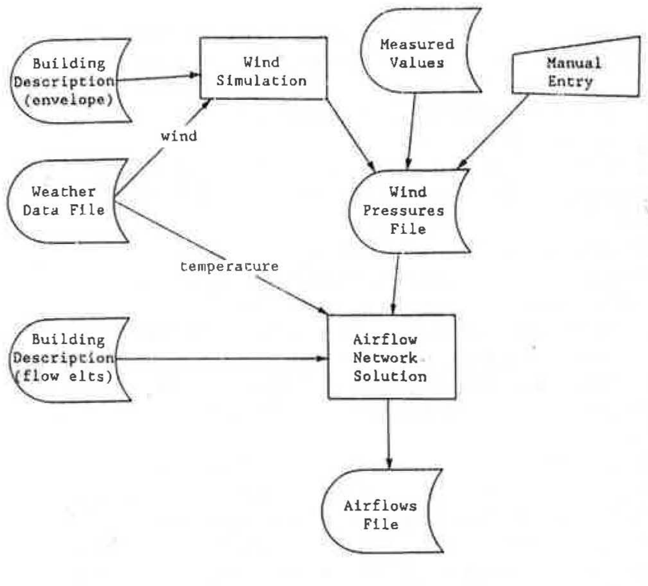

Figure 11 *Proposed structure of airflow network program*

General Limitations

The simple airflow network method outlined above has some inherent limitations. These include the inability to quickly model airflow patterns within a room or to model the transient airflows caused by short-term transients in wind pressure distributions. These effects can possibly be approximated by dividing rooms into several nodes and adding transient flows to the average flows, but the direct modeling of such effects would greatly increase calculation time and would probably be impractical for most engineering analysis. The existence of such known limitations, not to mention unknown factors, makes experimental validation of airflow network calculations essential.

It must be expected that uncertainty in the input parameters will always limit the absolute accuracy of airflow calculations. However, a network model based on physical laws will be useful for evaluating design alternatives because relative changes in flow values should be fairly accurate. Modularity can be used further in the design of an airflow analysis program. Figure 11 shows a structure for such a program similar to that used in ESP (ABACUS 1986). The program separates the evaluation of wind pressures from the airflow calculation to allow alternate inputs: manual entries, measured values, or simulated values. Input of airflow element data would involve a data base of element data. The computed airflows go to an output file, which could be used in either indoor air quality or loads calculations. The entire procedure could be incorporated into an energy analysis program.

SUMMARY AND CONCLUSIONS

This paper has discussed how an airflow network method can be used to provide a unified model of major building airflows. Of particular importance is the idea of modularity. ESP's modularity made the comparison test possible. It is often very difficult to isolate a single computational feature of a monolithic program. AIRNET included modularity of the airflow elements, allowing elements with greatly different flow characteristics to be connected to the core algorithm by a common interface. More airflow elements could be added. The sparse matrix solution of the simultaneous equations involving the Jacobian matrix allows larger systems of equations to be handled without the full execution time penalty of using the complete matrix. By separating the solution and matrix assembly processes, faster solution processes could be easily substituted.

The performance of AIRNET relative to ESPAIR indicates that it is practical to solve the flow network in detail. Solution of complex airflow networks for the steady-state case is practical on current small computers. Solution of the dynamic case for many timesteps is now possible. The use of small computers will make advanced user input features available, which could significantly aid in the airflow analysis process.

Research is still needed in several areas. These include determination of the most reliable airflow network solution method, a mathematical analysis of the network flow equations and the solution method, development of additional airflow elements (especially improved large opening models), experimental validation of the simplifying assumptions in the element models and network method, expansion of the wind pressure and airflow element performance database, and modeling of intraroom effects by simplified methods and by integration with microscopic modeling methods.

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