#3717

AT-90-5-2 (3369)

MULTIPLE TRACER GAS TECHNIQUES FOR MEASURING INTERZONAL AIRFLOWS FOR THREE INTERCONNECTED SPACES

M. Enai **C.Y. Shaw** **J.T. Reardon** **R.J. Magee**

ABSTRACT

The multiple tracer gas method is often used to predict interzonal airflows in buildings. In a previous study, this method was applied to a space consisting of two interconnected rooms where the airflows through the common wall were controlled and measured. The results indicated that the predicted airflows based on different sets of simultaneously measured tracer gas concentrations (obtained at different sampling times during a test) were not always the same. Guidelines are, therefore, needed to facilitate the selection of the appropriate set of tracer gas concentrations (from the measurements) for use with the multiple tracer gas method.

In this study, the test rooms were expanded to three interconnected rooms. A method was developed to determine the appropriate set of concentrations for interzonal airflow calculations. The proposed method was tested in the laboratory and the results are discussed.

INTRODUCTION

As concern for indoor air quality has grown, so too has the need to measure interzonal airflows to assess the distribution of outdoor air in buildings. These airflows can be evaluated using the multiple tracer gas method (Sinden 1978; Sherman et al. 1980; I'Anson et al. 1982; Perera 1983). It involves the injection of a different tracer gas into each of several interconnected spaces and the measurement of the tracer gas concentrations as a function of time. Based on the simultaneously measured tracer gas concentrations, the interzonal airflows can then be calculated from the mass conservation equations for each tracer gas and the mass flow balance equations for the air.

The application of the method is not straightforward, even for the simplest case, e.g., a building consisting of two interconnected chambers. In a previous study (Enai et al. 1990), the multiple tracer gas method was applied to a space consisting of two interconnected rooms where the airflows between the two rooms were controlled and measured. Two test procedures were used to introduce the tracer gases into the rooms: the decay technique for both rooms, and constant injection for one room and decay for the other room. The results indicated that, for any given test, the calculated airflow rates based on different sets of concentrations (measured at different times during a test) were not always the same and that the agreement between the calculated and measured airflow rates generally worsened when the concentrations used were measured earlier than 30 minutes after injection or later than 70 minutes after injection. These findings suggest that to achieve the best result, some guidelines are required for selection of the appropriate set of concentrations for calculating the interzonal airflows. A method was derived for calculating interzonal airflows for two interconnected rooms.

In this study, the test facility was expanded to three interconnected rooms, where the airflows between any two rooms were controlled at constant rates and measured during the experiments. The main objectives of the study were (1) to determine the appropriate set of concentrations that can be used for interzonal airflow calculations and (2) to determine the accuracy of the multiple tracer gas technique under laboratory conditions.

GOVERNING EQUATIONS

Figure 1 shows the case of three interconnected zones. If three tracer gases, denoted by A, B, and C, are injected into the zones (one for each zone), the rates of change in tracer gas concentrations in the three zones can be described by the following equations, assuming that the tracer gas concentrations outside the zones are negligible:

$$V_1(dC_{A1}/dt) = -(F_{10} + F_{12} + F_{13})C_{A1} + F_{21} * C_{A2} + F_{31} * C_{A3} + Q_{A1} \qquad (1)$$

$$V_1(dC_{B1}/dt) = -(F_{10} + F_{12} + F_{13})C_{B1} + F_{21} * C_{B2} + F_{31} * C_{B3} \qquad (2)$$

$$V_1(dC_{C1}/dt) = -(F_{10} + F_{12} + F_{13})C_{C1} + F_{21} * C_{C2} + F_{31} * C_{C3} \qquad (3)$$

$$V_2(dC_{A2}/dt) = F_{12} * C_{A1} - (F_{20} + F_{21} + F_{23})C_{A2} + F_{32} * C_{A3} \qquad (4)$$

M. Enai, Associate Professor, Architectural Department, Hokkaido University; **C.Y. Shaw**, **J.T. Reardon**, and **R.J. Magee**, Institute for Research in Construction, National Research Council, Canada.

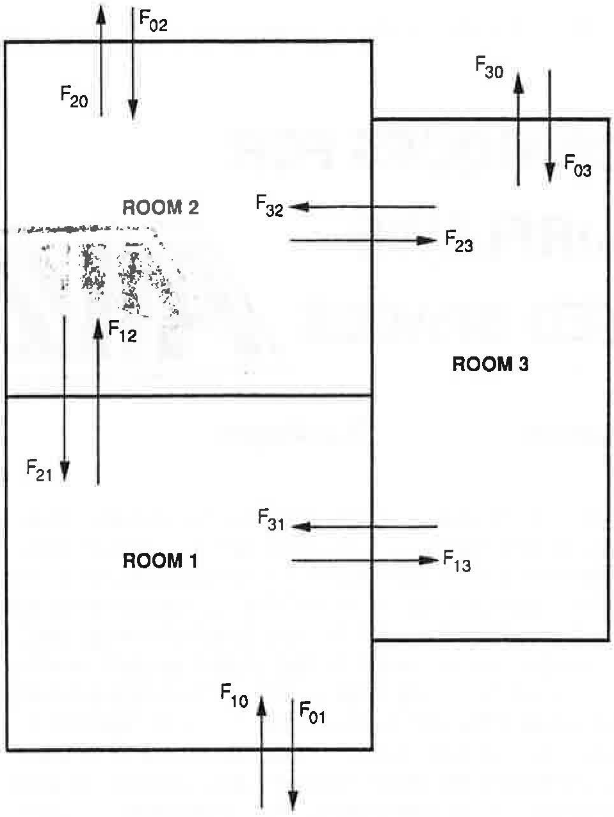

Figure 1 *Test rooms*

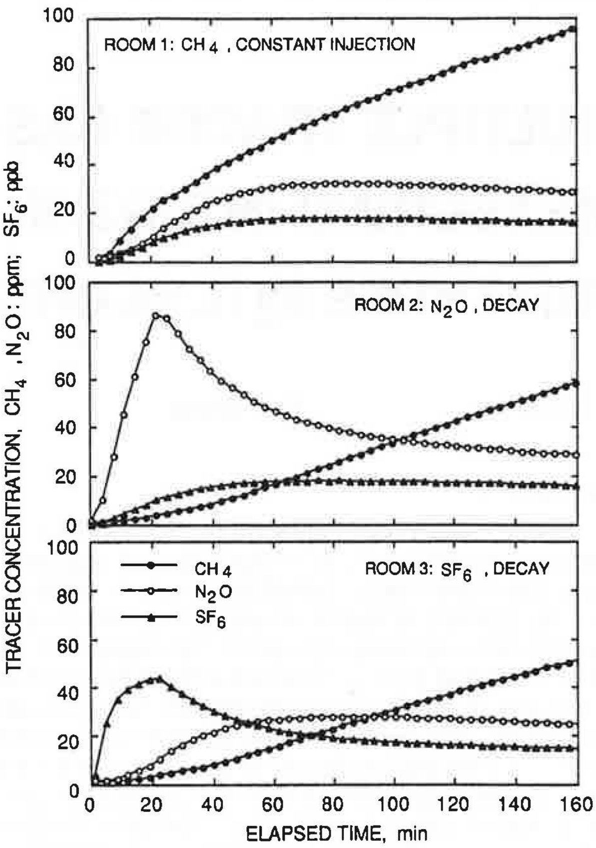

Figure 2 *Typical tracer gas concentration profiles*

$$V_2(dC_{B2}/dt) = F_{12} * C_{B1} - (F_{20} + F_{21} + F_{23})C_{B2} + F_{32} * C_{B3} + Q_{B2} \quad (5)$$

$$V_2(dC_{C2}/dt) = F_{12} * C_{C1} - (F_{20} + F_{21} + F_{23})C_{C2} + F_{32} * C_{C3} \quad (6)$$

$$V_3(dC_{A3}/dt) = F_{13} * C_{A1} + F_{23} * C_{A2} - (F_{30} + F_{31} + F_{32})C_{A3} \quad (7)$$

$$V_3(dC_{B3}/dt) = F_{13} * C_{B1} + F_{23} * C_{B2} - (F_{30} + F_{31} + F_{32})C_{B3} \quad (8)$$

$$V_3(dC_{C3}/dt) = F_{13} * C_{C1} + F_{23} * C_{C2} - (F_{30} + F_{31} + F_{32})C_{C3} + Q_{C3} \quad (9)$$

The mass flow balance equations for the three zones are:

$$-(F_{10} + F_{12} + F_{13}) + F_{01} + F_{21} + F_{31} = 0 \quad (10)$$

$$-(F_{20} + F_{21} + F_{23}) + F_{02} + F_{12} + F_{32} = 0 \quad (11)$$

$$-(F_{30} + F_{31} + F_{32}) + F_{03} + F_{23} + F_{13} = 0 \quad (12)$$

where

V = room volume

C = tracer gas concentration

t = time

F_{ij} = airflow rate from zone i to zone j

Q = tracer gas release rate

The subscripts 0, 1, 2, and 3 denote the outside and zones 1, 2, and 3, respectively, and A, B, and C refer to the tracer gases used. As an example, F_{10} represents the airflow from Zone 1 to the outside, and C_{A1} is the concentration of tracer gas A in Zone 1.

If three tracer gases, A, B, and C, are injected into Zones 1, 2, and 3, respectively, and their concentrations in each room are monitored over time, the 12 unknowns — F_{10}, F_{12}, F_{13}, F_{01}, F_{20}, F_{21}, F_{23}, F_{02}, F_{30}, F_{31}, F_{32}, and F_{03} — can be evaluated by solving Equations 1 through 12 simultaneously. Similarly, if all the airflow rates are known, they can be used in the solutions to the above set of simultaneous differential equations to calculate the tracer gas concentration profiles.

TEST METHODS

The test zones, as shown in Figure 1, were three interconnected rooms in a laboratory-office building. Rooms 1 and 2 were each 4.8 m wide, 4.8 m long, and 2.87 m high, and Room 3 was 2.33 m wide, 11 m long, and 2.57 m high. The walls, doors, and ceilings of the rooms were sealed to minimize air leakage. Each connecting doorway was sealed with plywood panels through which two airflow systems were installed (one for each flow direction). The airflow systems consisted of a fan, an airflow measuring device, and an airflow controller. Each individual airflow was controlled at a constant rate and continuously measured throughout each test. Except for those through the airflow systems,

TABLE 1
Test Conditions

Test No.	Injection Mode(*) Rm.1	Rm.2	Rm.3	Interzonal Airflows, ach F_{12}	F_{13}	F_{21}	F_{23}	F_{31}	F_{32}
101	D	D	D	0.79	0.79	0.80	0.79	0.80	0.78
102	D	I(87)	I(93)	0.80	0.80	0.80	0.80	0.77	0.80
103	D	C	I(94)	0.79	0.80	0.80	0.80	0.78	0.80
104	D	D	D	0.50	0.48	0.51	0.51	0.49	0.49
105	I(55)	D	D	0.50	0.48	0.51	0.47	0.49	0.50
106	I(78)	D	D	1.00	0.23	0.25	0.98	0.98	0.24
107	I(63)	D	D	0.30	0.90	0.61	0.51	0.61	0.79
108	D	D	D	0.30	0.89	0.60	0.51	0.60	0.79
109	D	D	D	1.00	0.23	0.25	0.99	0.98	0.26
110	I(26)	D	D	0.25	0.24	0.25	0.23	0.25	0.24
111	D	D	D	0.25	0.24	0.25	0.24	0.25	0.24
112	I(94)	D	D	0.79	0.79	0.80	0.79	0.80	0.79
113	D	D	D	0.78	0.79	0.79	0.78	0.77	0.80
114	I(71)	D	D	0.78	0.60	0.40	0.70	0.99	0.31

*C, D, and I denote constant concentration, decay, and constant injection, respectively (values in brackets are the injection rates of pure gas, mL/min).

the air leakage rates between the test rooms and their surroundings were not measured.

The tracer gas injection tube was located at the center of each room. The rooms were each divided into eight volumetrically equal regions with a sampling tube installed at the center of each region. Mixing of tracer gas in the test rooms has been studied previously (Enai et al. 1990). Based on the results, the sampling tubes in each room were connected to a manifold to produce an "average" sample.

Each test began by adjusting the airflows in the six systems to rates between 0.2 to 1 air change per hour (ach). Then, CH_4, SF_6, and N_2O were simultaneously introduced into Rooms 1, 2, and 3, respectively. Immediately after injection, the concentrations of the three gases at the manifolds in each room were measured at four-minute intervals for a period of two to five hours.

TEST RESULTS

Fourteen multiple tracer gas tests were conducted. The test conditions are listed in Table 1. Figure 2 shows a typical set of concentration profiles measured in the test rooms. Each set consists of nine profiles, one for each tracer gas in each room. From such profiles, the concentrations of CH_4, SF_6, and N_2O at about 30 minutes after injection of the tracer gases were used to calculate F_{12}, F_{21}, F_{13}, F_{31}, F_{23}, and F_{32} from Equations 1 through 12. Similar calculations were made using subsequent sets of concentration values measured at four-minute intervals for about one-and-a-half hours. These calculations were carried out for all tests.

To examine the effect of injection techniques on the calculated airflow rates, four tests using different combinations of tracer gas injection techniques and uniform interzonal airflow settings were conducted. The airflows of these tests were all set at about 0.8 ach. The combinations of injection techniques were (a) decay in all three rooms; (b) decay in Room 1 and constant injection in Rooms 2 and 3; (c) decay in Room 1, constant concentration in Room 2, and constant injection in Room 3; and (d) constant injection in Room 1 and decay in Rooms 2 and 3. Figures 3, 4, 5, and 6 show the calculated interzonal airflow rates as functions of time for these four tests. Tne results indicate that:

1. For the same test, the calculated airflow rates based on different sets of concentrations (measured at different times) were not always the same.
2. The calculated airflow rates, based on the concentrations measured between 30 and 70 minutes after injection, agreed with the measurement within 25% of the measured rates.
3. No clear evidence was found to suggest that the technique used to inject tracer gases (e.g., decay, constant injection, or constant concentration) has a significant or systematic effect on the calculated results. However, in some cases where the decay technique was used to inject tracer gases into more than one room, the agreement between the calculated and measured airflow rates worsened after 70 minutes.

These conclusions are not restricted to cases with uniform interzonal airflow settings. Similar behavior in the calculated results was observed for tests with nonuniform interzonal airflows.These findings suggest that concentrations selected from only a certain portion of the measured profiles can give a good estimate of interzonal airflows, as had been observed for the case of two interconnected rooms (Enai et al. 1990). To determine the appropriate set of concentrations for calculating interzonal airflows, the following method is proposed.

METHOD FOR CALCULATING INTERZONAL AIRFLOWS

Figures 3 through 6 show that the calculated interzonal airflow rates based on different sets of concentrations are not always the same, and a method of determining the appropriate set for use in calculations is needed.

Equations 1 through 9 define the tracer gas concentration profiles for a typical case of three interconnected zones. If all the airflows for such a case are known, the tracer gas concentration profiles can be calculated explicitly from these equations. The approach proposed here, therefore, is to estimate the airflow rates using a set of concentrations measured at some arbitrary time. These airflows are then used to estimate the

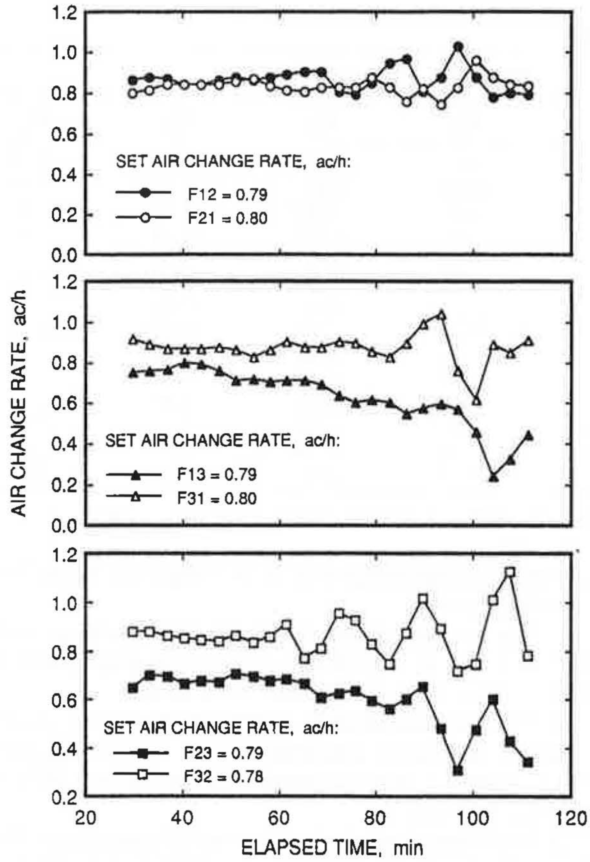

Figure 3 *Calculated interzonal airflow rates case (a) Room 1: decay, Room 2: decay, and Room 3: decay*

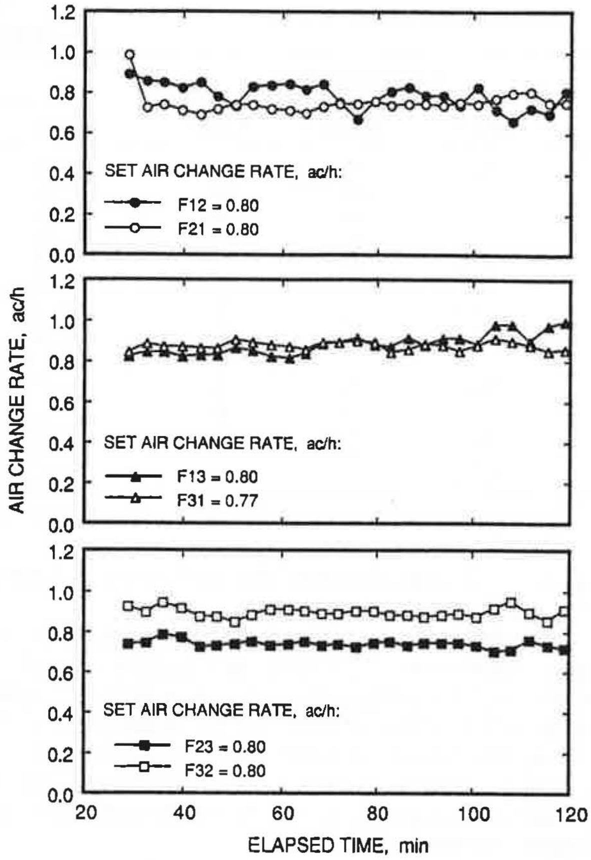

Figure 4 *Calculated interzonal airflow rates case (b) Room 1: decay, Room 2: constant injection, and Room 3: constant injection*

concentrations for a later time, using equations derived from Equations 1 through 9 (see Tracer Gas Concentration Profiles section). These estimated concentrations are then compared with the corresponding measured concentrations and, if the agreement is not satisfactory, the procedure is repeated with a set of concentrations measured at a later time. This comparison is made for concentrations at two different times, five sampling intervals apart, to ensure that the agreement between the calculated and measured concentration profiles is not accidental (e.g., the two profiles cross each other at one time but do not agree in general). The final calculated airflow rates are achieved when satisfactory agreement between the measured and calculated concentrations is reached at the two points (see Procedures for Calculating Interzonal Airflows section).

Tracer Gas Concentration Profiles

The concentration profiles of each tracer gas are defined by three equations, one for each room. Of the three equations, only one has a source term (e.g., Q_{A1} in Equation 1). Only one general solution to the three-equation set need be found for a single tracer gas, since the same solution will apply to the other two tracer gases. By dropping the subscript A and letting

$$N_1 = (F_{10} + F_{12} + F_{13})/V_1$$

$$N_2 = (F_{20} + F_{21} + F_{23})/V_2$$

$$N_3 = (F_{30} + F_{31} + F_{32})/V_3$$

Equations 1, 4, and 7 become the set of three general equations:

$$dC_1/dt = -N_1 * C_1 + (F_{21}/V_1)C_2 + (F_{31}/V_1)C_3 + Q_1/V_1 \quad (13)$$

$$dC_2/dt = (F_{12}/V_2)\, C_1 - N_2 * C_2 + (F_{32}/V_2)C_3 \quad (14)$$

$$dC_3/dt = (F_{13}/V_3)\, C_1 + (F_{23}/V_3)\, C_2 - N_3 * C_3 \quad (15)$$

From Equations 13, 14, and 15, it can be shown that the differential equation for the gas concentration profile C_1 is

$$\begin{aligned}(d^3C_1/dt^3) &+ (N_1 + N_2 + N_3) * (d^2C_1/dt^2) \\ &+ [N_1 * N_2 + N_2 * N_3 + N_3 * N_1 \\ &- (F_{13}/V_1) * (F_{31}/V_3) - (F_{32}/V_3) * (F_{23}/V_2) \\ &- (F_{21}/V_2) * (F_{12}/V_1)] * (dC_1/dt) \\ &+ [N_1 * N_2 * N_3 - N_1 * (F_{23}/V_2) * (F_{32}/V_3) \\ &- N_2 * (F_{31}/V_3) * (F_{13}/V_1) - N_3 * (F_{12}/V_1) \\ &* (F_{21}/V_2) - (F_{12}/V_1) * (F_{23}/V_2) * (F_{31}/V_3) \\ &- (F_{13}/V_1) * (F_{32}/V_3) * (F_{21}/V_2)] * C_1 + [(F_{23}/V_2) \\ &* (F_{32}/V_3) - N_2 * N_3] * (Q_1/V_1) = 0 \quad (16)\end{aligned}$$

(which is Equation A11 in Appendix A).

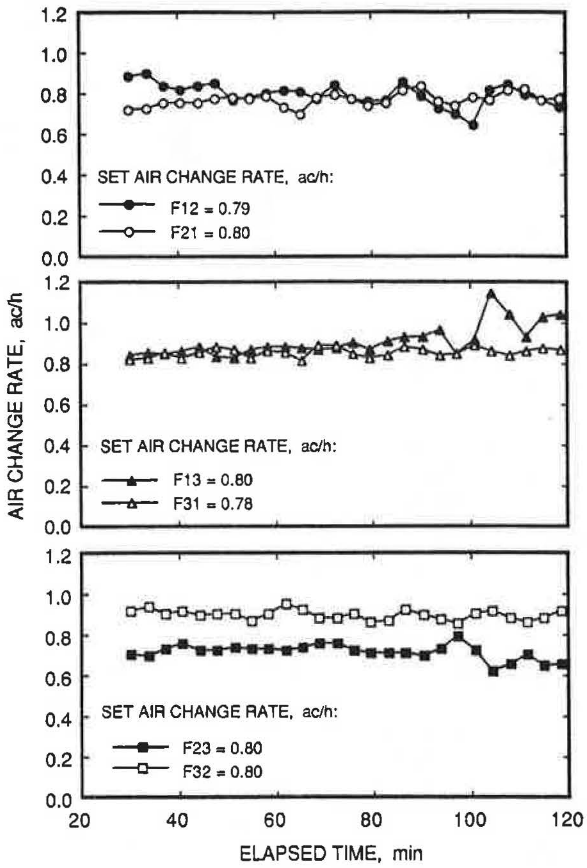

Figure 5 *Calculated interzonal airflow rates case (c) Room 1: decay, Room 2: constant concentration, and Room 3: constant injection*

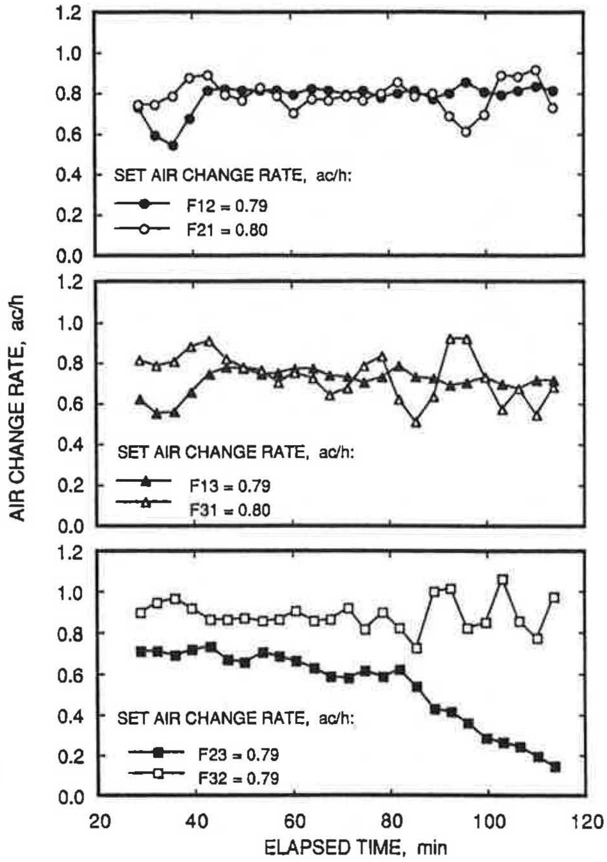

Figure 6 *Calculated interzonal airflow rates case (d) Room 1: constant injection, Room 2: decay, and Room 3: decay*

Since all the coefficients but one (for Q_1/V_1) in Equation 16 are common to the corresponding equations for C_2 and C_3, the general differential equation for the three tracer gas concentration profiles, C_i, where i = 1, 2, and 3, is

$$(d^3C_i/dt^3) + K_a * (d^2C_i/dt^2) + K_b * (dC_i/dt) + K_c * C_i + K_{di} = 0 \quad (17)$$

where

$$K_a = (N_1 + N_2 + N_3)$$

$$K_b = [N_1 * N_2 + N_2 * N_3 + N_3 * N_1 - (F_{12}/V_1) * (F_{21}/V_2) - (F_{23}/V_2) * (F_{32}/V_3) - (F_{31}/V_3) * (F_{13}/V_1)$$

$$K_c = [N_1 * N_2 * N_3 - N_1 * (F_{23}/V_2) * (F_{32}/V_3) - N_2 * (F_{31}/V_3) * (F_{13}/V_1) - N_3 * (F_{12}/V_1) * (F_{21}/V_2) - (F_{12}/V_1) * (F_{23}/V_2) * (F_{31}/V_3) - (F_{13}/V_1) * (F_{32}/V_3) * (F_{21}/V_2)]$$

$$K_{d1} = [(F_{23}/V_2) * (F_{32}/V_3) - N_2 * N_3] * (Q_1/V_1)$$

$$K_{d2} = -[N_3 * (F_{12}/V_1) + (F_{13}/V_1) * (F_{32}/V_3)] * (Q_1/V_2)$$

$$K_{d3} = -[N_2 * (F_{13}/V_1) + (F_{12}/V_1) * (F_{23}/V_2)] * (Q_1/V_3)$$

The solution to C_i can be obtained by the Laplace transformation method (see Appendix A). Thus,

$$C_i(t) = X_i * \exp(-at) + Y_i * \exp(-bt) + Z_i * \exp(-ct) + W_i \quad (18)$$

In the above equation, $C_i(t)$ is the tracer gas concentration in Room i at time t where i = 1, 2, and 3. The coefficients are

$$X_1 = -\{[(N_2 - a) * (N_3 - a) - (F_{32}/V_3) * (F_{23}/V_2)] * C_1(0) + [(N_3 - a) * (F_{21}/V_1) + (F_{23}/V_1) * (F_{31}/V_3)] * C_2(0) + [(N_2 - a) * (F_{31}/V_1) + (F_{21}/V_1) * (F_{32}/V_2)] * C_3(0) + (N_2 + N_3 - a) * (Q_1/V_1) + K_{d1}/a\}/[(c - a) * (a - b)]$$

$$Y_1 = -\{[(N_2 - b) * (N_3 - b) - (F_{32}/V_3) * (F_{23}/V_2)] * C_1(0) + [(N_3 - b) * (F_{21}/V_1) + (F_{23}/V_1) * (F_{31}/V_3)] * C_2(0) + [(N_2 - b) * (F_{31}/V_1) + (F_{21}/V_1) * (F_{32}/V_2)] * C_3(0) + (N_2 + N_3 - b) * (Q_1/V_1) + K_{d1}/b\}/[(a - b) * (b - c)]$$

$$Z_1 = -\{[(N_2 - c) * (N_3 - c) - (F_{32}/V_3) * (F_{23}/V_2)] * C_1(0) + [(N_3 - c) * (F_{21}/V_1) + (F_{23}/V_1) * (F_{31}/V_3)] * C_2(0) + [(N_2 - c) * (F_{31}/V_1) + (F_{21}/V_1) * (F_{32}/V_2)] * C_3(0) + (N_2 + N_3 - c) * (Q_1/V_1) + K_{d1}/c\}/[(b - c) * (c - a)]$$

$$W_1 = [(F_{23}/V_2) * (F_{32}/V_3) - N_2 * N_3] * (Q_1/V_1)/(-a * b * c)$$

$$X_2 = -\{[(N_3 - a) * (F_{12}/V_2) + (F_{13}/V_2) * (F_{32}/V_3)] * C_1(0) + [(N_1 - a) * (N_3 - a) - (F_{31}/V_3) * (F_{13}/V_1)] * C_2(0) + [(N_1 - a) * (F_{32}/V_2) + (F_{12}/V_1) * (F_{31}/V_2)] * C_3(0) + (F_{12}/V_1) * (Q_1/V_2) + K_{d2}/a\}/[(c - a) * (a - b)]$$

$$Y_2 = -\{[(N_3 - b) * (F_{12}/V_2) + (F_{13}/V_2) * (F_{32}/V_3)] * C_1(0) + [(N_1 - b) * (N_3 - b) - (F_{31}/V_3) * (F_{13}/V_1)] * C_2(0) + [(N_1 - b) * (F_{32}/V_2) + (F_{12}/V_1) * (F_{31}/V_2)] * C_3(0) + (F_{12}/V_1) * (Q_1/V_2) + K_{d2}/b\}/[(a - b) * (b - c)]$$

$$Z_2 = -\{[(N_3 - c) * (F_{12}/V_2) + (F_{13}/V_2) * (F_{32}/V_3)] * C_1(0) + [(N_1 - c) * (N_3 - c) - (F_{31}/V_3) * (F_{13}/V_1)] * C_2(0) + [(N_1 - c) * (F_{32}/V_2) + (F_{12}/V_1) * (F_{31}/V_2)] * C_3(0) + (F_{12}/V_1) * (Q_1/V_1) + K_{d2}/c\}/[(b - c) * (c - a)]$$

$$W_2 = -[N_3 * (F_{12}/V_1) + (F_{13}/V_1) * (F_{32}/V_3)] * (Q_1/V_2)/(-a * b * c)$$

$$X_3 = -\{[(N_2 - a) * (F_{13}/V_3) + (F_{12}/V_3) * (F_{23}/V_2)] * C_1(0) + [(N_1 - a) * (F_{23}/V_3) + (F_{13}/V_1) * (F_{21}/V_3)] * C_2(0) + [(N_1 - a) * (N_2 - a) - (F_{21}/V_2) * (F_{12}/V_1)] * C_3(0) + (F_{13}/V_1) * (Q_1/V_3) + K_{d3}/a\}/[(c - a) * (a - b)]$$

$$Y_3 = -\{[(N_2 - b) * (F_{13}/V_3) + (F_{12}/V_3) * (F_{23}/V_2)] * C_1(0) + [(N_1 - b) * (F_{23}/V_3) + (F_{13}/V_1) * (F_{21}/V_3)] * C_2(0) + [(N_1 - b) * (N_2 - b) - (F_{21}/V_2) * (F_{12}/V_1)] * C_3(0) + (F_{13}/V_1) * (Q_1/V_3) + K_{d3}/b\}/[(a - b) * (b - c)]$$

$$Z_3 = -\{[(N_2 - c) * (F_{13}/V_3) + (F_{12}/V_3) * (F_{23}/V_2)] * C_1(0) + [(N_1 - c) * (F_{23}/V_3) + (F_{13}/V_1) * (F_{21}/V_3)] * C_2(0) + [(N_1 - c) * (N_2 - c) - (F_{21}/V_2) * (F_{12}/V_1)] * C_3(0) + (F_{13}/V_1) * (Q_1/V_3) + K_{d3}/c\}/[(b - c) * (c - a)]$$

$$W_3 = -[N_2 * (F_{13}/V_1) + (F_{12}/V_1) * (F_{23}/V_2)] * (Q_1/V_3)/(-a * b * c)$$

Similar equations for concentration profiles under decay condition only have been derived by Irwin and Edwards (1987).

Procedures for Calculating Interzonal Airflows

The following procedures are proposed for calculating interzonal airflows:

(a) Let $t = t_s$. Since it normally takes more than 30 minutes to achieve adequate mixing, t_s should not be less than 30 minutes.

(b) Calculate the corresponding interzonal airflows from Equations 1 through 12 using the nine concentrations (one for each gas in each room).

(c) Calculate the concentrations C_i (where i = 1, 2, and 3) at T_1 where $T_1 = t + Dt$ (Dt is one sampling interval; four minutes was used in this study) from Equation 18 using the interzonal airflows obtained in (b).

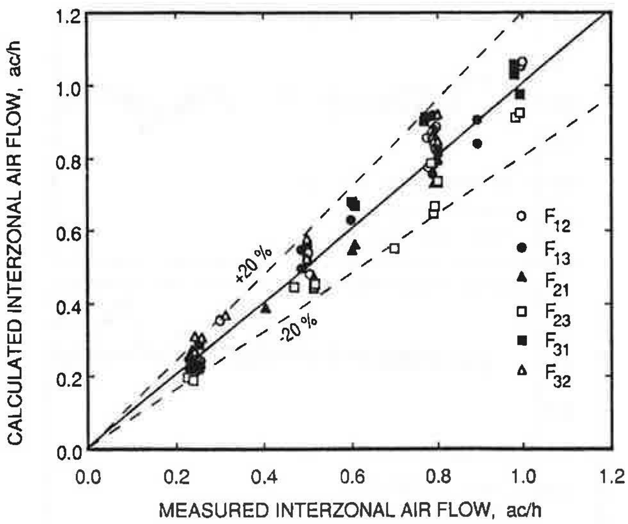

Figure 7 *Comparison of measured and calculated airflow rates in test rooms*

(d) Compare the calculated C_i values with the values for C_i measured at T_1.

(e) If the calculated and measured values do not agree within a preset criterion (e.g., 2% was used in this study; a more lenient criterion may be needed for field tests), let $t = t + Dt$ and repeat the procedures starting at (b).

(f) Otherwise, calculate C_i at T_2 where $T_2 = t + 5 * Dt$ and compare these with the corresponding measured values.

(g) If the calculated and measured values do not agree within a preset criterion (e.g., 2% was used in this study; a more lenient criterion may be needed for field tests), let $t = t + Dt$ and repeat the procedures starting at (b).

(h) Otherwise, the airflows used are the calculated airflows. A computer program has been developed for calculating interzonal airflows for three-room systems, which can accommodate any combination of tracer gas injection techniques used in this study.

COMPARISON BETWEEN CALCULATED AND MEASURED AIRFLOWS

The calculated and measured values for F_{12}, F_{21}, F_{13}, F_{31}, F_{23}, and F_{32} are given in Table 2 and Figure 7. Also included in Table 2 are the values of t at which the concentrations were selected for the calculated interzonal airflows. As shown in Table 2, the values of t were between 32.5 to 70.8 minutes, depending on the test conditions.

The standard errors of estimate were calculated for two different tracer gas injection techniques: (a) decay in all three rooms and (b) constant injection in one room and decay in the remaining rooms (other combinations of injection techniques were not considered due to limited data). They are 0.065 and 0.054 ach for cases (a) and (b), respectively, suggesting that neither injection technique appears to have a significant advantage over the other in terms of the accuracy of interzonal airflow calculations. Figure 7 and Table 2 show that the calculated and measured airflow rates agreed within 20% (of the measured value).

TABLE 2
Calculated and Measured Airflow Rates

Test No.	Result	Interzonal Airflows, ach						Run Time (min)
		F_{12}	F_{13}	F_{21}	F_{23}	F_{31}	F_{32}	
101	Measured	0.79	0.79	0.80	0.79	0.80	0.78	
	Calculated	0.86	0.76	0.80	0.65	0.92	0.88	33.2
102	Measured	0.80	0.80	0.80	0.80	0.77	0.80	
	Calculated	0.89	0.82	0.74	0.74	0.90	0.92	32.5
103	Measured	0.79	0.80	0.80	0.80	0.78	0.80	
	Calculated	0.81	0.87	0.70	0.74	0.82	0.93	65.5
104	Measured	0.50	0.48	0.51	0.51	0.49	0.49	
	Calculated	0.54	0.55	0.48	0.46	0.56	0.53	40.4
105	Measured	0.50	0.48	0.51	0.47	0.49	0.50	
	Calculated	0.48	0.50	0.44	0.45	0.52	0.58	56.8
106	Measured	1.00	0.23	0.25	0.98	0.98	0.24	
	Calculated	1.06	0.25	0.23	0.92	1.03	0.27	42.9
107	Measured	0.30	0.90	0.61	0.51	0.61	0.79	
	Calculated	0.36	0.91	0.57	0.45	0.67	0.86	40.7
108	Measured	0.30	0.89	0.60	0.51	0.60	0.79	
	Calculated	0.36	0.84	0.55	0.46	0.68	0.85	37.3
109	Measured	1.00	0.23	0.25	0.99	0.98	0.26	
	Calculated	1.06	0.22	0.23	0.93	1.06	0.31	47.7
110	Measured	0.25	0.24	0.25	0.23	0.25	0.24	
	Calculated	0.24	0.22	0.23	0.20	0.30	0.28	70.8
111	Measured	0.25	0.24	0.25	0.24	0.25	0.24	
	Calculated	0.25	0.23	0.23	0.19	0.29	0.31	32.5
112	Measured	0.79	0.79	0.80	0.79	0.80	0.79	
	Calculated	0.83	0.78	0.80	0.67	0.82	0.87	50.1
113	Measured	0.78	0.79	0.79	0.78	0.77	0.80	
	Calculated	0.86	0.88	0.73	0.79	0.92	0.85	36.2
114	Measured	0.78	0.60	0.40	0.70	0.99	0.31	
	Calculated	0.78	0.63	0.39	0.55	0.98	0.37	32.8

SUMMARY

1. A method has been developed for calculating the interzonal airflows for a space consisting of three interconnected zones. It includes a procedure for checking the accuracy of the calculated airflows, based on the measured tracer gas concentrations. A computer program has been developed for performing this calculation.

2. A comparison between the calculated and set airflows for the test conditions studied, where all three zones are similar in volume and shape, suggests that only concentrations measured between 30 and 70 minutes after tracer gas injection should be used for calculating interzonal airflows. Using the proposed method, the predicted airflow rates agreed with the measured values within about 20%.

3. No clear evidence was found to suggest that either technique used to inject tracer gases (decay only or a combination of decay and constant injection) has a significant advantage over the other in terms of the accuracy of the calculated airflows. However, in some cases where the decay technique was used to inject tracer gases into more than one room, the agreement between the calculated and set airflow rates deteriorates after 70 minutes.

ACKNOWLEDGMENTS

This work was undertaken in Ottawa at the Institute for Research in Construction, National Research Council of Canada. Prof. Masamichi Enai, Dr. Eng., is a guest researcher from Hokkaido University, Japan. The authors wish to acknowledge the cooperative effort of both organizations and particularly Prof. N. Aratani of Hokkaido University in supporting this project.

REFERENCES

Enai, M.; Shaw, C.Y.; and Reardon, J.T. 1990. "On multiple tracer gas techniques for measuring interzonal airflows in buildings." *ASHRAE Transactions*, Vol. 96, Part 1.

I'Anson, S.J.; Irwin, C.; and Howarth, A.T. 1982. "Airflow measurement using three tracer gases." *Building and Environment*, Vol. 17, No. 4, pp. 245-252.

Irwin, C., and Edwards, R.E. 1987. "Airflow measurement between three connected cells." *Building Services Engineering Research and Technology*, Vol. 8, pp. 91-96.

Kojima, N., and Machida, T. 1982. *Numerical analysis I—Basic for personal computer.* Tokai University Press.

Perera, M.D.A.E.S. 1983. "Review of techniques for measuring ventilation rates in multi-celled buildings." *Energy Conservation in Buildings — Heating, Ventilation and Insulation*, pp. 321-334. D. Reidel Publishing Company.

Sherman, M.; Grimsrud, D.T.; Condon, P.E.; and Smith, B.V. 1980. "Air infiltration measurement techniques." 1st Air Infiltration Centre Conference on Air Infiltration Instrumentation and Measuring Techniques, Windsor, England.

Sinden, F.W. 1978. "Multi-chamber theory of air infiltration." *Building and Environment*, Vol. 13, pp. 21–28.

APPENDIX A

TRACER GAS CONCENTRATION PROFILES

Derivation of Equations

The three basic differential equations are

$$dC_1/dt = -N_1 * C_1 + (F_{21}/V_1)C_2 + (F_{31}/V_1)C_3 + Q_1/V_1 \quad \text{(A1)}$$

$$dC_2/dt = (F_{12}/V_2)C_1 - N_2 * C_2 + (F_{32}/V_2)C_3 \quad \text{(A2)}$$

$$dC_3/dt = (F_{13}/V_3)C_1 + (F_{23}/V_3)C_2 - N_3 * C_3 \quad \text{(A3)}$$

First, Equation A1 can be rewritten as,

$$C_3 = (V_1/F_{31}) * (dC_1/dt) + C_1 * N_1 * V_1/F_{31} - C_2 * F_{21}/F_{31} - Q_1/F_{31} \quad \text{(A4)}$$

Substituting Equation A4 into Equation A2, we have

$$dC_2/dt = -[N_2 + (F_{21}/V_2) * (F_{32}/F_{31})] * C_2 + (V_1/V_2) * (F_{32}/F_{31}) * (dC_1/dt) + [(F_{12}/V_2) + N_1 * (V_1/V_2) * (F_{32}/F_{31})] * C_1 - (Q_1/V_2) * (F_{32}/F_{31})$$

or

$$dC_2/dt + [N_2 + (F_{21}/V_2) * (F_{32}/F_{31})] * C_2 = (V_1/V_2) * (F_{32}/F_{31}) * (dC_1/dt) + [(F_{12}/V_2) + N_1 * (V_1/V_2) * (F_{32}/F_{31})] * C_1 - (Q_1/V_2) * (F_{32}/F_{31}) \quad \text{(A5)}$$

Next, differentiating Equation A4 gives,

$$dC_3/dt = (V_1/F_{31}) * (d^2C_1/dt^2) + N_1 * (V_1/F_{31}) * (dC_1/dt) - (F_{21}/F_{31}) * (dC_2/dt) \quad \text{(A6)}$$

Substituting Equation A4 into Equation A3, we have,

$$dC_3/dt = (F_{13}/V_3) * C_1 + (F_{23}/V_3) * C_2 - N_3 * [(V_1/F_{31}) * (dC_1/dt) + C_1 * N_1 * V_1/F_{31} - C_2 * F_{21}/F_{31} - Q_1/F_{31}] \quad \text{(A7)}$$

Equating Equations A6 and A7, we have,

$$(V_1/F_{21}) * (d^2C_1/dt^2) + (N_1 + N_3) * (V_1/F_{21}) * (dC_1/dt) + [N_1 * N_3 * (V_1/F_{21}) - (F_{13}/F_{21}) * (F_{31}/V_3)] * C_1 - N_3 * Q_1/F_{21} = (dC_2/dt) + [N_3 + (F_{23}/F_{21}) * (F_{31}/V_3)] * C_2 \quad \text{(A8)}$$

Substituting Equation A5 into Equation A8 and letting

$$T = (N_3 - N_2) * (F_{21}/V_1) + (F_{23}/V_1) * (F_{31}/V_3) - (F_{32}/F_{31}) * (F_{21}/V_1) * (F_{21}/V_2)$$

Equation A8 becomes

$$C_2 = (d^2C_1/dt^2)/T + [N_1 + N_3 - (F_{21}/V_2) * (F_{32}/F_{31})] * (dC_1/dt)/T + [N_1 * N_3 - (F_{13}/V_1) * (F_{31}/V_3) - (F_{12}/V_1) * (F_{21}/V_2) - N_1 * (F_{21}/V_2) * (F_{32}/F_{31})] * C_1/T + [(F_{21}/V_2) * (F_{32}/F_{31}) - N_3] * (Q_1/V_1)/T \quad \text{(A9)}$$

Differentiating gives

$$(dC_2/dt) = (d^3C_1/dt^3)/T + [N_1 + N_3 - (F_{21}/V_2) * (F_{32}/F_{31})] * (d^2C_1/dt^2)/T + [N_1 * N_3 - (F_{13}/V_1) * (F_{31}/V_3) - (F_{12}/V_1) * (F_{21}/V_2) - N_1 * (F_{21}/V_2) * (F_{32}/F_{31})] * (dC_1/dt)/T \quad \text{(A10)}$$

Finally, combining Equations A5, A9, and A10 gives

$$(d^3C_1/dt^3) + (N_1 + N_2 + N_3) * (d^2C_1/dt^2) + [N_1 * N_2 + N_2 * N_3 + N_3 * N_1 - (F_{13}/V_1) * (F_{31}/V_3) - (F_{32}/V_3) * (F_{23}/V_2) - (F_{21}/V_2) * (F_{12}/V_1)] * (dC_1/dt) + [N_1 * N_2 * N_3 - N_1 * (F_{23}/V_2) * (F_{32}/V_3) - N_2 * (F_{31}/V_3) * (F_{13}/V_1) - N_3 * (F_{12}/V_1) * (F_{21}/V_2) - (F_{12}/V_1) * (F_{23}/V_2) * (F_{31}/V_3) - (F_{13}/V_1) * (F_{32}/V_3) * (F_{21}/V_2)] * C_1 + [(F_{23}/V_2) * (F_{32}/V_3) - N_1 * N_3] * (Q_1/V_1) = 0 \quad \text{(A11)}$$

Tracer Gas Concentration Profiles

Equation A11 can be rewritten in the general form (see Equation17)

$$(d^3C_i/dt^3) + K_a * (d^2C_i/dt^2) + K_b * (dC_i/dt) + K_c * C_i + K_{di} = 0 \quad \text{(A12)}$$

Taking Laplace transforms, we find

$$L\{C_i(t)\} = \{s^3 * C_i(0) + s^2 * [C_i'(0) + K_a * C_i(0)] + s * [C_i''(0) + K_a * C_i'(0) + K_b * C_i(0)] - K_{di}\}/[s * (s^3 + K_a * s^2 + K_b * s + K_c)\} \quad \text{(A13)}$$

The coefficients K_a, K_b, K_c, and K_{di} are defined in Equation 17. For $i = 1$, Equation A13 becomes

$$L\{C_1(t)\} = \{s^3 * C_1(0) + s^2 * [C_1'(0) + K_a * C_1(0)] + s * [C_1''(0) + K_a * C_1'(0) + K_b * C_1(0)] - K_{d1}\}/[s * (s^3 + K_a * s^2 + K_b * s + K_c)\} \quad \text{(A14)}$$

The initial conditions are as follows:

$$C_1'(0) = -N_1 * C_1(0) + (F_{21}/V_1) * C_2(0) + (F_{31}/V_1) * C_3(0) + Q_1/V_1$$

$$C_1''(0) = -(N_1 + N_3) * C_1'(0) - [N_1 * N_3 - (F_{13}/V_1) * (F_{31}/V_3)] * C_1(0) + N_3 * (Q_1/V_1) + (F_{21}/V_1) * C_2'(0) + [(F_{23}/V_1) * (F_{31}/V_3) + N_3 * (F_{21}/V_1)] * C_2(0)$$

$$C_2'(0) = (F_{12}/V_2) * C_1(0) - N_2 * C_2(0) + (F_{32}/V_2) * C_3(0)$$

Obtaining the roots (a', b', and c') of the equation $s^3 + K_a * s^2 + K_b * s + K_c$ using a numerical method (e.g., DKA method, Kojima and Machida 1982) and applying the initial conditions, Equation A14 becomes

$$L\{C_1(t)\} = W_1/s + X_1/(s - a') + Y_1/(s - b') + Z_1/(s - c') \quad \text{(A15)}$$

and

$$W_1 = K_{d1}/(a' * b' * c')$$

Letting $a' = -a$, $b' = -b$, and $c' = -c$, the solution of Equation A15 can be expressed by the typical form

$$C_1(t) = X_1 * \exp(-at) + Y_1 * \exp(-bt) + Z_1 * \exp(-ct) + W_1 \quad \text{(A16)}$$

Similarly, the general solution of A13 is

$$C_i(t) = X_i * \exp(-at) + Y_i * \exp(-bt) + Z_i * \exp(-ct) + W_i \quad \text{(A17)}$$