

# ON THE MULTIPLE TRACER GAS TECHNIQUES FOR MEASURING INTERZONAL AIRFLOWS IN BUILDINGS

M. Enai

C.Y. Shaw

J.T. Reardon



## ABSTRACT

The multiple tracer gas method is often used to predict interzonal airflows in buildings. In this study, this method was applied to an isolated space consisting of two interconnected rooms where the airflows through the common wall were controlled and measured. The results indicated that the calculated airflows based on different sets of simultaneously measured tracer gas concentrations (obtained at different sampling times during a test) were not always the same. To improve the accuracy of calculated results, a method was developed to facilitate the selection of appropriate sets of tracer gas concentrations (from the measurements) for use with the multiple tracer gas method. The proposed method was tested in the laboratory. The results are discussed.

## INTRODUCTION

As concern for indoor air quality has grown, so too has the need to measure interzonal airflows in buildings. These airflows can be evaluated using the multiple tracer gas method (Sinden 1978; Sherman et al. 1980; l'Anson et al. 1982; Perera 1983). It involves the injection of a different tracer gas into each of several interconnected spaces and the measurement of the tracer gas concentrations as a function of time. Based on the measured tracer gas concentrations, the interzonal airflows can then be calculated from the mass conservation equations for each tracer gas and the mass flow balance equations for the air.

The application of the method is not straightforward, even for the simplest case, e.g., a building consisting of two interconnected chambers. This is because the airflows are calculated from simultaneously measured tracer gas concentrations that are subject to errors caused by such factors as inadequate mixing. The technique used to release tracer gases in the test chambers may also affect the calculated results. Therefore, it is advisable to use different injection techniques to produce dissimilar concentration profiles for the calculation.

In this study, some of these problems were examined by applying the multiple tracer gas method to an isolated space consisting of two interconnected rooms

where the airflows between the two rooms were controlled and measured. Three tracer gases were used. Two were injected into the rooms under decay mode and the third gas was released into one of the rooms at a constant rate. The use of three tracer gases enabled a direct comparison between the results based on different tracer gas injection techniques (e.g., constant injection in one room and decay in the other room and decay in both rooms).

Since the interzonal airflows are calculated from the simultaneously measured tracer gas concentrations, and it takes time to achieve adequate mixing, erratic results can be expected if the concentrations are obtained before adequate mixing is achieved. Therefore, a method is proposed to assist the selection of appropriate sets of concentration measurements for interzonal airflow predictions. The main objectives of the study were (1) to determine the appropriate concentration measurements that can be used for interzonal airflow predictions and (2) to determine the accuracy of the multiple tracer gas technique under laboratory conditions.

## GOVERNING EQUATIONS

Figure 1 shows the experimental facility, which consisted of two interconnected rooms (Room 1 and Room 2). If a tracer gas,  $g$ , is injected into Room 1 and another tracer gas,  $s$ , is released in Room 2, the rates of change in tracer gas concentrations in the two rooms can be described by the following equations, assuming that the tracer gas concentrations outside the rooms are negligible:

Room 1

$$V_1(dC_{g1}/dt) = -(F_{10} + F_{12}) \cdot C_{g1} + F_{21} \cdot C_{g2} + Q_g \quad (1)$$

$$V_1(dC_{s1}/dt) = -(F_{10} + F_{12}) \cdot C_{s1} + F_{21} \cdot C_{s2} \quad (2)$$

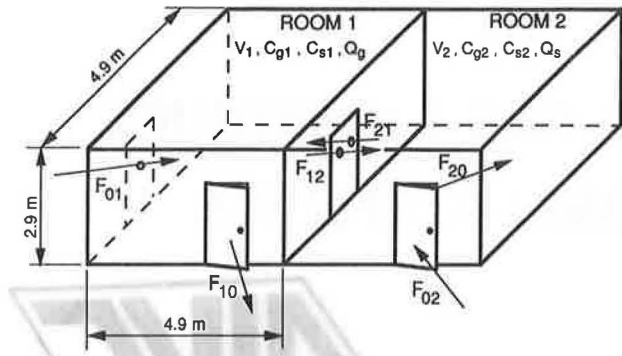
Room 2

$$V_2(dC_{g2}/dt) = -(F_{20} + F_{21}) \cdot C_{g2} + F_{12} \cdot C_{g1} \quad (3)$$

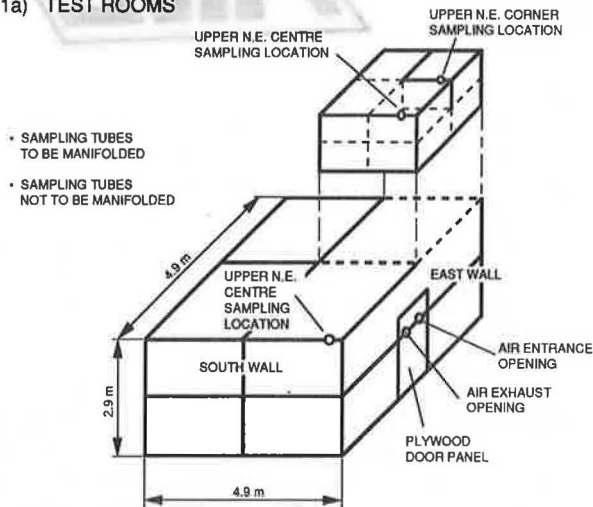
$$V_2(dC_{s2}/dt) = -(F_{20} + F_{21}) \cdot C_{s2} + F_{12} \cdot C_{s1} + Q_s \quad (4)$$

M. Enai is Associate Professor, Architectural Department, Hokkaido University, Japan; C.Y. Shaw and J.T. Reardon are Research Officers, Institute for Research in Construction, National Research Council, Canada.

THIS PREPRINT IS FOR DISCUSSION PURPOSES ONLY, FOR INCLUSION IN ASHRAE TRANSACTIONS 1990, V. 96, Pt. 1. Not to be reprinted in whole or in part without written permission of the American Society of Heating, Refrigerating and Air-Conditioning Engineers, Inc., 1791 Tullie Circle, NE, Atlanta, GA 30329. Opinions, findings, conclusions, or recommendations expressed in this paper are those of the author(s) and do not necessarily reflect the views of ASHRAE.



1a) TEST ROOMS



1b) SAMPLING LOCATIONS

Figure 1 Test rooms and sampling locations

The mass flow balance equations for the two rooms are:

Room 1

$$F_{01} + F_{21} - (F_{10} + F_{12}) = 0 \quad (5)$$

Room 2

$$F_{02} + F_{12} - (F_{20} + F_{21}) = 0 \quad (6)$$

where

$V$  = room volume

$C$  = tracer gas concentration

$t$  = time

$F$  = airflow rate

$Q$  = tracer gas release rate

The subscripts 0, 1, and 2 denote the outside, Room 1, and Room 2, respectively, and  $g$  and  $s$  refer to the tracer gases used.  $F_{10}$  indicates that the air flow is from Room 1 to the outside.

If two tracer gases,  $g$  and  $s$ , are injected into Rooms 1 and 2, respectively, and their concentrations are monitored, the six unknowns,  $F_{10}$ ,  $F_{12}$ ,  $F_{01}$ ,  $F_{20}$ ,  $F_{02}$ , and  $F_{21}$ , can be evaluated from Equations 1 through 6.

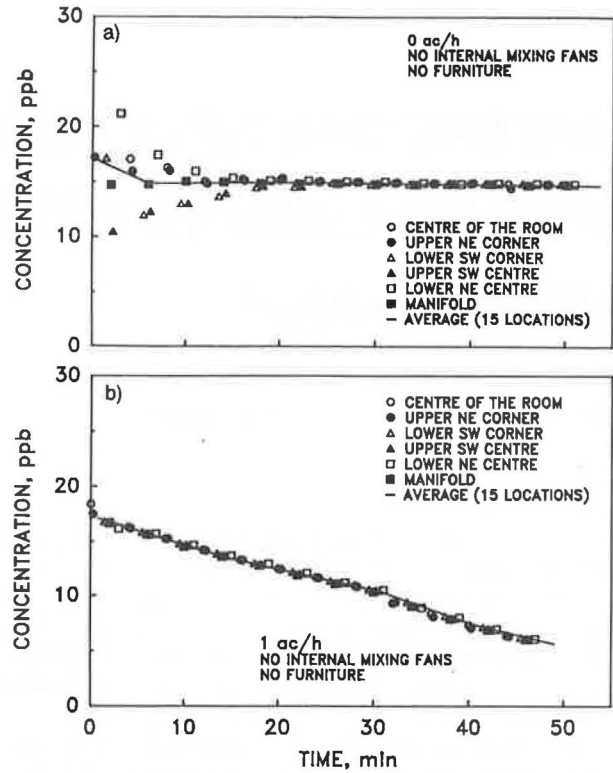


Figure 2  $SF_6$  concentrations at selected locations vs. time

## TEST ROOMS

The test rooms, as shown in Figure 1, were two identical interconnected rooms in a laboratory-office building. Each room was 4.8 m wide by 4.8 m long by 2.87 m high. The walls, doors, and ceilings of the rooms were tightly sealed to minimize air leakage. The doorway connecting the rooms was sealed with a plywood panel in which two airflow systems were installed (one for each flow direction). Each system consisted of a fan, a laminar flow element for measuring the airflow rate, and an airflow controller. Thus, during the tests, the airflows from one room to the other were controlled at constant rates and measured with an accuracy of 1% of the measured values. Similarly, a third airflow system was used to control the airflow from the surrounding area into Room 1. Except for those through the airflow systems, the air leakage rates between the test rooms and their surroundings were not measured. However, these air leakage rates were varied by opening or closing the connecting doors to see their effects on the calculated results. The door openings were approximately 5 cm wide.

The tracer gas injection tube was located at the center of each room. To determine the locations for the sampling stations, each room was divided into eight volumetrically equal regions with a sampling station installed at the center of each region (Figure 1b). Each region was further divided into eight volumetrically equal subregions with one additional sampling station installed at the center of each corner subregion but one to allow for one extra sampling station at the center of the room. Each sampling station consisted of a pair of sampling tubes, one of which was connected to a manifold to produce an "average" sample.

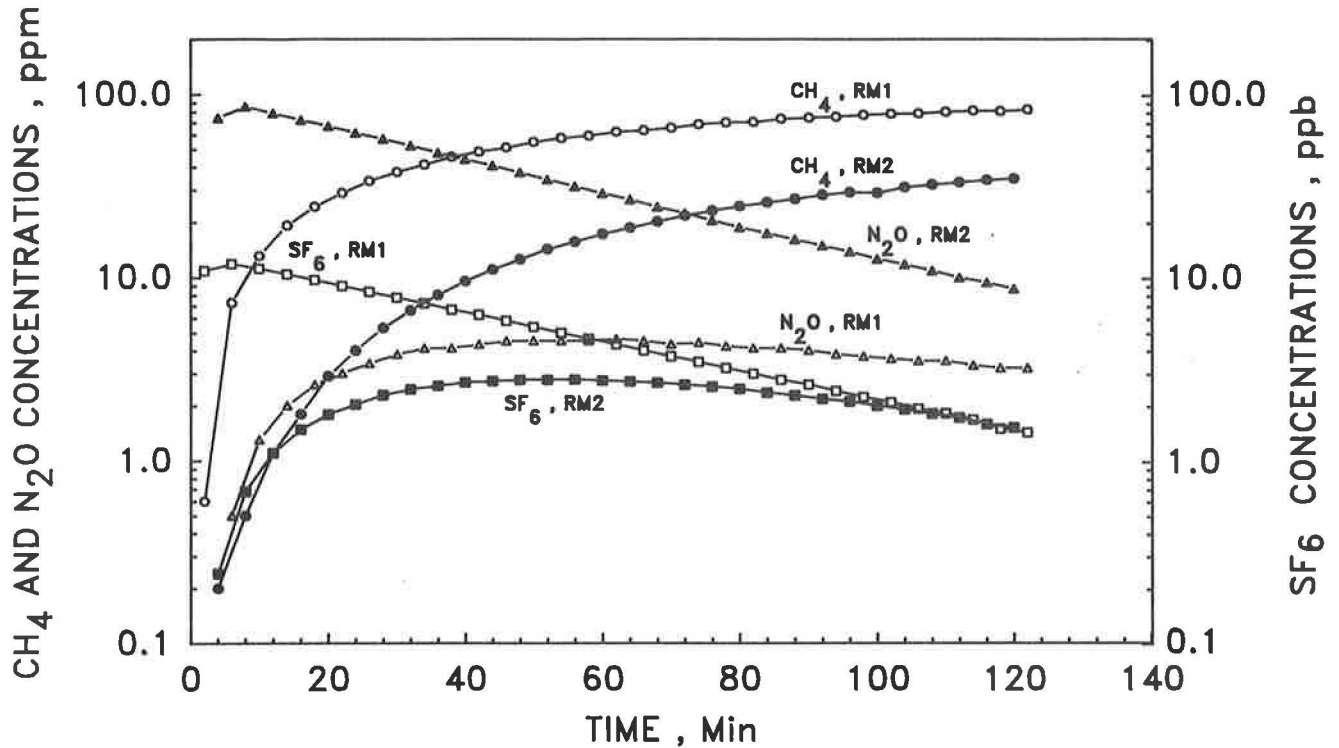


Figure 3 Typical tracer gas concentration profiles

## TEST RESULTS

### Preliminary Tests to Determine Sampling Locations

Prior to the experiment, a series of tests was conducted in Room 1 to check the condition of mixing between tracer gas and indoor air. The results were used to determine the sampling method for subsequent experiments. For these tests, after the supply and exhaust airflow rates for Room 1 were set, a small amount of SF<sub>6</sub> was released at the center. The tracer gas concentrations at the sampling locations and at the manifold were measured and recorded one after another at 15-second intervals beginning immediately after the injection and continuing for about 60 minutes.

TABLE 1  
Test Conditions

Test No.	F <sub>01</sub> ach	F <sub>12</sub> ach	F <sub>21</sub> ach	Door Opening		CH <sub>4</sub> Injection Rate, mL/min
				Rm 1	Rm 2	
101	0.0	1.00	1.00	Closed	Closed	0.0
102	0.0	0.75	0.56	Closed	Closed	0.0
103	0.0	1.00	1.00	Open	Open	0.0
104	0.25	1.00	1.00	Closed	Closed	169.0
105	0.25	1.00	1.00	Closed	Open	113.0
106	1.00	0.25	0.25	Closed	Open	83.8
107	1.00	0.75	0.25	Closed	Open	100.6
108	1.00	0.25	0.50	Closed	Open	96.8
109	1.00	0.25	0.75	Closed	Open	115.0
110	1.00	0.15	0.15	Closed	Open	114.0
111	1.00	0.15	0.50	Closed	Open	114.0
112	1.00	0.15	0.75	Closed	Open	117.0
113	1.00	0.75	0.15	Closed	Open	117.0
114	0.0	0.75	0.56	Closed	Open	115.0
115	0.0	0.75	0.56	Closed	Closed	97.9
116	0.0	1.00	1.00	Closed	Closed	97.8
117	0.0	1.00	1.00	Open	Open	97.1

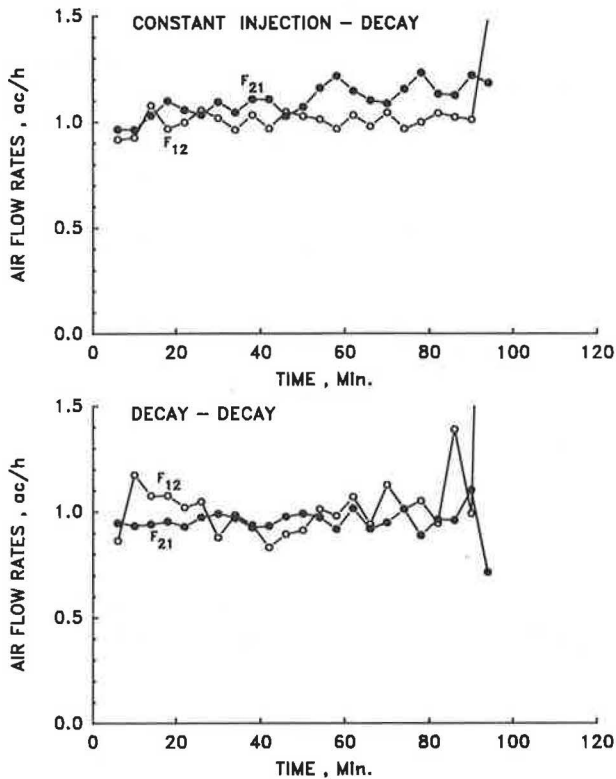
Door open denotes that the doors were open with an opening approximately 5 cm wide.

The results, which have been previously published (Evans and Shaw 1988), are shown in Figure 2. Figures 2a and b show the SF<sub>6</sub> concentrations at selected sampling locations as a function of time for air supply or exhaust rate of 0 and 1 air change per hour (ach), respectively. Also shown is the average concentration of all sampling locations (excluding the manifold). The results indicate that the SF<sub>6</sub> concentrations at various sampling locations approached a uniform value in less than 10 minutes for 1 ach and 30 minutes for 0 ach. The results also indicate that the concentration at the manifold closely represented the average concentration. Based on these tests, all the samples for subsequent multiple tracer gas tests were taken at the manifold.

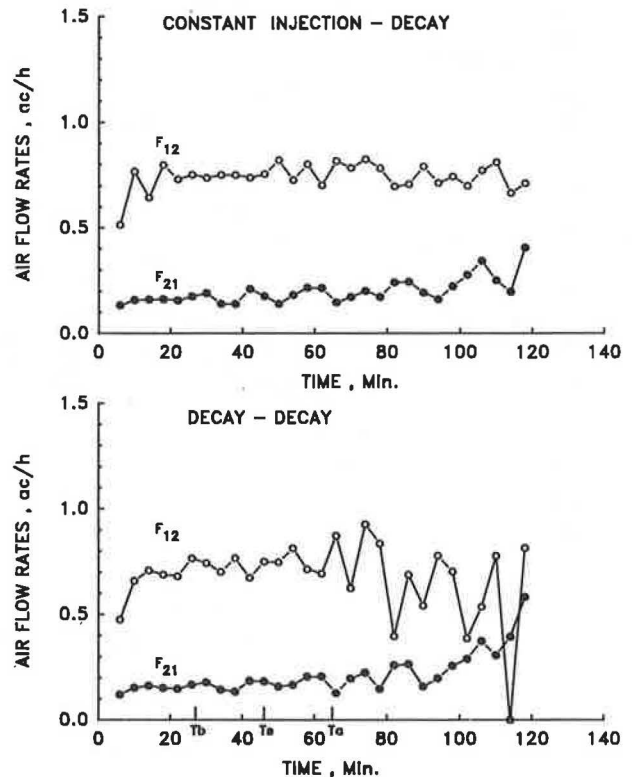
### Multiple Tracer Gas Tests

Seventeen multiple tracer gas tests were conducted. The test conditions are listed in Table 1. Each test began by adjusting the door openings and the airflow rates in the three airflow systems. Then, a small amount of SF<sub>6</sub> was injected into Room 1 and a small amount of N<sub>2</sub>O was injected into Room 2 simultaneously. Also, CH<sub>4</sub> was introduced into Room 1 using constant injection. Immediately after injection, the concentrations of the three gases at the manifolds in each room were measured at two-minute intervals for a period of approximately two hours. The concentrations outside the test rooms were periodically checked to ensure that they were negligible.

Figure 3 shows a typical set of concentration profiles measured in the test rooms. Each set consists of six profiles, one for each tracer gas in each room. From such profiles, the concentrations of CH<sub>4</sub> and N<sub>2</sub>O (representing a constant injection-decay test condition) corresponding to a sampling time six minutes after the



**Figure 4** Calculated interzonal airflow rates.  
Set airflow rates  $F_{12} = F_{21} = 1$  ach.



**Figure 5** Calculated interzonal airflow rates.  
Set airflow rates  $F_{12} = 0.75$ ,  $F_{21} = 0.15$  ach.

injection of the tracer gases were selected and used to calculate  $F_{12}$  and  $F_{21}$  from Equations 1 through 6. This calculation was repeated using sets of concentration values measured at four-minute intervals for about two hours. These calculations were carried out for all 17 different test conditions. Also, similar calculations were performed using the  $SF_6$  and  $N_2O$  concentrations (representing a decay-decay test condition).

Figures 4 and 5 show two examples of the calculated interzonal airflow rates as a function of time: one for  $F_{12} = F_{21} = 1$  ach (Test No. 116), and the other for  $F_{12} = 0.75$  ach and  $F_{21} = 0.15$  ach (Test No. 113). The results indicate that:

1. For the same test, the calculated airflow rates based on different sets of concentration measurements were not always the same.

2. If  $F_{12}$  and  $F_{21}$  were about equal, the calculated airflow rates, based on the concentrations measured between approximately 20 and 70 minutes after injection, agreed with the measurement within 20% of the measured rates (Figure 4).

3. If one of the two flows was much larger than the other (Figure 5), the agreement was better for the larger flow rate than for the smaller flow rate.

These findings suggest that only concentrations selected from a certain portion of the measured profiles can give a good estimate of interzonal airflows. For the case of two interconnected rooms, the recommended concentrations can be selected by the method described in the following paragraphs. An example is included in Appendix A to illustrate how to use the proposed method.

## METHOD FOR SELECTING APPROPRIATE CONCENTRATIONS

Because Equations 1 and 4 are similar, as are Equations 2 and 3, the solutions for the tracer gas concentration profiles (for tracer gas  $g$ ) using Equations 1 and 3 will be similar to those for tracer gas  $s$  using Equations 2 and 4. The basis for determining the appropriate tracer gas concentrations for use in interzonal airflow calculations can be derived by examining a typical pair of tracer gas concentration profiles for a single tracer gas released in one of the two rooms. Such profiles can be obtained analytically from Equations 1 and 3 or Equations 2 and 4.

## Tracer Gas Concentration Profiles and Time Constants

By dropping the subscript  $g$ , Equations 1 and 3 become:

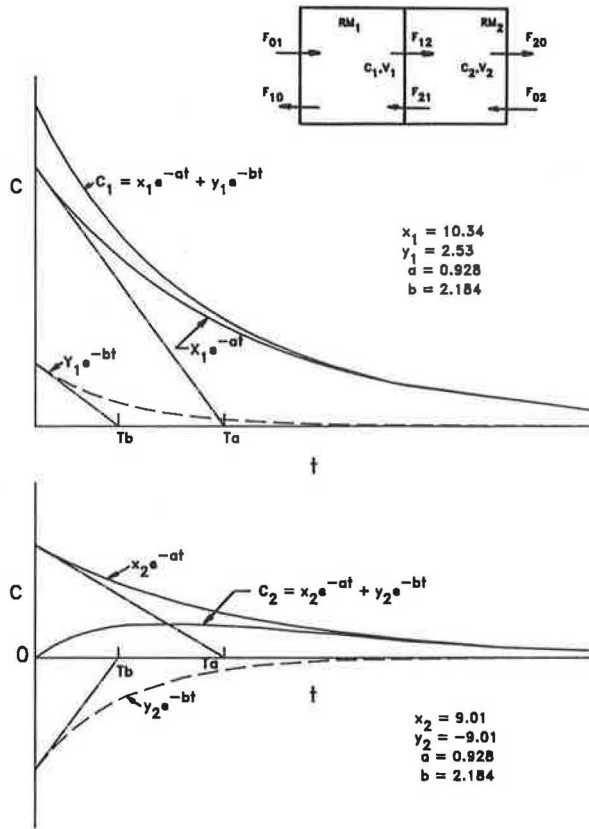
$$V_1(dC_1/dt) = -(F_{10} + F_{12}) \cdot C_1 + F_{21} \cdot C_2 + Q \quad (7)$$

$$V_2(dC_2/dt) = -(F_{20} + F_{21}) \cdot C_2 + F_{12} \cdot C_1 \quad (8)$$

Substituting for  $C_2$  from Equation 7 into Equation 8 and letting  $N_1 = (F_{12} + F_{10})/V_1$  and  $N_2 = (F_{21} + F_{20})/V_2$ , we have,

$$d^2C_1/dt^2 = -(N_1 + N_2) \cdot (dC_1/dt) - [N_1 \cdot N_2 - F_{12} \cdot F_{21}/(V_1 \cdot V_2)] \cdot C_1 + N_2 \cdot Q/V_1 \quad (9)$$

The solution to  $C_1$  can be obtained by the method of Laplace transformation. Thus,



**Figure 6** Typical concentration profiles and time constants

$$C_1(t) = X_1 \cdot \exp(-at) + Y_1 \cdot \exp(-bt) + Z_1 \quad (10)$$

where

$C_1(t)$  = tracer gas concentration in Room 1 at time  $t$

$$X_1 = [(b - N_1) \cdot C_1(0) + F_{21} \cdot C_2(0)/V_1]/(b - a)$$

$$Y_1 = [N_1 - a] \cdot C_1(0) - F_{21} \cdot C_2(0)/V_1/(b - a)$$

$$Z_1 = [Q \cdot G_1 \cdot b/(b - a)] \cdot [1 - \exp(-at)] - [Q \cdot G_1 \cdot a/(b - a)] \cdot [1 - \exp(-bt)] + \{Q/[V_1 \cdot (b - a)]\} \cdot [\exp(-at) - \exp(-bt)]$$

$$A = N_1 + N_2$$

$$B = N_1 \cdot N_2 - F_{12} \cdot F_{21}/(V_1 \cdot V_2)$$

$$a = [A - (A^2 - 4B)^{0.5}]/2$$

$$b = [A + (A^2 - 4B)^{0.5}]/2$$

$$G_1 = (F_{21} + F_{20})/[(F_{12} + F_{10}) \cdot (F_{21} + F_{20}) - F_{21} \cdot F_{12}]$$

Similarly,  $C_2(t)$  can be expressed by the equation,

$$C_2(t) = X_2 \cdot \exp(-at) + Y_2 \cdot \exp(-bt) + Z_2 \quad (11)$$

where

$C_2(t)$  = tracer gas concentration in Room 2 at time  $t$

$$X_2 = [(b - N_2) \cdot C_2(0) + F_{12} \cdot C_1(0)/V_2]/(b - a)$$

$$Y_2 = [N_2 - a] \cdot C_2(0) - F_{12} \cdot C_1(0)/V_2/(b - a)$$

$$Z_2 = [Q \cdot G_2 \cdot b/(b - a)] \cdot [1 - \exp(-at)] - [Q \cdot G_2 \cdot a/(b - a)] \cdot [1 - \exp(-bt)]$$

$$G_2 = F_{12}/[(F_{12} + F_{10}) \cdot (F_{21} + F_{20}) - F_{21} \cdot F_{12}]$$

If a single tracer gas ( $\text{SF}_6$ ) is released in Room 1 using the decay mode (Test 113), then  $Q = 0$ ;  $Z_1 = 0$ ,  $Z_2 = 0$  and Equations 10 and 11 become,

$$C_1(t) = X_1 \cdot \exp(-at) + Y_1 \cdot \exp(-bt) \quad (12)$$

$$C_2(t) = X_2 \cdot \exp(-at) + Y_2 \cdot \exp(-bt) \quad (13)$$

From these equations, two time constants,  $T_a$  and  $T_b$ , can be defined (Figure 6),

$$T_a = 1/a \quad (14)$$

$$T_b = 1/b \quad (15)$$

The significance of  $T_a$  can be explained using Equations 12 and 13. As indicated by their definitions (given in Equation 10),  $b$  is always greater than  $a$ . Therefore, as  $t$  becomes very large, the term containing  $\exp(-bt)$  in Equations 12 and 13 approaches zero first, and  $C_1(t)$  equals  $(X_1/X_2) \cdot C_2(t)$ . When this occurs, large errors can be expected if  $C_1(t)$  and  $C_2(t)$  are used as independent input data for calculating interzonal airflows.

This can be illustrated by examining a typical pair of concentration profiles such as the  $\text{SF}_6$  concentration profiles shown in Figures 3 and 6.  $T_a$  for these profiles was calculated to be 65 minutes (using the method described in the following sections). As shown in Figure 6, at  $t > T_a$ , the term containing  $\exp(-bt)$  becomes negligible, and the ratio of  $C_1$  and  $C_2$  approaches a constant. Figure 5 correspondingly indicates that errors in the calculated interzonal airflows based on the profiles generally become unacceptable at  $t > T_a$  (65 min). Examinations made on other concentration profiles measured under 17 different test conditions (Table 2) produced similar results. Therefore,  $T_a$  may be used as an indication of the upper time limit for the useful portion of the concentration profiles.

The significance of  $T_b$  cannot be readily explained using the definitions of  $a$  and  $b$ . However, since  $T_b$  relates to the term containing  $\exp(-bt)$ , which occurs mainly in the initial stage of the concentration profile (Figure 6), it is expected to be related to the conditions of mixing between the tracer gases and indoor air in the test rooms. Because the values of  $T_b$  for the concentration profiles measured under various test conditions varied from 20 to 44 minutes (Table 2), which would be the time required to achieve adequate mixing in the test room (Figure 2),  $T_b$  may be used as an indicator for the minimum time required to achieve adequate mixing.

The two time constants, therefore, specify the time interval in which the measured concentrations can be used for interzonal airflow predictions. These time constants can be estimated using the application of the successive integration method (Aratani et al. 1971), to be described below. The proposed method applies only to cases where at least one tracer gas is introduced using the decay technique.

**TABLE 2**  
**Calculated and Measured Airflow Rates**

Test No.	Measured		Case 1		Case 2		Case 3		$T_b$ min	$T_a$ min	$T_s$ min
	$F_{12}$ ach	$F_{21}$ ach	$F_{12}$ ach	$F_{21}$ ach	$F_{12}$ ach	$F_{21}$ ach	$F_{12}$ ach	$F_{21}$ ach			
101	1.00	1.00	—	—	0.97	0.98	—	—	30	1154	66
102	0.75	0.56	—	—	0.78	0.55	—	—	42	545	62
103	1.00	1.00	—	—	1.00	0.95	—	—	24	125	34
104	1.00	1.00	1.06	0.88	0.98	0.98	1.03	0.95	29	559	52
105	1.00	1.00	0.99	3.22	1.10	0.88	1.04	1.27	20	95	42
106	0.25	0.25	0.37	0.88	0.23	0.24	0.31	0.26	42	77	60
107	0.75	0.25	0.82	0.52	0.71	0.13	0.82	0.14	37	62	44
108	0.25	0.50	0.24	1.17	0.31	0.40	0.27	0.43	33	57	48
109	0.25	0.75	0.38	2.42	0.34	0.71	0.36	0.81	26	53	44
110	0.15	0.15	0.27	1.54	0.11	0.17	0.18	0.17	44	60	52
111	0.15	0.50	0.29	- 0.4	0.13	0.56	0.22	0.53	35	70	50
112	0.15	0.75	0.32	0.40	0.19	0.84	0.26	0.82	29	67	44
113*	0.75	0.15	0.86	- 0.2	0.72	0.18	0.79	0.16	27	65	46
114	0.75	0.56	0.76	1.31	0.76	0.52	0.76	0.61	30	135	48
115	0.75	0.56	0.75	0.75	0.71	0.59	0.74	0.64	40	521	80
116	1.00	1.00	1.00	1.31	1.03	0.93	1.01	1.09	28	1166	68
117	1.00	1.00	0.96	0.73	1.15	1.06	1.03	0.99	22	79	42

Case 1: Two tracer gases, both injected into Room 1. One gas was injected under decay mode and the other under constant injection mode.  
 Case 2: Two tracer gases, one for each room. Both gases were injected under decay mode.  
 Case 3: Two tracer gases, one for each room. One gas was injected under constant injection mode (Room 1), and the other under decay mode.  
 \* See Appendix A sample calculations.

**Determining Time Constants By Successive Integration Method**

Consider a typical pair of tracer gas profiles.

$$C_1(t) = X_1 \cdot \exp(-at) + Y_1 \cdot \exp(-bt) \quad (12)$$

$$C_2(t) = X_2 \cdot \exp(-at) + Y_2 \cdot \exp(-bt) \quad (13)$$

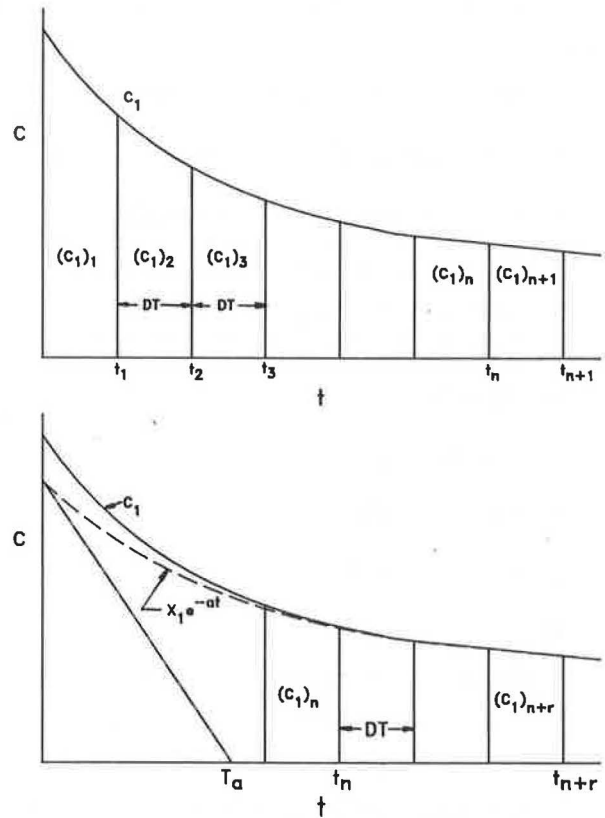
The time constants can be determined by evaluating  $X_1$ ,  $Y_1$ ,  $X_2$ ,  $Y_2$ ,  $a$ , and  $b$  from the measured concentrations using the following procedures.

**Exponent  $a$  and Time Constant  $T_a$ .** If Equation 12 is integrated successively at equal time intervals,  $Dt$ , from 0 to  $t_1$ ,  $t_1$  to  $t_2$ , ..., and from  $t_n$  to  $t_{n+1}$  (Figures 7a and b), we have,

$$\begin{aligned} (C_1)_1 &= \int_0^{t_1} C_1(t) dt \\ &= (-X_1/a) \cdot [\exp(-a \cdot Dt) - 1] \\ &\quad + (-Y_1/b) \cdot [\exp(-b \cdot Dt) - 1] \\ &= (R_x)_1 + (R_y)_1 \\ (C_1)_2 &= \int_{t_1}^{t_2} C_1(t) dt \\ &= (-X_1/a) \cdot \exp(-a \cdot Dt) \cdot [\exp(-a \cdot Dt) - 1] \\ &\quad + (-Y_1/b) \cdot \exp(-b \cdot Dt) \cdot [\exp(-b \cdot Dt) - 1] \\ &= (R_x)_1 \cdot \exp(-a \cdot Dt) + (R_y)_1 \cdot \exp(-b \cdot Dt) \\ &= (R_x)_2 + (R_y)_2 \\ (C_1)_3 &= \int_{t_2}^{t_3} C_1(t) dt \\ &= (R_x)_1 \cdot [\exp(-a \cdot Dt)]^2 + (R_y)_1 \cdot [\exp(-b \cdot Dt)]^2 \\ &= (R_x)_2 \cdot \exp(-a \cdot Dt) + (R_y)_2 \cdot \exp(-b \cdot Dt) \end{aligned}$$

and

$$\begin{aligned} (C_1)_n &= \int_{t_{n-1}}^{t_n} C_1(t) dt \\ &= (R_x)_1 \cdot [\exp(-a \cdot Dt)]^{n-1} + (R_y)_1 \cdot [\exp(-b \cdot Dt)]^{n-1} \\ &= (R_x)_{n-1} \cdot \exp(-a \cdot Dt) + (R_y)_{n-1} \cdot \exp(-b \cdot Dt) \end{aligned}$$



**Figure 7** Successive integration of typical concentration profiles

For typical tracer gas concentration profiles, such as those for SF<sub>6</sub> in Figures 3 and 6, where  $b$  is approximately equal to  $3a$ , it can be shown that as  $n$  increases,  $[\exp(-b \cdot Dt)]^{n-1}$  rapidly becomes negligible compared to  $[\exp(-a \cdot Dt)]^{n-1}$  (Figure 6). Therefore,

$$(C_1)_n = (R_x)_{n-1} \cdot \exp(-a \cdot Dt) \\ = (R_x)_n$$

and

$$(C_1)_{n+1} = (R_x)_n \cdot \exp(-a \cdot Dt)$$

Therefore,

$$[(C_1)_{n+1} / (C_1)_n] = \exp(-a \cdot Dt) = E_{n+1} \quad (16)$$

Similarly,

$$[(C_2)_{n+1} / (C_2)_n] = \exp(-a \cdot Dt) = E_{n+1} \quad (17)$$

Hence,

$$a = -\ln(E_{n+1}) / Dt \quad (18)$$

and

$$T_a = 1/a$$

**Coefficients  $X_1$  and  $X_2$**  When  $t$  is large,  $\exp(-bt)$  becomes negligible compared to  $\exp(-at)$  (Figure 6) and Equations 12 and 13 become

$$C_1(t) = X_1 \cdot \exp(-at)$$

$$C_2(t) = X_2 \cdot \exp(-at)$$

Integrating  $C_1(t)$  from  $t_n$  to  $t_{n+r}$ , we have

$$\int_{t_n}^{t_{n+r}} C_1(t) dt = (-X_1/a) \cdot [\exp(-a \cdot t_{n+r}) - \exp(-a \cdot t_n)]$$

or

$$X_1 = (-a) \cdot \int_{t_n}^{t_{n+r}} C_1(t) dt / [\exp(-a \cdot t_{n+r}) \\ - \exp(-a \cdot t_n)] \quad (19)$$

Similarly,

$$X_2 = (-a) \cdot \int_{t_n}^{t_{n+r}} C_2(t) dt / [\exp(-a \cdot t_{n+r}) \\ - \exp(-a \cdot t_n)] \quad (20)$$

**Coefficient  $Y_2$**  If a tracer gas is injected into Room 1 using the decay mode, the concentration of this tracer gas in Room 2 at  $t = 0$  is zero. Therefore, for  $t = 0$ , Equation 13 becomes

$$C_2(0) = X_2 + Y_2 \\ = 0$$

or

$$Y_2 = -X_2 \quad (21)$$

**Exponent  $b$  and Time Constant  $T_b$**  Integrating Equation 13 from  $t = 0$  to  $t = n \cdot Dt$  where  $Dt$  is a finite time interval, we have

$$\int_0^{n \cdot Dt} C_2(t) dt = (-X_2/a) \cdot [\exp(-a \cdot n \cdot Dt) - 1] \\ + (-Y_2/b) \cdot [\exp(-b \cdot n \cdot Dt) - 1]$$

or

$$b = Y_2 \cdot [1 - \exp(-b \cdot n \cdot Dt)]$$

$$/ \left\{ \int_0^{n \cdot Dt} C_2(t) dt - (-X_2/a) \cdot [\exp(-a \cdot n \cdot Dt) - 1] \right\} \quad (22)$$

and

$$T_b = 1/b$$

In the above equation,  $a$ ,  $X_2$ , and  $Y_2$  can be evaluated from Equations 18, 20, and 21, and the definite integral of  $C_2(t)$  can be evaluated numerically from the measured concentrations. The exponent,  $b$ , can then be evaluated by either graphical or trial-and-error methods.

**Coefficient  $Y_1$**  Finally,  $Y_1$  can be evaluated by integrating Equation 12 from  $t = 0$  to  $t = n \cdot Dt$ . Thus,

$$\int_0^{n \cdot Dt} C_1(t) dt = (-X_1/a) \cdot [\exp(-a \cdot n \cdot Dt) - 1] \\ + (-Y_1/b) \cdot [\exp(-b \cdot n \cdot Dt) - 1]$$

or

$$Y_1 = b \cdot \left\{ \int_0^{n \cdot Dt} C_1(t) dt - (-X_1/a) \cdot [\exp(-a \cdot n \cdot Dt) - 1] \right\} \\ / [1 - \exp(-b \cdot n \cdot Dt)] \quad (23)$$

Again, in the previous equation, the definite integral of  $C_1(t)$  can be evaluated numerically from the measured concentrations, and  $a$ ,  $X_1$ , and  $b$  are defined by Equations 18, 19, and 22.

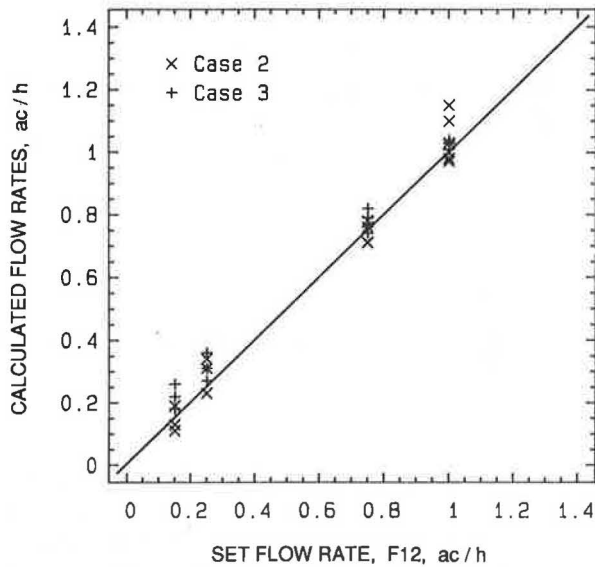
### Selecting Appropriate Concentration Measurements for Calculating Interzonal Airflows

With  $T_a$  and  $T_b$  known, the interzonal airflows can be calculated from Equations 1 through 6 using a set of simultaneous concentrations measured at any time  $t$  between  $T_a$  and  $T_b$ . This calculation can then be repeated for several values of  $t$ . The average values of the calculated airflows are used as the calculated results.

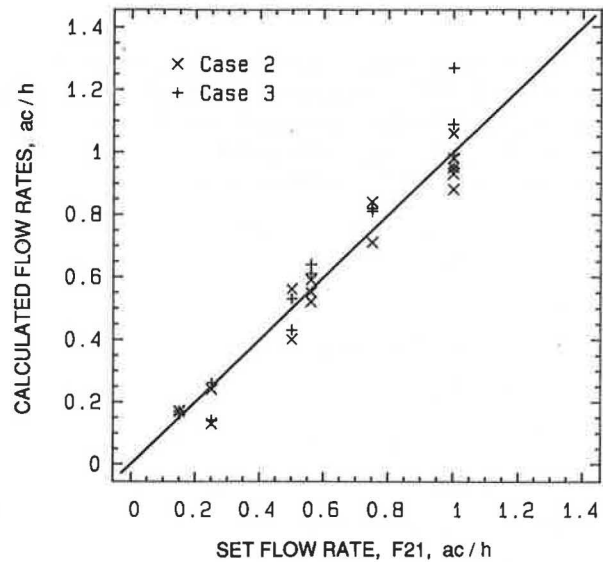
Alternatively, it may be possible to obtain the calculated airflows based on a single set of concentrations taken at a specific time  $T_s$ . The time  $T_s$  is chosen on the basis that it falls between  $T_a$  and  $T_b$  and that at  $T_s$  Equation 13 predicts the same or nearly the same concentration as the measured value.

### COMPARISON BETWEEN CALCULATED AND MEASURED AIRFLOWS

Table 2 gives the calculated airflows,  $F_{12}$  and  $F_{21}$ , based on the concentrations measured at  $T_s$  and the measured results. It also includes the calculated values of  $T_a$ ,  $T_b$ , and  $T_s$  based on the measured SF<sub>6</sub> concentrations. Three cases were examined. In Case 1, two tracer gases were injected into Room 1, SF<sub>6</sub> using the decay



**Figure 8** Comparison of set and calculated airflow rates in test rooms



**Figure 9** Comparison of set and calculated airflow rates in test rooms

mode and  $\text{CH}_4$  the constant injection mode. In Case 2, a different tracer gas was injected into each room using the decay mode,  $\text{SF}_6$  in Room 1 and  $\text{N}_2\text{O}$  in Room 2. In Case 3,  $\text{CH}_4$  was injected into Room 1 using the constant injection mode, and  $\text{N}_2\text{O}$  was released in Room 2 using the decay mode.

The standard errors of estimate were calculated for the three cases. For  $F_{12}$ , they were 0.019, 0.014, and 0.010 for Cases 1, 2, and 3, respectively. For  $F_{21}$ , they were 0.232, 0.015, and 0.024. This indicates that Case 1 (both gases injected into Room 1) gave the worst results. An examination of Table 2 reveals that Case 1 calculated  $F_{12}$  as well as the other two cases. However, for some tests (e.g., Tests 105 and 109), Case 1 overestimated  $F_{21}$  by a factor of three. In Tests 111 and 113, Case 1 gave the wrong flow direction for  $F_{21}$ .

Based on the standard errors of estimate, Cases 2 and 3 appeared to predict airflows equally well. Figures 8 and 9 and Table 2 show that for larger values of  $F_{12}$  and  $F_{21}$  (i.e., 0.5 ach and greater), the calculated and measured airflow rates for all but one test (Test 105) agreed within 15% (of the measured value). They also show that for smaller values of  $F_{12}$  and  $F_{21}$  (i.e., smaller than 0.5 ach), the agreement was within 50% of the measured value.

## SUMMARY

1. The results obtained from the test room suggest that, for a rectangular room, adequate mixing between the tracer gas and indoor air may be achieved within 30 minutes. During the mixing period, the average concentration may be obtained by pumping the samples from several locations to a manifold and taking the sample at the manifold.

2. The coefficients and exponents for equations representing the tracer gas concentration profiles in two interconnected rooms can be obtained using the successive integration method on the measured concentration profiles. These equations, in turn, can provide the

upper and lower time limits for choosing appropriate concentrations for interzonal airflow predictions.

3. No clear evidence was found to suggest that the technique used to inject tracer gases (e.g., decay or constant injection) has a significant effect on the calculated results. However, results do suggest that a different tracer gas injected into each room gives better results than injecting several gases into one room only.

4. A comparison between the calculated and measured results suggests that only concentrations measured between  $T_b$  and  $T_a$  (the lower and upper time constants defining the tracer gas behavior, respectively) should be used for calculating interzonal airflows. Based on the concentrations measured at  $T_s$  (the time between  $T_b$  and  $T_a$  at which calculated and measured tracer gas concentrations most closely match), the calculated airflow rates agreed with the measured values within about 15% if the two flow rates are of similar magnitude. The agreement where one flow rate is significantly lower than the other was not as good for the lower flow rate.

## ACKNOWLEDGMENTS

This work was undertaken in Ottawa at the Institute for Research in Construction, National Research Council of Canada. The senior author, Prof. Masamichi Enai, Dr. Eng., is a guest worker from Hokkaido University, Japan. The authors wish to acknowledge the cooperative effort of both organizations, and particularly Prof. N. Aratani of Hokkaido University, in supporting this project.

## REFERENCES

- Aratani, N.; Sasaki, K.; and Enai, M. 1971. "A successive integration method for the analysis of the thermal environment of buildings." NBS Building Science Series 39, pp. 305-316.
- Evans, R.G., and Shaw, C.Y. 1988. "A multiposition tracer gas sampling system for building air movement studies." Air Infiltration and Ventilation Centre Measurement Techniques Workshop, Paper 5, Koge, Denmark, pp. 73-93.
- l'Anson, S.J.; Irwin, C.; and Howarth, A.T. 1982. "Airflow measurement using three tracer gases." *Building and Environment*, Vol. 17, No. 4, pp. 245-252.



Perera, M.D.A.E.S. 1983. "Review of techniques for measuring ventilation rates in multi-celled buildings." *Energy Conservation in Buildings—Heating, Ventilation and Insulation*. Dordrecht: D. Reidel Publishing Company, pp. 321-334.

Sherman, M.; Grimsrud, D.T.; Condon, P.E.; and Smith, B.V. 1980. "Air infiltration measurement techniques." *Proceedings of the 1st Air Infiltration Centre Conference on Air Infiltration Instrumentation and Measuring Techniques*, Windsor, England.

Sinden, F.W. 1978. "Multi-chamber theory of air infiltration." *Building and Environment*, Vol. 13, pp. 21-28.

## APPENDIX A

### SAMPLE CALCULATIONS

#### Measured SF<sub>6</sub> Concentrations Used in the Calculation

Measured SF <sub>6</sub> Concentrations Test No. 113, Dt = 4 min							
n	t min	C <sub>1</sub> (t) ppb	(C <sub>1</sub> ) <sub>n</sub> = C <sub>1</sub> (t) · Dt	E <sub>n</sub>	C <sub>2</sub> (t) ppb	(C <sub>2</sub> ) <sub>n</sub> = C <sub>2</sub> (t) · Dt	E <sub>n</sub>
1	3	11.08	44.32		0.18	0.72	
2	7	11.69	46.76	1.056	0.57	2.28	3.617
3	11	11.08	44.32	0.947	1.00	4.00	1.746
4	15	10.20	40.80	0.921	1.39	5.56	1.392
5	19	9.47	37.88	0.929	1.71	6.84	1.236
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
10	39	6.55			2.64		
11	43	6.13			2.71		
12	47	5.68			2.75		
13	51	5.25			2.76		
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
20	79	3.15	12.60	0.928	2.48	9.92	0.973
21	83	2.93	11.72	0.930	2.38	9.52	0.960
22	87	2.72	10.88	0.930	2.29	9.16	0.963
23	91	2.55	10.20	0.938	2.20	8.80	0.959
24	95	2.36	9.44	0.925	2.12	8.48	0.965
25	99	2.19	8.76	0.930	2.02	8.08	0.953
26	103	2.04	8.16	0.930	1.92	7.68	0.953
27	107	1.89	7.56	0.928	1.83	7.28	0.949

#### Calculating a, T<sub>a</sub>, X<sub>1</sub>, X<sub>2</sub>, Y<sub>2</sub>, b, T<sub>b</sub>, Y<sub>1</sub>, and T<sub>s</sub>

##### (A) a and T<sub>a</sub>

For large n,

$$E_{n+1} = (C_1)_{n+1} / (C_1)_n \quad (16)$$

$$E_{26} = 0.93 \text{ for } C_1, \text{ and } 0.953 \text{ for } C_2$$

$$E_{27} = 0.928 \text{ for } C_1, \text{ and } 0.949 \text{ for } C_2$$

$$E_{avg} = (0.930 + 0.953 + 0.928 + 0.949) / 4 = 0.94$$

$$a = -\ln(E) / Dt$$

$$= -\ln(0.94) / [4 \text{ min} / (60 \text{ min/h})]$$

$$= 0.928 \text{ h}^{-1}$$

$$T_a = 1/a$$

$$= 1 / (0.928 \text{ h}^{-1}) = 1.0776 \text{ h or } 65 \text{ min}$$

##### (B) X<sub>1</sub> and X<sub>2</sub>

Integrating the measured tracer gas concentration profile numerically at equal time intervals, from t<sub>1</sub> to t<sub>n</sub>, we have:

$$\int_{t_1}^{t_n} (C_1) dt = Dt \cdot [C_1(t_1)/2 + C_1(t_2) + C_1(t_3) + \dots + C_1(t_n)/2]$$

$$= [(C_1)_1/2 + (C_1)_2 + (C_1)_3 + \dots + (C_1)_n/2]$$

For n = 20 to 27, t<sub>n</sub> = 79/60 = 1.317 h to t<sub>n+r</sub> = 107/60 = 1.783 h,

$$\int_{t_{20}}^{t_{27}} (C_1) dt = [12.6/2 + 11.72 + 10.88 + 10.2 + 9.44 + 8.76 + 8.16 + 7.56/2] / (60 \text{ min/h})$$

$$= 1.154 \text{ ppb-h}$$

$$X_1 = (-a) \cdot \int_{t_{20}}^{t_{27}} (C_1) dt / [\exp(-a \cdot t_{n+r}) - \exp(-a \cdot t_n)]$$

$$= -0.928 \cdot 1.154 / [\exp(-0.928 \cdot 1.783) - \exp(-0.928 \cdot 1.317)]$$

$$= 10.34$$

$$\int_{t_{20}}^{t_{27}} (C_2) dt = [9.92/2 + 9.52 + 9.16 + 8.8 + 8.48 + 8.08 + 7.68 + 7.28/2] / (60 \text{ min/h})$$

$$= 1.005 \text{ ppb-h}$$

$$X_2 = -0.928 \cdot 1.005 / [\exp(-0.928 \cdot 1.783) - \exp(-0.928 \cdot 1.317)]$$

$$= 9.01$$

##### (C) Y<sub>2</sub>

$$Y_2 = -X_2$$

$$= -9.01$$

##### (D) b and T<sub>b</sub>

Integrating the measured tracer gas concentration profile numerically at equal time intervals, from t<sub>1</sub> to t<sub>27</sub>, we have:

$$\int_{t_1}^{t_n} (C_2) dt = Dt \cdot [C_2(t_1)/2 + C_2(t_2) + C_2(t_3) + \dots + C_2(t_n)/2]$$

$$= [(C_2)_1/2 + (C_2)_2 + (C_2)_3 + \dots + (C_2)_n/2]$$

$$= [0.72/2 + 2.28 + 4.00 + 5.56 + \dots + 8.08 + 7.68 + 7.28/2] / (60 \text{ min/h})$$

$$= 3.810 \text{ ppb-h}$$

For t = 1 to 27, n = 27 and Dt = 4 min,

$$b = Y_2 \cdot [1 - \exp(-b \cdot n \cdot Dt)]$$

$$/ \left\{ \int_{t_1}^{t_n} C_2(t) dt - (-X_2/a) \cdot [\exp(-a \cdot n \cdot Dt) - 1] \right\}$$

$$= -9.01 \cdot [1 - \exp[-b \cdot (27 \cdot 4/60)]] / \{3.810 - (-9.01/0.928) \cdot [\exp(-0.928 \cdot 27 \cdot 4/60) - 1]\}$$

$$= 2.214 \cdot [1 - \exp(-1.8 \cdot b)]$$

The value of b can be obtained by trial-and-error method. Thus, b = 2.184 h<sup>-1</sup>.

$$T_b = 1/b$$

$$= 1 / (2.184 \text{ h}^{-1}) = 0.46 \text{ h or } 27.5 \text{ min}$$

(E)  $Y_1$

$$\int_{t_1}^{t_n} C_1(t_n) dt = Dt \cdot [C_1(t_1)/2 + C_1(t_2) + C_1(t_3) + \dots + C_1(t_n)/2]$$

$$= [(C_1)_1/2 + (C_1)_2 + (C_1)_3 + \dots + (C_1)_n/2]$$

$$= [44.32/2 + 46.76 + 44.32 + 40.8 + \dots + 8.76 + 8.16 + 7.56/2]/(60 \text{ min/h})$$

$$= 10.147$$

and

$$Y_1 = b \cdot \left\{ \int_{t_r}^{t_n} C_1(t) dt - (-X_1/a) \cdot [\exp(-a \cdot n \cdot Dt) - 1] \right\} / [1 - \exp(-b \cdot n \cdot Dt)]$$

For  $n = 27$  and  $Dt = 4$  min,  $n \cdot Dt = 27 \cdot 4/60 = 1.8$  h,

$$Y_1 = 2.184 \cdot \{10.147 - (-10.34/0.928) \cdot [\exp(-0.928 \cdot 1.8) - 1]\} / [1 - \exp(-2.184 \cdot 1.8)]$$

$$= 2.53$$

(F)  $T_s$

$$C_2(t) = X_2(t)\exp(-at) + Y_2(t)\exp(-bt)$$

$$= 9.01 \cdot \exp(-0.928 \cdot t) + (-9.01) \cdot \exp(-2.184 \cdot t)$$

At  $t = 39, 43, 47,$  and  $51$  min, the calculated concentrations from the previous equation and the measured concentrations are:

$t, \text{min}$	$(C_2)_{\text{calculated}}, \text{ppb}$	$(C_2)_{\text{measured}}, \text{ppb}$
39	2.75	2.64
43	2.74	2.71
47	2.73	2.75
51	2.69	2.76

It can be shown that at  $t = 46$  min, the calculated and measured concentrations are equal. Therefore,  $T_s$  is 46 min.

(G)  $F_{21}$  and  $F_{12}$

As there were no measured concentrations at  $t = T_s = 46$  min, values interpolated from the measured profiles must be used to calculate  $F_{12}$  and  $F_{21}$ .

Measured Tracer Gas Concentrations  
Test 113

$t, \text{min}$	SF <sub>6</sub> , ppb		$dC_{g1}/dt$	$dC_{g2}/dt$	N <sub>2</sub> O, ppm		$dC_{s1}/dt$	$dC_{s2}/dt$
	$C_{g1}$	$C_{g2}$			$C_{s1}$	$C_{s2}$		
44	6.02	2.72			4.4	40.4		
46	5.79	2.74	-0.1125	0.0075	4.5	38.75	0.025	-0.825
48	5.57	2.75			4.5	37.1		

For  $V_1 = V_2 = 4.8 \cdot 4.8 \cdot 2.87 = 66 \text{ m}^3$  and  $Q_g = Q_s = 0$  (decay mode only),  $F_{21}$  and  $F_{12}$  at  $T_s = 46$  min can be calculated from Equations 1 through 4, repeated below.

$$V_1(dC_{g1}/dt) = -(F_{10} + F_{12}) \cdot C_{g1} + F_{21} \cdot C_{g2} + Q_g \quad (1)$$

$$V_1(dC_{s1}/dt) = -(F_{10} + F_{12}) \cdot C_{s1} + F_{21} \cdot C_{s2} \quad (2)$$

$$V_2(dC_{g2}/dt) = -(F_{20} + F_{21}) \cdot C_{g2} + F_{12} \cdot C_{g1} \quad (3)$$

$$V_2(dC_{s2}/dt) = -(F_{20} + F_{21}) \cdot C_{s2} + F_{12} \cdot C_{s1} + Q_s \quad (4)$$

Substituting  $V_1, dC_{g1}/dt, C_{g1}, C_{g2}, dC_{s1}/dt, C_{s1},$  and  $C_{s2}$  into Equations 1 and 2, we have:

$$66 \cdot (-0.1125) = -5.79 \cdot (F_{10} + F_{12}) + 2.74 \cdot (F_{21})$$

$$66 \cdot (0.025) = -4.5 \cdot (F_{10} + F_{12}) + 38.75 \cdot (F_{21})$$

Therefore,

$$F_{21} = 0.2026 \text{ m}^3/\text{min}, \text{ or } 0.18 \text{ ach}$$

Similarly, substituting  $V_2, dC_{g2}/dt, C_{g2}, C_{g1}, dC_{s2}/dt, C_{s2},$  and  $C_{s1}$  into Equations 3 and 4, we have:

$$66 \cdot (0.0075) = -2.74 \cdot (F_{20} + F_{21}) + 5.79 \cdot (F_{12})$$

$$66 \cdot (-0.825) = -38.75 \cdot (F_{20} + F_{21}) + 4.5 \cdot (F_{12})$$

Therefore,

$$F_{12} = 0.794 \text{ m}^3/\text{min}, \text{ or } 0.72 \text{ ach}$$

The two interzonal flow rates are  $F_{12} = 0.72$  ach and  $F_{21} = 0.18$  ach.